The Korean Economic Review
Volume 41, Number 1, Winter 2025, 111-140.
DOI : 10.22841/kerdoi.2025.41.1.004

Banking, Costly Credit, and Interest Rates with Limited Commitment*

Hyung Sun Choi**

A model for banking is constructed to explore the role of a payment system with limited commitment as another determinant of a spread between the loan and deposit rates. Limited commitment constrains credit settlements. In equilibrium, collateral is required in the payment system and affects the loan rate, the distribution of money, consumption, and output. The optimal policy mix minimizes the interest rate spread and increases output. The Friedman rule is generally not optimal.

JEL Classification: E42, E44, E50
Keywords: Banking, Costly Credit, Payment System, Interest Rates, Limited Commitment

I. Introduction

This study aims to explore the effects of a payment system with limited commitment on the loan rate, the distribution of money, and social welfare. The payment system needs liquidity to proceed with credit settlements established through goods trades among economic agents. The demand for settlement liquidity varies depending on the types of the settlements—gross and net settlements—and the timing of the settlements—real time and deferred settlements. This variation in the amount of the settlement liquidity could affect the loan rate, bank reserves, and the distribution of money.

For example, the net settlement—offsetting settlement positions with other banks is known for demanding less liquidity in the payment system than the gross

Received: Dec. 29, 2023. Revised: June 1, 2024. Accepted: Sept. 6, 2024.

* I am very grateful to Steve Williamson, Gabriele Camera, Kee-Youn Kang, Changjoon Kim, two anonymous referees, and the Coeditor Yoonsoo Lee for their helpful comments and constructive suggestions. This paper was previously circulated under the title, "Banking, Costly Credit, and Interest Rates."

** Professor, Department of Economics, Kyung Hee University, Email: hyungsunchoi@khu.ac.kr

settlement—transferring money individually to counterparty banks. Additionally, a deferred settlement in which a transaction is settled later at the end of a predefined date promotes intertemporal trades and economic activities in a highly sophisticated economy in contrast to a real-time settlement in which a transaction is settled as it takes place. The deferred settlement is subject to limited commitment, which requires some eligible assets to serve as collateral to secure the credit provided. Assets eligible as collateral are usually liquid and risk free, such as money and government bonds.[1] However, the collateral would come at a price by decreasing bank investment. The loan rate would rise, and economic activity would decline.

Our baseline model explores the effects of the payment system with limited commitment on banking and economic activities through the variation of settlement liquidity. A household involves multiple means of payment—cash and one-period credit. In a financial market, a bank issues one-period credit to the household. The household's credit balances are subject to limited commitment, and money serves as collateral. The bank settles credit balances with money via the following two settlement processes in an interbank payment system: the gross settlement system, a proxy for a wholesale payment system; and the net settlement system, a proxy for a traditional retail payment system. Bank liabilities are subject to limited commitment. Credit settlements hold up money as collateral in the payment system. Unlike Boel and Camera (2020), the gross settlement demands more liquidity than the net settlement and also requires more collateral. In the goods market, the household exchanges money or credit for consumption goods with others. Money serves as a means of payment in a goods market and a means of settlement within the banking system. The government controls the collateral requirement rate as collateral policy and the money growth rate as monetary policy.

In a symmetric equilibrium, the net settlement does not need collateral deposits given that no positive net-settlement balances are left to transfer once offsetting settlement positions with other banks. The gross settlement demands collateral deposits in the payment system. If the collateral constraint does not bind, then, the bank holds enough collateral deposits in the payment system. The collateral policy does not influence the loan and deposit rate similar to Boel and Camera (2020). The gross settlement can be efficient contrary to Boel and Camera (2020). By contrast, if the collateral constraint binds, then, collateral deposits become scarce in the payment system and distort the loan rate. The collateral policy may unwind the distortion. Owing to the distortion arising from the gross settlement, the net settlement is efficient. The higher demand for settlement liquidity and collateral distorts the loan rate and bank investment, hence resulting in negative effects on consumption and output. An increase in the spread may suppress consumption and

[1] According to the Bank for International Settlements, a payment system must take safe assets as collateral including money and nominal government bonds.

output in line with Bigio (2023). Next, monetary policy affects both interest rates whether the collateral constraint binds or not. Inflation reduces the purchasing power of cash and compels the household to substitute cash for credit, as spending on the latter is more beneficial. The demand for settlement liquidity increases, whereas the demand for money as a means of payment decreases. This substitution of cash for credit in goods trades pushes up the loan rate and reduces labor supply. Inflation may be somewhat welfare-improving as long as the positive effect of an increase in credit-good consumption dominates the negative effects. Finally, the policy mix may minimize the interest rate spread to enhance welfare.

The key novelty of this paper is to capture the role of collateral assets in banking and economic activities under limited commitment. Limited commitment constrains credit settlements. Collateral is required in the payment system as an insurance device for a smooth settlement process. The constrained credit settlement distorts the interest rate spread, the distribution of money, consumption, output, and social welfare. The related theoretical work on financial intermediation and liquidity premia includes Williamson (2016) and Boel and Camera (2020).[2] Williamson (2016) discusses the effects of collateral pledgeability on a term premium when aggregate collateralizable assets are scarce. The different degrees of pledgeability across assets and the scarcity of collateralizable assets reflected in inefficiency distort banking activity and lead to a term premium. Boel and Camera (2020) are particularly relevant in that they explore the effects of costly banking on the positive spread between the loan and deposit rates and social welfare. Competitive banks make out within-period cash loans to their customers by bearing labor resource costs. In equilibrium, the inefficiency in banking induces a positive interest rate spread to cover the labor resource costs. The bank may not improve welfare in a deflationary or sufficiently low inflationary economy. According to Williamson (2016) and Boel and Camera (2020), financial intermediaries settle credit payments solely on a net basis and do not require any settlement liquidity. However, these models do not focus on the real effects of limited commitment in the payment system on the loan rate, the distribution of money, and economic activities.

The following growing body of research focuses on the real effects of credit settlements: Callado-Muñoz (2007), Lester (2009), Kahn (2013), Tomura (2018), Choi (2023a), and Bigio (2023).[3] Particularly, Kahn (2013) emphasizes the role of collateral in competition between public and private credit arrangements and the

[2] Literature on other issues of banking includes Diamond and Dybvig (1983), Berentsen et al. (2007), Becivenga and Camera (2011), Chiu and Meh (2011), Williamson (2012), and Bianchi and Bigio (2022).

[3] A large literature also exists on the optimal design of the payment system, including Freeman (1996), Kahn and Roberds (1998; 2001), Williamson (2003), Koeppl et al. (2008), Kahn and Roberds (2009), Jurgilas and Martin (2010), Callado-Muñoz (2013), and Choi (2018; 2023b).

central bank's policy decisions. A government's monopoly power over the assets for payments and collateral induces real effects on economic activity. However, Kahn (2013) does not consider any explicit frictions on the rise of collateral deposits. Choi (2023a) examines the effect of multiple types of collaterals in credit settlements on consumption and social welfare in an endowment economy. A bank may hold money reserves or one-period government bonds as collateral. A greater demand for money reserves functioning as collateral reduces the amount of currency in circulation as a means of payment and consumption with cash. The composition of collateral may have real effects on consumption and social welfare. Banking activity is efficient and does not induce any positive interest rate spread. However, these models do not shed much light on the welfare implications of financial frictions in the payment system.

The remainder of the paper is structured as follows. The second section outlines the baseline model. The third section characterizes a set of equilibrium allocations. The fourth section examines government-policy implications. The last section concludes.

II. Model

The background environment of this model is based on Andolfatto and Williamson (2015) and Boel and Camera (2020) with elements of credit settlements taken from Choi (2023a).

2.1. Households

An existing unit mass of infinitely lived identical households is indexed by $i \in [0,1]$. Each household consists of a continuum of buyers with unit mass and a seller. The representative household has preferences given by

$$U(\{c_{m,t}, c_{c,t}, n_t\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t [\tau u(c_{m,t}) + (1-\tau) u(c_{c,t}) - v(n_t)], \tag{1}$$

where β is the discount factor, $c_{m,t}$ is the perishable consumption goods purchased by cash, $c_{c,t}$ is the perishable consumption goods purchased by credit, and n_t is the total supply of labor which adds up $n_{s,t}$ units of labor for financial-service provision and $n_{w,t}$ for consumption-good production, i.e., $n_t = n_{s,t} + n_{w,t}$. In (1), $u(\cdot)$ is a twice continuously differentiable and strictly concave utility function with $u(0) = 0$, $u'(0) = \infty$, $u'(\infty) = 0$. Additionally, $v(\cdot)$ is a twice continuously differentiable and strictly convex disutility function with $v(0) = v'(0) = 0$ and $v'(\infty) = \infty$.

