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Payment Systems, Multiple Types of Collateral, Banking, and Collateral Policy*

Hyung Sun Choi**

A monetary model is constructed to explore the real effects of multiple collateral requirements for credit settlements on the choice of cash and credit, banking activity, and consumption. Money serves not only as the means of payments but as the means of settlements. A collateral policy may be welfare-improving in a steady-state equilibrium when the nominal interest rate is positive. Reducing cash-collateral requirements increases the amount of cash as the means of payments and consumption with cash but adds up the cost of foregone interest. The optimal collateral policy is to balance out the marginal benefit and cost of holding cash.

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I. Introduction

An interbank payment system facilitates the transfer of funds among banks and finalizes settlements. To ensure the stability of the payment system a bank generally posts collateral. The eligible types of collateral assets may include reserves and government and private securities, but the composition of collateral varies from one system to another around the world.[1] Because each type of collateral may entail a different opportunity cost, for example, the foregone nominal interest of reserves

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** Professor, Department of Economics, Kyung Hee University, Email: hyungsunchoi@ khu.ac.kr

[1] See Bank for International Settlement (2003, 2005) for more discussion of the categories of eligible collateral and Williamson (2016), Tomura (2018), and Fiore et al. (2022) for theoretical applications. Also, see the following links for "Payment, clearing and settlement in various countries" provided by the Bank for International Settlements and "Financial services" provided by the Federal Reserve Bank Services of the US: https://www.bis.org/cpmi/paysysinfo.html, https://stats.bis.org/statx/toc/CPMI.html, and https://www.frbservices.org.

and the liquidity costs of securities, the composition of collateral would be potentially important for the government to implement collateral requirements on the interbank payment system. Further, collateral policy may have real effects on the demand for money as the means of payments, banking activity, and consumption. However, the implications of multiple collateral requirements in the payment system have not yet been studied much.

The objective of this study is to study the real effects of multiple collateral requirements including cash reserves and government nominal bonds on the choice of multiple means of payments, banking activity, the choice of settlement types, and consumption.

Traditionally, collateral policy on the interbank payment system is known for its role in stabilizing the system. However, in this paper, we will shed light on the mechanism of collateral policy outside the system. It explores how liquid and illiquid collateral may have asymmetric influences on economic individuals' decisions and banking activity to answer the following question: Do the types of collateral for the interbank payment system matter when the collateral policy has real effects?

The novelty of this paper is to explore the welfare-improving role of cash collateral in the form of reserves in the interbank payment system when money serves not only as the means of payments in goods trades but the means of settlements. For example, if the government would reduce the degree of cash-collateral requirements, then a greater amount of money available as the means of payments would increase consumption with cash and social welfare to some extent.

A set of literature involves research on banking, liquidity, reserves, and asset exchange. Some key contributions include Berentsen et al. (2007), Bianchi and Bigio (2018), Williamson (2012), Choi and Lee (2016), Tomura (2018), and Boel and Camera (2019). For example, Tomura (2018) is quite relevant in that his model is explicit about an interbank payment system. A net settlement – an arrangement in which a settlement occurs by offsetting positions by trading partners – can avoid a hold-up problem arising from bank's fund transfers in the over-the-counter interbank money market at no cost and economize liquid collateral for bank's reserves. Like Kahn et al. (2003), net settlement can also avoid a gridlock problem of a gross settlement—an arrangement in which a settlement occurs by transferring funds individually and simultaneously with goods trades. The net settlement promotes an optimal allocation. However, most of these studies focus on the relationship among bank's liquidity management, the supply of credit, and monetary policy and they are silent on the interaction between multiple types of collateral and the costly settlement process of credit payments. Our model is different from Tomura (2018) in a couple of aspects. First, money and within-period credit are circulated as medium of exchange. Next, in the payment system, the gross settlement entails multiple types of collateral and the net settlement is costly to manage. Finally, collateral policy has real effects. Therefore, the main implications on the interactions among the choice of multiple means of payments, banking activity, and the endogenous choice of settlement processes would be quite different from Tomura (2018).

Another strand of literature on the optimal design of the gross and net settlement includes Freeman (1996), Kahn and Roberds (1998, 2001), Kahn et al. (2003), Williamson (2003), Kahn (2013), Tomura (2018), and Choi (2019, 2021).[2] In particular, Choi (2021) studies the risk-sharing role of a collateral-requirement rate for a deferred gross settlement in a pure credit economy. The deferred gross settlement involves forgone interest arising from government-nominal-bond collateral, and a deferred net settlement incurs a transactions cost. In equilibrium, an increase in the collateral-requirement rate may increase the demand for bonds and dampen a consumption loss against interest-rate risk arising from inflation. Hence, collateral policy has real effects and may improve social welfare. However, in the pure credit economy, money solely serves as the means of settlements within the interbank payment system and the implications for currency in circulation as the means of payments and the stock of $M1$, which would affect the optimal design of the payment system, are limited.