In each period, every buyer in the household receives preference shock $\tau \in [0,1]$ in line with Andolfatto and Williamson (2015). With the probability of τ, a buyer wants to use cash to acquire consumption goods. However, the buyer wants to spend on credit with the probability of $1-\tau$. A seller is endowed with one unit of labor and decides how much to produce financial services in a bank and consumption goods in a goods market. Any member of the household cannot consume her own goods.

At the beginning of period t, the household carries M_t units of money in a transactions account, D_t units of money in a one-period savings account, H_t units of money in a one-period collateral account, and L_t amount of outstanding credit balances issued at period $t-1$. Here, we assume that $M_0 > 0$, $D_0 = 0$, $H_0 = 0$, and $L_0 = 0$ are given. From the transactions account, the household can withdraw money freely at any time without bearing any cost. The household cannot liquidate other accounts before they mature. Then, buyers receive the preference shocks.

In the financial market, the bank provides three types of financial services to its customer household: a money transfer from one account to another for goods trades in the subsequent goods market, an asset transformation of savings deposits into interest-bearing assets, and credit management including an issue of a new line of credit and credit settlements through an interbank payment system. First, the household transfers cash X_t between the transactions account and the matured savings account D_t. The remaining funds are deposited into the new interest-bearing one-period savings account D_{t+1}. Each deposit of D_{t+1} in period t will earn a net interest rate of $r_{D,t+1} \geq 0$ in period $t+1$.

Next, the household obtains a new line of bank credit L_{t+1} against D_{t+1}. A unit of credit of L_{t+1} is issued at a price of one unit of money in period t and is a claim to $1+r_{L,t+1}$ units of money in period $t+1$ given the net loan rate $r_{L,t+1} \geq 0$. Moreover, the household decides the fraction $\hat{\theta}_{t+1} \in [0,1]$ of L_{t+1} that will be settled on a gross basis through the wholesale or large-value payment system in period $t+1$ and the rest $1-\hat{\theta}_{t+1}$ on a net basis through the retail payment system.[4] Against L_{t+1}, the household must submit a collateral of H_{t+1} in the collateral account ahead of a period.[5] Considering that L_{t+1} is settled later in

[4] In practice, the household does not explicitly choose the settlement type which is determined by the value of credit payments. However, in this large-household framework, a unit mass of buyers in each household represents a variety of economic individuals with different values of credit payments. Hence, the gross settlement share $\hat{\theta}$ is a proxy of a fraction of buyers who make large-value payments, and the net-settlement share $1-\hat{\theta}$ is a proxy of another fraction of buyers who make small-value payments. For example, large-value and high-priority payments are generally settled on a gross basis in a wholesale or large-value payment system. Small-value payments are settled on a net basis in a retail payment system.

[5] The household separately holds D_{t+1} and H_{t+1} to emphasize the role of collateral deposits in credit settlements and the household's real activities. Particularly, the demand for H_{t+1} will depend

period $t+1$, this collateral deposit serves as an insurance tool to prevent potential default risk from arising. Each collateral earns a net interest rate of $r_{H,t+1} \geq 0$ in period $t+1$. No aggregate scarcity of the supply of collateralizable assets exists unlike Andolfatto and Williamson (2015) and Williamson (2016).

Third, the household settles the matured credit balances L_t issued from period $t-1$ through the bank. Concerning payment-system maintenances and upgrades, the household pays off some resource costs— a fixed fee γ_g for per-unit gross settlement and γ_n for per-unit net settlement. We assume $\gamma_n > 2\gamma_g$ to capture a cost reduction in the gross settlement due to advances in information and communication technology in line with Choi (2023a).[6]

The household's financial transactions are summarized by the cash-in-advance constraint and the credit constraint, respectively, as follows:

$$P_t \tau c_{m,t} = M_t + X_t, \tag{2}$$

$$P_t (1-\tau) c_{c,t} = L_{t+1}, \tag{3}$$

where P_t is the average price level of consumption goods, $c_{m,t}$ is consumption goods purchased by cash, and $c_{c,t}$ is consumption goods purchased by credit. The financial constraint is given by

$$X_t + D_{t+1} + (1+r_{L,t})L_t + H_{t+1} + P_t[\gamma_g + (1-\hat{\theta}_t)\Upsilon] = (1+r_{D,t})D_t + (1+r_{H,t})H_t, \tag{4}$$

where $(1-r_{L,t})L_t$ is the amount of credit loan paid off in period t, $(1+r_{D,t})D_t$ is the amount of savings deposits matured in period t, $(1+r_{H,t})H_t$ is the amount of collateral deposits matured in period t, and $\gamma_g + (1-\hat{\theta}_t)\Upsilon$ denotes the aggregate fees of payment-system maintenances and upgrades for $\Upsilon = \gamma_n - \gamma_g > 0$ and $\gamma_g + (1-\hat{\theta}_t)\Upsilon = \hat{\theta}_t\gamma_g + (1-\hat{\theta}_t)\gamma_n$. In (4), the household demands money for consumption-good trades (X_t) and credit settlements $\{(1+r_{L,t})L_t + H_{t+1} + P_t[\gamma_g + (1-\hat{\theta}_t)\Upsilon]\}$. Money serves not only as a means of payment for goods trades

---

on the choice of settlement types, whereas the demand for D_{t+1} will not.

[6] According to Bech et al. (2008), Allsopp et al. (2009), Bech et al. (2017), and Bank for International Settlements (2021), advances in information and communication technology have dramatically reduced the costs of operating the gross settlement system and have many countries to replace the net settlement system with the gross settlement system. For example, as of December 2021, the development of retail fast payment systems (FPS) is operational in 27 countries including Australia, Canada, China, the Euro Area, Hong Kong, Japan, South Korea, Singapore, and the United States according to Bech et al. (2017) and Bank for International Settlements (2021). However, as Allsopp et al. (2009) point out, the information on the costs of gross and net settlements is not generally publicly available. Hence, based on the discussions in Bech et al. (2008), Allsopp et al. (2009), Bech et al. (2017), and Bank for International Settlements (2021), we assume that the operating costs of the gross settlement are smaller than those of the net settlement to capture the recent adoption of the gross settlement system.

but also a means of settlements. Notably, the household receives the matured collateral deposits after the settlement is completed. Therefore, the consolidated liquidity-in-advance constraint is given by (2) and (4),

$$P_t \tau c_{m,t} + D_{t+1} + (1+r_{L,t})L_t + H_{t+1} + P_t[\gamma_g + (1-\hat{\theta}_t)\Upsilon]$$
$$= M_t + (1+r_{D,t})D_t + (1+r_{H,t})H_t . \quad (5)$$

In the goods market, a seller produces one unit of the consumption good for each unit of labor supplied. The production technology is identical across sellers and is independent from i. The goods market consists of two competitive submarkets similar to those in Williamson (2009) and Andolfatto and Williamson (2015). One is a cash market in which cash is solely accepted in trades as no memory and no record-keeping technology exist. The other is a credit market in which credit is accepted in trades as the bank manages perfect record-keeping technology and debt enforcement technology.[7] In a cash transaction, a constant fraction τ of the buyers purchase consumption goods with cash from sellers incurring no transactions cost. In a credit transaction, the rest $1-\tau$ of the buyers spend on credit to acquire consumption goods. After the credit transaction, banks transfer cash to sellers on behalf of their buyers. Therefore, τ plays as a proxy of a cash-credit choice made by a unit mass of buyers across the goods markets. Finally, each seller does not bear any transactions costs arising during the goods trades. Therefore, the identical production technology across sellers without bearing any costs in the goods market implies that the price of consumption good across the markets would equal P_t.

At the end of the period, all agents return home with the revenue of sales. The household deposits earned cash into the bank and receives a dividend income from the bank. No further exchange occurs, and no barter is allowed. The household's budget constraint is given by

$$P_t[\tau c_{m,t} + (1-\tau)c_{c,t}] + D_{t+1} + (1+r_{L,t})L_t + H_{t+1} + P_t[\gamma_g + (1-\hat{\theta}_t)\Upsilon] + M_{t+1}$$
$$= M_t + L_{t+1} + (1+r_{D,t})D_t + (1+r_{H,t})H_t + W_{w,t}n_{w,t} + W_{s,t}n_{s,t} + P_t\pi_t , \quad (6)$$

where M_{t+1} is the demand for money brought over in the next period, $W_{w,t}n_{w,t}$ + $W_{s,t}n_{s,t}$ is aggregate labor income, $W_{w,t}$ is the nominal wage rate for consumption-good production, $W_{s,t}$ is the nominal wage rate for financial-service production, and $P_t\pi_t$ is the dividend income from the bank.

---

[7] We could introduce the risk of theft of cash in the model to assure that credit is the sole medium of exchange in the credit market. However, it does not alter any main implications of the model, but it makes our equilibrium analysis less tractable.

2.2. Private Banks

A continuum of competitive private banks exists in the financial market. Financial intermediation is assumed to be costly in that financial service management incurs labor costs as in Boel and Camera (2020). The bank is assumed to be the sole entity that can manage a record-keeping technology of financial histories and trading histories in the credit-good market similar to Berentsen et al. (2007) and Boel and Camera (2020). Additionally, the bank is able to force credit repayment at no cost by using debt enforcement technology.

The bank facilitates financial services to customer households by managing four assets, namely, money, credit balances, collateral, and nominal government bonds. First, in period t, the bank offers a deposit contract (D_{t+1}) to its customer household and invests all received deposits into newly issued one-period nominal government bonds (B_{t+1}). A unit of nominal government bonds sells for one unit of money in period t and is a claim to $1+r_{t+1}$ units of money in period $t+1$. Given that the government bond is a book-entry bond, it cannot be circulated as a medium of exchange. In addition, it cannot be liquidated before its maturity. In other words, the household's savings deposit cannot be circulated as a medium of exchange either. No reserve requirements are required for savings deposits.

Next, the bank issues a new line of one-period credit (L_{t+1}) for the household against D_{t+1} and takes collateral H_{t+1} ahead of a period to prevent default risk from arising.[8] Then, the bank settles credit balances from period $t-1$ through the interbank payment system. Once the settlement is completed, the bank returns collateral H_t with an interest $r_{H,t}$ to the household. Notably, the bank works like a narrow bank in that the bank's investment in interest-bearing government bonds is risk-free, and the maturity of the investment matches the maturity of the savings deposit. We assume that the bank pays off the return on collateral as much as the return on government bonds, i.e., $r_{H,t}=r_t$, given that the collateral deposit also serves as a one-period asset.[9]

Finally, the bank requires labor resources for financial management including credit settlements. In period t, the bank can employ $n_{s,t}$ units of labor to manage $f(n_{s,t})$ units of credit that will be matured at period $t+1$:

[8] Given that the bank takes collateral from the household and does not face any uncertainty of banking activity, it does not demand intraday/overnight overdrafts from the government unlike Freeman (1996; 1999). Considering that the bank keeps collateral as reserves for a period, household collateral works much like overnight overdrafts in our model.

[9] We could assume that the bank may impose the collateral-deposit rate smaller than the nominal interest rate $(r_{H,t}<r_t)$ similar to interest on reserves. Then, owing to the smaller return on the collateral deposit, the share of the gross settlement would decrease. However, this modification does not significantly alter the main results and their implications.

$$L_{t+1} = P_t f(n_{s,t}), \tag{7}$$

where $n_{s,t}$ is the labor supply of the seller for the bank, and $f(\cdot)$ is twice continuously differentiable and concave with $f(0) = 0$. From (7), the bank's profit-maximization problem in period t is characterized by

$$\max_{n_{s,t}} [r_{L,t+1} L_{t+1} + r_{t+1} B_{t+1} - r_{D,t+1} D_{t+1} - r_{H,t+1} H_{t+1} - W_{s,t} n_{s,t}], \tag{8}$$

subject to

$$L_{t+1} = P_t f(n_{s,t}) \le D_{t+1}, \tag{9}$$
$$B_{t+1} \le D_{t+1} \tag{10}$$
$$(1 + r_{L,t+1}) \hat{\theta}_{t+1} L_{t+1} \le (1 + r_{H,t+1}) H_{t+1}. \tag{11}$$

The bank's profit in (8) consists of the expected net return from credit management $(r_{L,t+1} L_{t+1} - r_{H,t+1} H_{t+1})$, the expected net return from investment $(r_{t+1} B_{t+1} - r_{D,t+1} D_{t+1})$, and the labor costs in banking activity $(W_{s,t} n_{s,t})$. The constraint of (9) denotes the household's credit limit against its savings deposits D_{t+1}. In (10), the bank's investment in nominal government bonds must be less than or equal to D_{t+1}. Finally, the constraint of (11) is the collateral constraint for credit settlements in period $t+1$ implied by limited commitment. To transfer actual credit payments individually to other banks on a gross basis, the household must deposit enough collateral that covers the gross-settlement funds in period $t+1$ $(\hat{\theta}_{t+1} L_{t+1})$. By symmetry, no positive net-settlement balances will be left to transfer once offsetting settlement positions with other banks.

2.3. Government

The government controls a couple of policy variables—the rate of money growth $\mu > -1$ for monetary policy and the collateral requirement rate $\kappa \ge 1$ for collateral policy. First, the government conducts open market operations to control μ and affect the supply of money. The government budget constraint is satisfied with

$$(1 - r_t) B_t^s - B_{t+1}^s = M_{t+1}^s - M_t^s = \mu M_t^s, \tag{12}$$

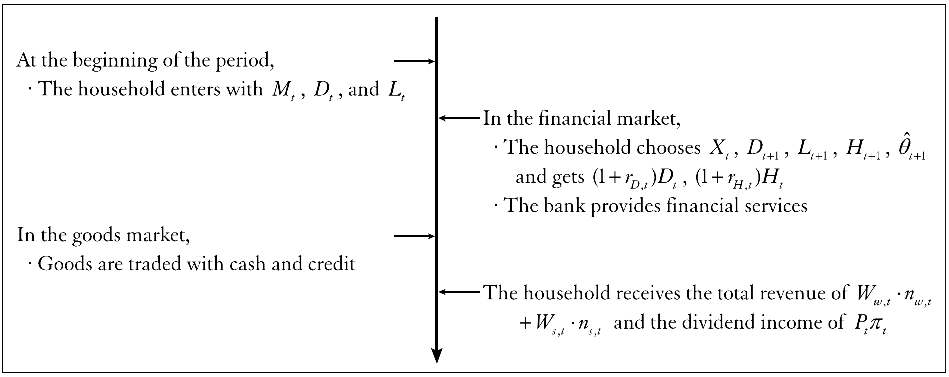
where B_{t+1}^s is the amount of the newly issued nominal bond supply, and M_{t+1}^s is the money stock after open market operation in period t.

Next, the government sets the collateral requirement rate κ for the stability of the interbank payment system affecting the bank collateral constraint for credit settlements in (11) and the household's collateral deposits,

$$H_{t+1} = \kappa \hat{\theta}_{t+1} L_{t+1} . \tag{13}$$

Figure 1 sums up the itinerary of events of the household and the bank in period t .

[Figure 1] Itinerary of the Household and the Bank in Period t



III. Symmetric Equilibrium

__Definition:__ A symmetric competitive equilibrium is given by the sequences $\{\hat{\theta}_{t+1}, c_{m,t}, c_{c,t}, n_t, n_{w,t}, n_{s,t}, M_{t+1}, D_{t+1}, H_{t+1}, B_{t+1}, L_{t+1}, M_t^s, P_t, W_{w,t}, W_{s,t}, r_{L,t}, r_{D,t}, r_{H,t}, r_t\}_{t=0}^{\infty}$ given $\{\gamma_g, \gamma_n, \tau, \kappa, \mu\}$ such that,

1. The household maximizes (1) subject to (5) and (6) and the nonnegativity constraints, $L_{t+1} \geq 0$ and $M_{t+1} \geq 0$.
2. The bank maximizes (8) subject to (7), (9), and (11).
3. The government budget constraint is satisfied with (12).
4. The markets clear every period.
   (a) Goods market: (i) Aggregate consumption: $\tau c_{m,t} + (1-\tau) c_{c,t} + \gamma_g + (1-\hat{\theta}_t)\Upsilon = n_{w,t}$ given $\Upsilon = \gamma_n - \gamma_g > 0$ and (ii) Credit-good consumption: $(1-\tau) c_{c,t} L_{t+1} / P_t$.
   (b) Labor market: $n_t = n_{w,t} + n_{s,t}$ and $W_{w,t} = W_{s,t} = P_t$.
   (c) Money market: $M_t + X_t = M_{t+1} = M_{t+1}^s$ and $M_{t+1} = (1+\mu) M_t$.
   (d) Bonds market: $D_t = B_t = B_t^s$, $(1+r_t) B_t - B_{t+1} = \mu M_t$, and $r_{D,t} = r_{H,t} = r_t$.