In Choi (2019), he discusses a new source of inefficiency of a realtime gross settlement in a economy with cash and debit cards. The realtime gross settlement involves foregone interest by holding cash collateral and a realtime net settlement incurs a transaction cost. During inflation, the gross settlement is disadvantageous to the net settlement and induces a consumption loss. Hence, it is optimal for the government to reduce the burden of a collateral-requirement rate on the gross settlement unlike Choi (2021).

The baseline model is built on Ireland (1994) with some elements taken from the studies by Choi and Lee (2016) and Choi (2019). There are a unit mass of households and a bank. The bank offers a one-period transaction-deposit contract with no interest and a one-period saving-deposit contract with the positive rate of interest and issues within-period credit. In a financial market, the bank delivers several financial services to the household: a financial-portfolio management, a credit management and settlement via a gross and net settlement.

The collateral policy may be welfare-improving in a steady-state equilibrium when the nominal interest rate is positive. Reducing a cash-collateral proportion in aggregate collateral decreases bank's reserves and encourages bank's investment in nominal government bonds. A larger amount of cash serves as the means of payments in the goods market. This policy action results in two opposite effects. One is a positive effect from having more cash as the means of payments. The other

[2] See Nosal and Rocheteau (2006), Chiu and Lai (2007), and Kahn and Roberds (2009) for a detailed literature survey.

is a negative effect from substituting the means of payments from credit into cash, which adds up the cost of foregone interest. Consumption with cash increases while consumption with credit decreases. Hence, the optimal collateral policy is to balance out the marginal benefit and cost of holding cash as the means of payments.

The remainder of the paper is organized as follows. In the second section, the basic elements of the model is described. In the third section, a symmetric steady-state equilibrium is characterized and analyzed. The fourth section studies an economy under the Friedman rule. The fifth section analyzes an economy under a positive interest rate and discusses collateral-policy implications. The last section concludes.

II. Model

2.1. Households

Time is discrete and indexed by $t = 0,1,2,\ldots$. There is a continuum of infinitely-lived households with unit mass and a continuum of uniformly distributed consumption goods indexed by $i \in [0,1]$. Each household consists of a worker and a shopper. The worker sells consumption goods to shoppers from other households in a goods market and the shopper purchases them with either cash or costly credit across market i. In line with Ireland (1994), the household has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [(1-\hat{\imath}_t)\ln(c_t^m) + \hat{\imath}_t \ln(c_t^c) - h_t], \tag{1}$$

where β is the discount factor, $c_t^m(c_t^c)$ denotes distinct and perishable consumption goods purchased with cash (credit) for some markets in period t, h_t is transactions costs as disutility arising from a credit management as in Camera and Li (2008) and Choi (2011). In (1), $(1-\hat{\imath}_t)\ln(c_t^m)+\hat{\imath}_t\ln(c_t^c)$ represents a weighted average of utility in period t, where $1-\hat{\imath}_t$ denotes a threshold for the number of markets of cash transactions and $\hat{\imath}_t$ for the number of markets of credit transactions for $\hat{\imath}_t \in \{i \mid i \in [0,1]\}$.[3]

At the beginning of each period t, the household starts with M_t units of money in a transactions account and D_t amount of money in a one-period savings account. Assume that $M_0 > 0$ is exogenously given. The household receives constant endowments y that cannot be consumed by any members of the

[3] A detailed process of the endogenous choice of cash and credit ($\hat{\imath}_t$) will be discussed later in this section.

household.

On opening the financial market, the household involves the three types of financial transactions through a bank that can have access to a bonds market for exchanging money and one-period government nominal bonds and to a centralized interbank payment system for credit settlement.[4]

One is to determine household's portfolio (D_{t+1}, M_{t+1}). The household transfers some cash from its matured savings account (D_t) to its transactions account in which it can withdraw cash at any time for consumption-goods purchases $[(1-\hat{\imath}_t)c_t^m]$ in the subsequent goods market. Then, the household deposits the remaining cash into a new one-period savings account (D_{t+1}) that will mature at period $t+1$. Assume that D_{t+1} cannot be liquidated in cash before its maturity at period $t+1$ and cannot be circulated as a medium of exchange.

Another is to open a new line of bank-issued within-period credit up to a household's income for consumption-good purchases $(\hat{\imath}_t c_t^c)$ and to decide the share of a gross and net settlement within the interbank payment system managed by the government by comparing the nominal per-unit opportunity costs of the gross settlement of $\hat{\theta}_t \hat{\imath}_t c_t^c$ and the net settlement of $(1-\hat{\theta}_t)\hat{\imath}_t c_t^c$, where $\hat{\theta}_t \in [0,1]$ denotes the share of the gross settlement.