In the symmetric equilibrium, the price level is identical across the goods markets as P_t because each seller has the same production technology and does not bear any transactions costs in goods trades. The wage rate for consumption-good production is identical to the price level, i.e., $P_t = W_{w,t}$, too. Moreover, owing to the linear relationship between n_w and n_s , the wage rate of consumption-good production and financial-service production must be identical as $W_{w,t} = W_{s,t}$. Therefore, all wage rates equal the price level, i.e., $W_{w,t} = W_{s,t} = P_t$.[10] Next, the bank invests all savings deposits into one-period nominal government bonds $(D_t = B_t)$ which implies that $r_{D,t} = r_t$ for all t. Given that the bank pays r_t on collateral deposits, the return on savings deposits equals the return on collateral deposits $(r_{D,t} = r_{H,t} = r_t)$.

3.1. Household's Decision

Suppose $\lambda_{1,t} > 0$ and $\lambda_{2,t} > 0$ are the Lagrange multipliers at period t associated with the consolidated liquidity-in-advance constraint of (5) and the budget constraint of (6), respectively. The choices for $(c_{m,t}, (1-\hat{\theta}_{t+1})c_{c,t}, n_{w,t}, M_{t+1}, D_{t+1})$ are determined by

$$\tau u'(c_{m,t}) = P_t \tau (\lambda_{1,t} + \lambda_{2,t}), \tag{14}$$

$$(1-\tau)(1-\hat{\theta}_{t+1}) u'(c_{c,t}) = P_t (1-\tau)(1-\hat{\theta}_{t+1}) \lambda_{2,t}, \tag{15}$$

$$v'(n_t) = P_t \lambda_{2,t}, \tag{16}$$

$$\lambda_{2,t} = \beta \mathbb{E}_t [\lambda_{1,t+1} + \lambda_{2,t+1}], \tag{17}$$

$$\lambda_{1,t} + \lambda_{2,t} = \beta \mathbb{E}_t [(1+r_{t+1})(\lambda_{1,t+1} + \lambda_{2,t+1})]. \tag{18}$$

In equilibrium, much like standard cash-in-advance models, the marginal utility of cash-good consumption in (14) is related to both shadow prices of the liquidity-in-advance and budget constraints. The marginal utility of credit-good consumption in (15) is related to the share of the net settlement $(1-\hat{\theta}_{t+1})$ and the shadow price of the liquidity-in-advance constraint. The marginal disutility of labor supply in (16) is dependent of the shadow price of the budget constraint.

Now, the share of the gross settlement $\hat{\theta}_{t+1}$ is determined by

$$\begin{aligned} &(1+r_{t+1}) \left[\frac{P_{t+1}(1-\hat{\theta}_{t+1})\gamma_n}{(1+r_{L,t+1})(1-\hat{\theta}_{t+1})L_{t+1}} \right] \\ &= \frac{(1+r_{H,t+1})H_{t+1}}{(1+r_{L,t+1})\hat{\theta}_{t+1}L_{t+1}} + (1+r_{t+1}) \left[\frac{P_{t+1}\hat{\theta}_{t+1}\gamma_g}{(1+r_{L,t+1})\hat{\theta}_{t+1}L_{t+1}} \right], \end{aligned} \tag{19}$$

[10] Notably, the curvature of the loan production function $f(n_s)$ is a determinant of the bank's loan rate but is independent from the nominal wage rate $W_{s,t}$.

where the opportunity cost of per-unit net settlement equals that of per-unit gross settlement. On the left side of (19), the opportunity cost of per-unit net settlement only involves the foregone interest rate from the transactions costs γ_n. Collateral deposits are no longer in use for the net settlement because no positive net-settlement balances are left to transfer once offsetting settlement positions with other banks. On the right side of (19), the opportunity cost of per-unit gross settlement is the unit cost from collateral deposits plus the foregone interest rate from γ_g. Then, from (13) and (19), the real demand for loans is characterized by

$$\frac{L_{t+1}}{P_{t+1}} = \frac{\Upsilon}{\kappa}, \tag{20}$$

where $\Upsilon = \gamma_n - \gamma_g$. The total credit issued in period t is dependent of the price level in period $t+1$ at the time of the settlement P_{t+1}, Υ, and κ.

3.2. Bank's Decision

The bank invests all savings deposits in one-period government nominal bonds and holds no reserves against the savings deposits. To secure settlement funds in the payment system, the bank takes the collateral deposit from each household.[11]

3.2.1. Unbinding Collateral Constraint

Suppose the collateral constraint of (11) does not bind. Then, the bank holds enough collateral deposits for the gross settlement. The bank's profit-maximization problem in period t from (8)–(11) with (13) is reduced to

$$\max_{n_{s,t}}[P_t f(n_{s,t})(r_{L,t+1} - \kappa\hat{\theta}_{t+1} r_{t+1}) - P_t n_{s,t}].$$

The choice for $n_{s,t}$ is given by

$$r_{L,t+1} - \kappa\hat{\theta}_{t+1} r_{t+1} = \frac{1}{f'(n_{s,t})}. \tag{21}$$

The payment system plays an explicit role as a determinant of the loan rate. The

[11] If the bank holds 100% reserves against the savings deposit, then, the bank would not take the collateral deposit from the household. In this case, the bank's decision would be much different from those that will be studied in the current model. See Appendix A for further discussion. Furthermore, as long as the returns on savings and collateral deposits are identical, partial reserve requirements on savings deposits will not alter our key results.

loan rate depends not only on the deposit rate and the marginal cost of financial services as in Boel and Camera (2020) but also the share of the gross settlement. For example, if the gross settlement share $\hat{\theta}_{t+1}$ increases, then, the loan rate increases in (21). A greater $\hat{\theta}_{t+1}$ increases the household collateral deposits and reduces deposit funds in the savings account. Bank investment in government bonds decreases. The bank starts charging a higher loan rate for credit management to compensate for the investment loss. Hence, the spread between the loan rate and the deposit rate $(r_{L,t+1} - r_{t+1})$ widens up. This spread is generally positive under the condition of $\kappa\hat{\theta}_{t+1} > 1$. However, contrary to Boel and Camera (2020), the spread may be negative if the gross settlement share becomes too low that the loan rate is smaller than the deposit rate.

3.2.2. Binding Collateral Constraint

Suppose the collateral constraint of (11) binds. Then, from (13) with $r_{t+1} = r_{H,t+1}$ for all t, the collateral constraint is given by

$$1 + r_{L,t+1} = (1 + r_{t+1})\kappa . \tag{22}$$

The loan rate primarily depends on the deposit rate and the collateral requirement rate (κ). The loan rate is greater than the deposit rate as long as the government sets the collateral requirement rate greater than one. Under this circumstance, the government may play an important role in controlling the loan rate by adjusting κ. Contrary to (21), the share of the gross settlement does not directly affect the loan rate.

IV. Steady-State Allocations

Hereinafter, we focus on a steady-state equilibrium in which for all t, $\lambda_1 = \lambda_{1,t}$, $\lambda_2 = \lambda_{2,t}$, $\hat{\theta} = \hat{\theta}_{t+1}$, $c_m = c_{m,t}$, $c_c = c_{c,t}$, $n = n_t$, $n_w = n_{w,t}$, $n_s = n_{s,t}$, $m = M_{t+1} / P_t$, $l = L_{t+1} / P_t$, $r_L = r_{L,t}$, $r = r_t = r_{D,t} = r_{H,t}$, and $P_t / P_{t-1} = 1 + \mu$.

Suppose the bank has a linear credit-production function $f(n_s) = n_s$ for tractability. Then, the bank does not earn any profit, and the household does not receive any dividend income. In the steady-state equilibrium, from (14) and (16)–(18), the marginal rate of substitution of cash-good consumption for aggregate labor supply equals the return on nominal government bonds,

$$\frac{u'(c_m)}{v'(n)} = 1 + r , \tag{23}$$

where the nominal interest rate is given by

$$1+r=\frac{1+\mu}{\beta}. \tag{24}$$

From (15) and (16), the marginal rate of substitution of credit-good consumption for aggregate labor supply equals one:

$$\frac{u'(c_c)}{v'(n)}=1. \tag{25}$$

Next, from (3) and (7) with $f(n_s)=n_s$, the household's credit balances (l), the labor demand for the bank (n_s), and the aggregate credit-good consumption $[(1-\tau)c_c]$ are identical,

$$l=n_s=(1-\tau)c_c. \tag{26}$$

Hence, from (20) and (26), they can be expressed in terms of the money growth rate μ and the collateral requirement rate κ:

$$l=n_s=(1-\tau)c_c=(1+\mu)\frac{\Upsilon}{\kappa}. \tag{27}$$

If κ increases or μ decreases, then, in (27), the amount of bank loan, the labor supply for banking activity, and credit-good consumption decrease.

Lastly, from (5), (6), and (13) with the money- and bonds-market clearing conditions, the real demand for money (m) that equals the aggregate labor income (n) is given by[12]

$$\underbrace{\tau c_m}_{\text{For goods trades}}+\underbrace{\left[\frac{1+r_L}{1+\mu}-\kappa\hat{\theta}\left(\frac{1}{\beta}-1\right)\right]l+\gamma_g+(1-\hat{\theta})\Upsilon}_{\text{For credit settlements}}=m=n. \tag{28}$$

The money demand for the household has the following two types: demand money as a means of payment for goods trades and as a means of settlement for credit payments and collateral deposits. From (26), (28), and the goods-market clearing condition $[\tau c_m+(1-\tau)c_c+\gamma_g+(1-\hat{\theta})\Upsilon=n_w]$, the gross settlement share $\hat{\theta}$ can

[12] See Appendix B for the derivation of (28).

be determined by[13]

$$0 = \kappa\hat{\theta}\left(\frac{1}{\beta} - 1\right) + 2 - \frac{(1 + r_L)}{1 + \mu}. \quad (29)$$

The fraction $\hat{\theta}$ of credit payments that go through the payment system on a gross basis depends on the loan rate, the collateral requirement rate, and the money growth rate. From (28) and (29), $\hat{\theta}$ interacts with the loan rate and real variables similar to Bigio (2023). Hence, the payment system may influence banking and economic activities.

4.1. Unbinding Collateral Constraint

Suppose the collateral constraint of (11) does not bind. Then, the bank operates similarly to Boel and Camera (2020) soaking up labor resources similar to Boel and Camera (2020). Hence, collateral policy does not influence the loan and deposit rates.

Proposition 1 *Under the condition of* $\beta \in (0,1)$,

$$\kappa \geq 2, \quad (30)$$
$$\mu \geq \beta - 1, \quad (31)$$

$(\hat{\theta}, r_L)$ *is determined by (21), (24), (26), and (29):*

$$\hat{\theta} = \frac{2}{\kappa}, \quad (32)$$

$$1 + r_L = 2(1 + r) = 2\left(\frac{1 + \mu}{\beta}\right). \quad (33)$$

From (23)–(33) with the labor-market clearing condition $(n = n_w + n_s)$, $(l, n_s, c_c, c_m, n, n_w)$ *is determined.*

Proof. In a steady state, (32) is given by inserting (21) into (29), and (33) is given by (21) and (32). The proof of $(l, n_s, c_c, c_m, n, n_w)$ is straightforward from (23)–(33) and $n = n_w + n_s$. ■

When the collateral constraint unbinds the share of the gross settlement, $\hat{\theta}$ is

[13] See Appendix C for the derivation of (29).

negatively related to the collateral requirement rate κ from (32). A higher κ increases the burden of the gross settlement and decreases $\hat{\theta}$. From (24) and (33), the loan rate is independent of the collateral requirement rate but dependent on the money growth rate. Hence, the government can influence the interest rate spread solely through monetary policy.

4.1.1. Collateral-policy Effects

Suppose the government permanently increases κ holding μ constant under the condition of (30). Then, the increased burden of the gross settlement distorts the choice of $\hat{\theta}$ and yieldsa negative effect on consumption.

Lemma 1 *From Proposition 1, the effect of* κ *on* $(\hat{\theta}, l, n_s, c_c, c_m, n, n_w)$ *is given by (1)* $\partial\hat{\theta}/\partial\kappa < 0$*, (2)* $\partial l/\partial\kappa = \partial n_s/\partial\kappa = \partial(1-\tau)c_c/\partial\kappa < 0$*, (3)* $\partial\tau c_m/\partial\kappa < 0$*, (4)* $\partial n/\partial\kappa > 0$*, and (5)* $\partial n_w/\partial\kappa > 0$.

Proof. See Appendix D. ■

Proposition 2 *From (24) and (33), no effect exists on* $(r_L, r, r_L - r)$*: (1)* $\partial r_L/\partial\kappa = 0$*, (2)* $\partial r/\partial\kappa = 0$*, and (3)* $\partial(r_L - r)/\partial\kappa = 0$.

Proof. From (24) and (33), the proof is straightforward. ■

An increase in κ exacerbates the burden on the gross settlement and reduces $\hat{\theta}$. Consequently, the aggregate costs of the settlement increase $[\lambda_g + (1-\hat{\theta})\Upsilon]$. This cost-raising policy negatively reduces bank loan and the demand for loan production, thereby reducing consumption. To compensate for consumption loss, the labor supply of consumption-good production and hence, the aggregate labor supply increase. There arise welfare costs of an increase in κ. Suppose social welfare is defined by

$$W = \tau u(c_m) + (1-\tau)u(c_c) - v(n). \tag{34}$$

Then, from Lemma 1, the effect of an increase in κ on the social welfare of (34) is negative:

$$\frac{\partial W}{\partial\kappa} = u'(c_m)\frac{\partial\tau c_m}{\partial\kappa} + u'(c_c)\frac{\partial(1-\tau)c_c}{\partial\kappa} - v'(n)\frac{\partial n}{\partial\kappa} < 0. \tag{35}$$

Proposition 3 *From (30)–(32) and (35), the optimal allocation of the collateral requirement rate and the gross settlement share is given by* $(\kappa^*, \hat{\theta}^*) = (2,1)$.

Given (30), reducing the collateral requirement rate to the lower bound $(\kappa^{*}=2)$ is optimal for the government to improve the social welfare. The optimal rate of collateral requirements derives the net settlement out of the payment system and minimizes the costs of the latter. The interest rate spread remains constant from Proposition 2.

4.1.2. Monetary-policy Effects

Now, suppose the government permanently increases μ through open market purchases under the condition of (31) holding κ constant. Then, inflation arises and distorts banking and economic activities. However, unlike collateral policy, monetary policy results in redistribution between cash-good consumption and credit-good consumption.

Lemma 2 *From Proposition 1, the effect of* μ *on* $(\hat{\theta},l,n_s,c_c,c_m,n,n_w)$ *is given by (1)* $\partial\hat{\theta}/\partial\mu=0$*, (2)* $\partial l/\partial\mu=\partial n_s/\partial\mu=\partial(1-\tau)c_c/\partial\mu>0$*, (3)* $\partial\tau c_m/\partial\mu<0$*, (4)* $\partial n/\partial\mu<0$*, and (5)* $\partial n_w/\partial\mu<0$.

Proof. See Appendix E. ■

Proposition 4 *From (24) and (33), the effect on* (r_L,r,r_L-r) *is given by (1)* $\partial r_L/\partial\mu>0$*, (2)* $\partial r/\partial\mu>0$*, and (3)* $\partial(r_L-r)/\partial\mu>0$.

Proof. From (24) and (33), the proof is straightforward. ■

Inflation decreases the value of money. The household economizes cash and spends more on credit. Both bank loan and the demand for labor for loan production increase. Credit-good consumption increases, but cash-good consumption decreases. The household holds less money as a means of payment and more money as a means of settlement. Inflation redistributes money between cash- and credit-good consumptions. This redistributive effect is not subject to market segmentation as in Williamson (2009), Choi (2011b, 2023a), and Breu (2013) but is subject to the demand for liquidity in credit settlements. Finally, the supply of labor of loan production increases, whereas the labor supply of consumption-good production decreases. In sum, the aggregate labor supply decreases. Inflation induces the substitution of cash for credit in goods trades and exhibits a negative relationship between the interest-rate spread and the labor supply.

A change in μ also exhibits the following two opposite effects of inflation on social welfare: the welfare benefits of inflation from an increase in credit-good consumption and the welfare costs of inflation from a decrease in cash-good consumption. From Lemma 2, the effects on the social welfare of (34) depend on the size of redistribution:[14]

$$\frac{\partial W}{\partial \mu}=\frac{v'(n)}{1-\tau}\left[r\frac{u''(c_c)}{v''(n)}-(1-\tau)\right]\frac{\partial(1-\tau)c_c}{\partial \mu}=0\,, \tag{36}$$

where $u''(c_c)<0$ and $v''(n)>0$.

Proposition 5 *From (25), (27), (30), (31), and (36), the optimal rate of money growth* μ^* *is determined in terms of* κ, τ, *and* β:

$$\mu^* \in \left\{\mu \left| \frac{u''(c_c)}{v''(n)}=-\frac{(1-\tau)\beta}{1+\mu-\beta}\right.\right\}, \tag{37}$$

where $c_c=(1-\mu)\Upsilon/\kappa$ *and* $u'(c_c)=v'(n)$. *From (24), (33), and (31), the optimal interest rates and their spread are given by* $(r_L^*, r^*, r_L^*-r^*)=(2(1+\mu^*)/\beta-1,\ (1+\mu^*)/\beta-1,\ (1+\mu^*)/\beta)$.

The optimal rate of money growth depends on the size of redistribution, which is related to the fraction of cash-good consumption (τ), the nominal interest rate $[1+r=(1+\mu)/\beta]$, and the curvature of $u(\cdot)$ and $v(\cdot)$. Hence, the optimal rate is not necessarily negative which is in line with Boel and Camera (2020). The redistribution of money makes the household ease the distortion through credit management against inflation. Inflation can be somewhat beneficial for those who can substitute cash for credit. The Friedman rule is generally not optimal.

4.1.3. Optimal Policy Combination

The optimal policy mix that is given by $\kappa^*=2$ and μ^* from (37) would improve social welfare by minimizing both the cost of the gross settlement and the opportunity cost of holding money. The optimal share of the gross settlement is one, which drives the net settlement out of the payment system. The gross settlement is efficient if the bank holds enough collateral. The optimal set of the interest rates is determined solely by the optimal money growth from Proposition 5.

4.2. Binding Collateral Constraint

Suppose the collateral constraint of (11) binds. Then, the binding collateral constraint causes scarcity in the collateral deposits for the gross settlement and

[14] See Appendix F for the derivation.

distorts the loan rate and banking and economics activities.

Proposition 6 *Under the condition of* $2\beta > 1$*, (31), and*

$$\kappa \in (2\beta, 2), \tag{38}$$

$(\hat{\theta}, r_L)$ *is determined from (22), (26), (27), and (29):*

$$\hat{\theta} = \left(1 - \frac{2\beta}{\kappa}\right)\left(\frac{1}{1-\beta}\right), \tag{39}$$

$$1 - r_L = (1+r)\kappa = \kappa\left(\frac{1+\mu}{\beta}\right). \tag{40}$$

From (23)–(27), (39), and (40) with the labor-market clearing condition, $(l, n_s, c_c, c_m, n, n_w)$ *is determined.*

Proof. In a steady state, (39) is given by inserting (22) and (27) into (29). Next, (40) is given by (22), (26), and (39). The proof of $(l, n_s, c_c, c_m, n, n_w)$ is straightforward from (23)–(27), (39), and (40) with $n = n_w + n_s$. ■

From (40), constrained collateral deposits imply that the loan rate depends on κ and μ. The government can influence the interest rate spread through collateral and monetary policies contrary to the government with the unbinding collateral constraint. From (24) and (40), the spread is constantly positive, given that the loan rate is greater than the deposit rate. In (39), the share of the gross settlement $\hat{\theta}$ is positively related to the collateral requirement rate κ.

4.2.1. Collateral-policy Effects

Suppose the government permanently increases κ holding μ constant under the condition of (38). Then, it increases the loan rate and the cost of bank loan. A negative effect occurs on consumption.

Lemma 3 *From Proposition 6, the effect of* κ *on* $(\hat{\theta}, l, n_s, c_c, c_m, n, n_w)$ *is given by (1)* $\partial\hat{\theta}/\partial\kappa > 0$*, (2)* $\partial l/\partial\kappa = \partial n_s/\partial\kappa = \partial(1-\tau)c_c/\partial\kappa < 0$*, (3)* $\partial\tau c_m/\partial\kappa < 0$*, (4)* $\partial n/\partial\kappa > 0$*, and (5)* $\partial n_w/\partial\kappa > 0$.

Proof. See Appendix G. ■

Proposition 7 *From (24) and (40), the effect on* $(r_L, r, r_L - r)$ *is given by (1)*

$\partial r_L / \partial \kappa > 0$, *(2)* $\partial r / \partial \kappa = 0$, *and (3)* $\partial (r_L - r) / \partial \kappa > 0$.

Proof. From (24) and (40), the proof is straightforward. ■

An increase in κ increases the loan rate, which exacerbates the cost of bank loan. Hence, $\hat{\theta}$ increases to reduce the aggregate costs of the settlement $[\gamma_g + (1-\hat{\theta})\Upsilon]$. A greater loan rate from the tight collateral policy reduces bank loan and the demand for labor for loan production. Consequently, consumption decreases. To compensate for consumption loss, both the labor supply of consumption-good production and the aggregate labor supply increase. Hence, an increase in κ deteriorates social welfare from Lemma 3 and (34):

$$\frac{\partial W}{\partial \kappa} = u'(c_m)\frac{\partial \tau c_m}{\partial \kappa} + u'(c_c)\frac{\partial (1-\tau) c_c}{\partial \kappa} - v'(n)\frac{\partial n}{\partial \kappa} < 0. \tag{41}$$

Proposition 8 *From (39), (38), and (41), the optimal allocation of the collateral requirement rate and the gross settlement share is given by* $(\kappa^{**}, \hat{\theta}^{**}) = (2\beta, 0)$ *for* $2\beta > 1$. *From (24) and (40), the optimal interest rates and their spread are given by* $(r_L, r, r_L - r) = (\kappa^{**}(1+\mu)/\beta - 1, (1+\mu-\beta)/\beta, (\kappa^{**}-1)(1+\mu)/\beta)$.

Given (38), the government must lower the collateral requirement rate as much as possible, that is, $\kappa^{**} = 2\beta$, to minimize the distortion arising from the loan rate. This policy action would drive the gross settlement out of the payment system, thereby reducing the limit of the gross settlement. The net settlement serves as an efficient settlement system which is in line with Boel and Camera (2020). The optimal interest rate spread is positive.

4.2.2. Monetary-policy Effects

Suppose the government permanently increases μ through open market purchases under the condition of (31) holding κ constant. Then, the monetary policy induces the following two opposite effects: the inflation effect and the interest rate effect. These effects are similar to those with the unbinding collateral constraint.

Lemma 4 *From Proposition 6, the effect of* μ *on* $(\hat{\theta}, l, n_s, c_c, c_m, n, n_w)$ *is given by (1)* $\partial \hat{\theta} / \partial \mu = 0$, *(2)* $\partial l / \partial \mu = \partial n_s / \partial \mu = \partial (1-\tau) c_c / \partial \mu > 0$, *(3)* $\partial \tau c_m / \partial \mu < 0$, *(4)* $\partial n / \partial \mu < 0$, *and (5)* $\partial n_w / \partial \mu < 0$.

Proof. See Appendix H. ■

Proposition 9 *From (24) and (40), the effect on* $(r_L, r, r_L - r)$ *is given by (1)*

$\partial r_L / \partial \mu > 0$, *(2)* $\partial r / \partial \mu > 0$, *and (3)* $\partial (r_L - r) / \partial \mu > 0$.

Proof. From (24) and (40), the proof is straightforward. ■

Inflation induces the substitution of cash for credit in goods trades and exhibits a negative relationship between the interest-rate spread and the labor supply. Credit-good consumption increases, whereas cash-good consumption decreases. Bank loan and the demand for labor for loan production increase. The household holds less money as a means of payment and more money as a means of settlement. This redistributive effect is similar to one with the unbinding collateral constraint. The supply of labor of loan production increases, whereas the labor supply of consumption-good production decreases. The aggregate labor supply decreases. Hence, from Lemma 4, (34), and (36), the effect of inflation on welfare depends on the size of redistribution.

Proposition 10 *From (25), (27), (36), and (38), the optimal rate of money growth* μ^{**} *may be positive given by*

$$\mu^{**} \in \left\{ \mu \left| \frac{u''(c_c)}{v''(n)} = -\frac{(1-\tau)\beta}{1+\mu-\beta} \right. \right\}, \tag{42}$$

where $c_c = (1+\mu)\Upsilon / \kappa$ *and* $u'(c_c) = v'(n)$. *From (24), (40), and (38), the interest rates and their spread are given by* $(r_L, r, r_L - r) = (\kappa(1+\mu^{**})/\beta - 1,\ (1+\mu^{**})/\beta - 1,\ (\kappa - 1)(1+\mu^{**})/\beta)$.

4.2.3. Optimal Policy Combination

The optimal policy mix given by $\kappa^{**} = 2\beta$ and μ^{**} from (42) minimizes the cost of the gross settlement and the opportunity cost of holding money. The optimal share of the gross settlement is 0. The net settlement is efficient because the bank does not hold enough collateral against it. The optimal interest rates and their spread are generally positive.

V. Conclusion

In this paper, the bank's function is similar to a narrow bank in that it takes a savings deposit from the household and invests it into risk-free interest-bearing government bonds. Additionally, no bank's maturity risk that concerns maturity transformation exists by matching the maturity of the deposit and the maturity of the bank's investment. Therefore, this work is worth more rigorous investigation in an extended version of the model with a set of longer-term assets, in which the payment system may affect the term structure of interest rates. This extended model may provide new insights into the relationship among the payment system, liquidity premia, and the term structure of interest rates. Another interesting extension of the model is to introduce a risky asset and explore systemic risk in netting arrangements. If the bank can invest in riskier assets, then, the uncertainty of asset returns may cause interest rate and systemic risk and potentially disrupt banking activity. Intraday/overnight overdrafts can reduce the risk and the negative real effects on household and bank portfolios.

Appendix

A. Full-reserve/100%—reserve banking

The bank holds all savings deposits as reserves and does not pay any interest on them. The household does not earn any interest on the savings deposit but hold the savings account as collateral just enough to cover credit payments. That is, $H_{t+1}=0$ and $(1+r_{L,t+1})\hat{\theta}_{t+1}L_{t+1}\leq D_{t+1}$ for all t. The bank's profit-maximization problem in period t from (8)–(11) with (13) is reduced to

$$\max_{n_{s,t}}[r_{L,t+1}L_{t+1}-P_t n_{s,t}], \tag{43}$$

subject to

$$L_{t+1}=P_t f(n_{s,t})<D_{t+1}, \tag{44}$$

$$(1+r_{L,t+1})\hat{\theta}_{t+1}L_{t+1}=D_{t+1}. \tag{45}$$

In the credit limit in (44), the bank issues a new line of credit strictly less than the amount of savings deposits which should cover total credit payments in the payment system from (45). In equilibrium, the bank's profit-maximization problem in period t from (43) and (44) is given by

$$\max_{n_{s,t}}[r_{L,t+1}P_t f(n_{s,t})-P_t n_{s,t}].$$

The choice for $n_{s,t}$ is given by

$$r_{L,t+1}=\frac{1}{f'(n_{s,t})}.$$

The payment system does not play any explicit role as a determinant of the loan rate. The loan rate entirely depends on the decision on $n_{s,t}$. Additionally, if $f(n_{s,t})=n_{s,t}$, then, the loan rate is simply given by one. Collateral and monetary policy will not have any effect on $r_{L,t}$. The interest rate spread would be managed only through the return on government nominal bonds r_t.

B. Derivation of (28)

In a steady state equilibrium, the real demand for money equals aggregate labor supply, i.e., $m=n$, from (5) and (6) with $\pi=0$. Then, from (5) and (13) with the money- and bonds-market clearing conditions, the consolidated liquidity-in-advance constraint is expressed in real terms as (28).

C. Derivation of (29)

Subtract the goods-market clearing condition $[\tau c_m + (1-\tau)c_c + \gamma_g + (1-\hat{\theta})\Upsilon = n_w]$ from (28) and from (26) and (28), the following holds:

$$l = l\left[\frac{1+r_L}{1+\mu} + \kappa\hat{\theta} - \left(\frac{1+\mu}{\beta}\right)\frac{\kappa\hat{\theta}}{1+\mu} - 1\right]. \tag{46}$$

Then, rearranging terms of (46) gives (29).

D. Proof of Lemma 1

From (27) and (32), the effect of κ on $(\hat{\theta}, l, n_s, c_c)$ is determined by

$$\begin{aligned} &\frac{\partial\hat{\theta}}{\partial\kappa} = -\frac{2}{\kappa^2} < 0. \\ &\frac{\partial l}{\partial\kappa} = \frac{\partial n_s}{\partial\kappa} = \frac{\partial(1-\tau)c_c}{\partial\kappa} = -\frac{(1+\mu)\Upsilon}{\kappa^2} < 0. \end{aligned} \tag{47}$$

Then, from (25) and (47), the effect on n is positive:

$$\frac{\partial n}{\partial\kappa} = \left[\frac{u''(c_c)}{v''(n)}\right]\frac{\partial c_c}{\partial\kappa} > 0. \tag{48}$$

From the labor-market clearing condition, (47), and (48), the effect on n_w is positive:

$$\frac{\partial n_w}{\partial\kappa} = \frac{\partial n}{\partial\kappa} - \frac{\partial n_s}{\partial\kappa} > 0. \tag{49}$$

From (23) and (25), the following holds:

$$u'(c_m) = (1+r)u'(c_c). \tag{50}$$

Then, from (47) and (50), the effect on c_m is negative:

$$\frac{\partial c_m}{\partial \kappa} = (1+r)\left[\frac{u''(c_c)}{u''(c_m)}\right]\frac{\partial c_c}{\partial \kappa} < 0 .$$

E. Proof of Lemma 2

From (27) and (32), the effect of μ on $\hat{\theta}$ is 0. The effect on (l, n_s, c_c) is determined by

$$\frac{\partial l}{\partial \mu} = \frac{\partial n_s}{\partial \mu} = \frac{\partial (1-\tau) c_c}{\partial \mu} = \frac{\Upsilon}{\kappa} > 0 . \tag{51}$$

Then, from (25) and (51), the effect on n is negative:

$$\frac{\partial n}{\partial \mu} = \left[\frac{u''(c_c)}{v''(n)}\right]\frac{\partial c_c}{\partial \mu} < 0 . \tag{52}$$

From the labor-market clearing condition, (51), and (52), the

$$\frac{\partial n_w}{\partial \mu} = \frac{\partial n}{\partial \mu} - \frac{\partial n_s}{\partial \mu} < 0 . \tag{53}$$

Now, from (28) and (52), the effect on c_m is negative:

$$\frac{\partial \tau c_m}{\partial \mu} = \frac{\partial n}{\partial \mu} < 0 .$$

F. Derivation of (36)

From (23), (25), (26), and (28), the effect of μ on social welfare of (34) is given by

$$\begin{aligned}
\frac{\partial W}{\partial \mu} &= u'(c_m)\frac{\partial \tau c_m}{\partial \mu} + u'(c_c)\frac{\partial (1-\tau) c_c}{\partial \mu} - v'(n)\frac{\partial n}{\partial \mu} \\
&= \left[\frac{(1+r)v'(n)}{1-\tau}\right]\left(\frac{u''(c_c)}{v''(n)}\right)\frac{\partial (1-\tau) c_c}{\partial \mu} + v'(n)\frac{\partial (1-\tau) c_c}{\partial \mu} \\
&\quad -\left(\frac{v'(n)}{1-\tau}\right)\left[\frac{u''(c_c)}{v''(n)}\right]\frac{\partial (1-\tau) c_c}{\partial \mu} \\
&= \frac{v'(n)}{1-\tau}\left[r\frac{u''(c_c)}{v''(n)} - (1-\tau)\right]\frac{\partial (1-\tau) c_c}{\partial \mu} ,
\end{aligned}$$

where $u'(c_m) = (1+r)v'(n)$ and

$$\frac{\partial \tau c_m}{\partial \mu} = \frac{\partial n}{\partial \mu} = \left[\frac{u''(c_c)}{v''(n)}\right]\frac{\partial c_c}{\partial \mu}.$$

G. Proof of Lemma 3

From (39), the effect of κ on $(\hat{\theta}, l, n_s, c_c)$ is determined by

$$\frac{\partial \hat{\theta}}{\partial \kappa} = -\frac{2(1+\mu)\hat{\theta}}{\kappa^2\left[(1+\mu)\left(\frac{1}{\beta}-\frac{2}{\kappa}\right)+1\right]} < 0.$$

$$\frac{\partial l}{\partial \kappa} = \frac{\partial n_s}{\partial \kappa} = \frac{\partial (1-\tau)c_c}{\partial \kappa} = -\frac{\left(\frac{1}{\beta}+\frac{1}{1+\mu}\right)l}{\kappa\left(\frac{1}{\beta}+\frac{1}{1+\mu}\right)-2} < 0. \quad (54)$$

Then, from (25) and (54), the effect on n is positive:

$$\frac{\partial n}{\partial \kappa} = \left[\frac{u''(c_c)}{v''(n)}\right]\frac{\partial c_c}{\partial \kappa} > 0. \quad (55)$$

From the labor-market clearing condition, (54), and (55), the effect

$$\frac{\partial n_w}{\partial \kappa} = \frac{\partial n}{\partial \kappa} - \frac{\partial n_s}{\partial \kappa} > 0. \quad (56)$$

Then, from (50) and (55), the effect on c_m is negative:

$$\frac{\partial c_m}{\partial \kappa} = (1+r)\left[\frac{u''(c_c)}{u''(c_m)}\right]\frac{\partial c_c}{\partial \kappa} < 0.$$

H. Proof of Lemma 4

From (27) and (38), the effect of μ on $\hat{\theta}$ is 0. The effect on (l, n_s, c_c) is determined by

$$\frac{\partial l}{\partial \mu} = \frac{\partial n_s}{\partial \mu} = \frac{\partial (1-\tau)c_c}{\partial \mu} = \frac{\Upsilon}{\kappa} > 0. \quad (57)$$

Then, from (25) and (57), the effect on n is negative:

$$\frac{\partial n}{\partial \mu} = \left[\frac{u''(c_c)}{v''(n)} \right] \frac{\partial c_c}{\partial \mu} < 0 \,. \tag{58}$$

From the labor-market clearing condition, (57), and (58), the effect

$$\frac{\partial n_w}{\partial \mu} = \frac{\partial n}{\partial \mu} - \frac{\partial n_s}{\partial \mu} < 0 \,. \tag{59}$$

Now, from (28) and (58), the effect on c_m is negative:

$$\frac{\partial \tau c_m}{\partial \mu} = \frac{\partial n}{\partial \mu} < 0 \,.$$

References

Allsopp, P., B. Summers, and J. Veale (2009), "The Evolution of Real-time Gross Settlement," Financial Infrastructure Series, Payment Systems Policy and Research, The World Bank.

Andolfatto, D., and S. D. Williamson (2015), "Scarcity of Safe Assets, Inflation, and the Policy Trap," *Journal of Monetary Economics*, 73, 70–92.

Committee on Payments and Market Infrastructure (2021), "Developments in Retail Fast Payments and Implications for RTGS Systems," Bank for International Settlements.

Bech, M., C. Preisig, and K. Soramäki (2008), "Global Trends in Large-value Payments," *FRBNY Economic Policy Review*, Federal Reserve Bank of Ney York, September, 59–81.

Bech, M., Y. Shimizu, and P. Wong (2017), "The Quest for Speed in Payments," *BIS Quarterly Review*, Bank for International Settlements, March, 57–68.

Becivenga, V., and G. Camera (2011), "Banking in a Matching Model of Money and Capital," *Journal of Money, Credit, and Banking*, 43, 449–476.

Berentsen, A., G. Camera, and C. Waller (2007), "Money, Credit, and Banking," *Journal of Economic Theory*, 135, 171–195.

Bianchi, J., and S. Bigio (2022), "Banks, Liquidity Management and Monetary Policy," *Econometrica*, 90, 391–454.

Bigio, S. (2023), "A Theory of Payments-chain Crises," Working Paper 30859, NBER Working Paper Series.

Boel, P., and G. Camera (2020), "Monetary Equilibrium and the Cost of Banking Activity," *Journal of Money, Credit, and Banking*, 52, 653–683.

Breu, M. (2013), "The Welfare Effect of Access to Credit," *Economic Inquiry*, 51, 235–247.

Callado-Muñoz, F. (2007), "The Use of Collateral in Gross and Net Payment Systems," *The European Journal of Finance*, 459–481.

________________ (2013), "Intrda Liquidity and Central Bank Credit in Gross Pyament Systems," *International Finance*, 363–392.

Chiu, J., and C. Meh (2011), "Financial Intermediation, Liquidity, and Inflation," *Macroeconomic Dynamics*, 15, 83–118.

Choi, H. (2011b), "Money, Credit, and Limited Participation," *Macroeconomic Dynamics*, 15, 616–655.

_______ (2018), "Inflation Risk, Settlement Cycles, and Monetary Policy," *The Korean Economic Review*, 34, 267–282.

_______ (2023a), "Money, Payments Systems, Limited Participation, and Central Banking," *North American Journal of Economics and Finance*, 64, 101874.

_______ (2023b), "Payment Systems, Multiple Types of Collateral, Banking, and Collateral Policy," *The Korean Economic Review*, 39, 469–493.

Diamond, D., and P. Dybvig (1983), "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91, 401–419.

Freeman, S. (1996), "The Payments System, Liquidity, and Rediscounting," *American*

Economic Review, 86, 1126–1138.

Freeman, S. (1999), "Rediscounting Under Aggregate Risk," *Journal of Monetary Economics*, 43, 197–216.

Jurgilas, M., and A. Martin (2010), "Liquidity-saving Mechanisms in Collateral-based RTGS Payment Systems," *Federal Reserve Bank of New York Staff Reports*, No.438, Federal Reserve Bank of New York.

Kahn, C. M. (2013), "Private Payment Systems, Collateral, and Interest Rates," *Annals of Finance*, 9, 83–114.

Kahn, C. M., J. McAndrews, and W. Roberds (2003), "Settlement Risk Under Gross and Net Settlement," *Journal of Money, Credit and Banking*, 35, 591–608.

Kahn, C. M., and W. Roberds (1998), "Payment System Settlement and Bank Incentives," *Review of Financial Studies*, 11, 845–870.

____________________ (2001), "Real-time Gross Settlement and the Cost of Immediacy," *Journal of Monetary Economics*, 47, 299–319.

Kahn, C. M. and W. Roberds (2009), "Payments Settlement: Tiering in Private and Public Systems," *Journal of Money, Credit, and Banking*, 41, 855–884.

Koeppl, T., C. Monnet, and T. Temzelides (2008), "A Dynamic Model of Settlement," *Journal of Economic Theory*, 142, 233–246.

Lester, B. (2009), "Settlement Systems," *The B.E. Journal of Macroeconomics*, 9, Article 17.

Lotz, S., and C. Zhang (2016), "Money and Credit as Means of Payment: A New Monetarist Approach," *Journal of Economic Theory*, 164, 68–100.

Nosal, E., and G. Rocheteau (2006), "Economics of Payments," *Policy Discussion Paper*, No.14, Federal Reserve Bank of Cleveland.

Tomura, H. (2018), "Payment Instruments and Collateral in the Interbank Payment System," *Journal of Economic Theory*, 178, 82–104.

Williamson, S. D. (2003), "Payments Systems and Monetary Policy," *Journal of Monetary Economics*, 50, 475–495.

____________ (2009), "Transactions, Credit, and Central Banking in a Model of Segmented Markets," *Review of Economic Dynamics*, 12, 344–362.

____________ (2012), "Liquidity, Financial Intermediation, and Monetary Policy in a New Monetarist Model," *American Economic Review*, 102, 2570–2605.

____________ (2016), "Scarece Collateral, the Term Premium, and Quantitative Easing," *Journal of Economic Theory*, 164, 136–165.

유한책임 계약하에서의 은행업, 신용, 그리고 이자율*

최 형 선**

초 록 본 논문은 유한책임 신용계약모형을 기반으로 지급결제시스템이 이자율 스프레드의 결정에 미치는 영향을 이론적으로 분석하고 있다. 유한책임 신용계약은 신용결제를 제한 한다. 균형에서 지급결제시스템의 운영을 위해 신용담보가 필요하게 되고, 이는 대출금리, 통화분배, 소비 및 생산에 영향을 주게 된다. 최적 정책의 조합은 이자율 스프레드를 최소화하고, 생산을 극대화하는 것이다. 따라서 프리드먼 법칙의 최적성은 항상 성립하지 않는다.

핵심 주제어: 은행업, 신용, 지급결제시스템, 이자율, 유한책임 계약
경제학문헌목록 주제분류: E42, E44, E50

투고 일자: 2023. 12. 29. 심사 및 수정 일자: 2024. 6. 1. 게재 확정 일자: 2024. 9. 6.
* 본 논문의 작성 및 내용에 유익한 조언을 해주신 Steve Williamson 교수님, Gabriele Camera 교수님, 강기윤 교수님, 김창준 교수님, 익명의 두 심사위원분들, 그리고 수석부편집위원장님이신 이윤수 교수님께 깊이 감사드립니다.
** 경희대학교 경제학과 교수, e-mail: hyungsunchoi@khu.ac.kr