The other is to make some financial transactions, in advance, for credit arrangements which will occur at the end of each period in order to secure the finality of settlements. For the gross settlement, the household is required to deposit two types of collateral to the bank which will be redeposited to a government account in spirit of Williamson (2016) and Tomura (2018). The collateral deposits are comprised of cash collateral $\alpha(1-\sigma)L_{t+1}^m$ in the form of reserves and bond collateral $\alpha\sigma q_t L_{t+1}^b$ in the form of one-period government nominal bonds, where $\alpha \in [0,1]$ denotes the collateral-requirement rate, $\sigma \in [0,1]$ is the collateral distribution ratio between cash- and bond-collateral deposits, L_{t+1}^m is the nominal value of credit payments associated with cash collateral, q_t is the price of a unit of bond, and L_{t+1}^b is the nominal value of credit payments associated with bond collateral. A bond sells for q_t units of money in period t and is a claim to one unit of money in period $t+1$. The bond cannot be liquidated in cash before its maturity. It is a book-entry bond and cannot be circulated as a medium of exchange. Assume that the return on bond collateral will be paid to the household and will not be taken away by the government. Hence, the per-unit opportunity cost of the gross settlement depends on foregone interest on cash collateral.

For the net settlement, the household is required to bear the management costs $\gamma\Gamma(i)$, for a credit-risk control, where $\gamma \in (0, \alpha(1-\sigma)y]$ denotes the level of

[4] Generally, the major participants in the bonds market are government institutions, financial intermediaries, and corporate entities rather than individual investors. Also, the participants of the centralized interbank payment system are mainly financial institutions.

transactions costs in the net-settlement process, which would decrease with a technological innovation, and $\Gamma(i)$ is an increasing function on the number of markets $i \in [0,1]$, which demands a greater computing power, with $\Gamma(0)=0$ and $\Gamma(1) > (1+\mu)/\beta$ given the net rate of money growth $(\mu > -1)$. Hence, the per-unit opportunity cost of the net settlement is foregone per-unit management costs. The aggregate costs are given by $(1-\hat{\theta}_t)\int_0^{\hat{\iota}_t} \gamma\Gamma(i)di$, where $1-\hat{\theta}_t$ denotes the weight of the net settlement on the costs and $\int_0^1 \Gamma(i)di \geq \alpha(1-\sigma)y/\gamma$. As in Camera and Li (2008) and Choi (2011), this aggregate management costs generate disutility in the household's preferences:[5]

$$h_t = (1-\hat{\theta}_t)\int_0^{\hat{\iota}_t} \gamma\Gamma(i)di . \tag{2}$$

Furthermore, because the costs $\gamma\Gamma(i)$ increase with i credit is used for markets $i < \hat{\iota}_t$ and cash for markets $i > \hat{\iota}_t$, denoting $\hat{\iota}_t \in (0,1)$ as the threshold of the number of markets i for credit purchases.

The cash-in-advance constraint is given by

$$(1-\hat{\iota}_t)c_t^m = \frac{M_t}{P_t} + \frac{X_t}{P_t}, \tag{3}$$

and the financial constraint is given by

$$\frac{X_t}{P_t} + \frac{\alpha\sigma q_t L_{t+1}^b}{P_t} + \frac{\alpha(1-\sigma)L_{t+1}^m}{P_t} + \frac{D_{t+1}}{P_t(1+r_t)} = \frac{\alpha\sigma L_t^b}{P_t} + \frac{D_t}{P_t}, \tag{4}$$

where X_t is the amount of cash transferred between the transactions and savings account, P_t is the average price level of consumption goods, $\alpha\sigma L_t^b$ is the nominal value of matured-bond collateral from period $t-1$, and r_t is the net return on the savings account. From (3) and (4), the consolidated liquidity-in-advance constraint is expressed as

$$(1-\hat{\iota}_t)c_t^m + \frac{\alpha\sigma q_t L_{t+1}^b}{P_t} + \frac{\alpha(1-\sigma)L_{t+1}^m}{P_t} + \frac{D_{t+1}}{P_t(1+r_t)} = \frac{M_t}{P_t} + \frac{\alpha\sigma L_t^b}{p_t} + \frac{D_t}{P_t}. \tag{5}$$

On opening the goods market, shoppers purchase consumption goods from workers by using either cash or the costly credit across the market i. Credit

[5] The transactions costs of credit settlements in terms of disutility instead of resource costs will make our equilibrium analysis more tractable without altering any key results and policy implications.

purchases incur foregone interest in the market i whereas cash does not incur any transactions costs.

At the end of each period, all the household members return home with the revenue from sales. No further trade or barter is allowed. Then, the household pays off credit debt. The bank finalizes the settlement of household's credit balances issued earlier in this period simultaneously with other banks through the interbank payment system. Once the settlement is completed at the end of period t, cash collateral returns to the household whereas bond collateral deposited in period t returns in the next-period financial market. The budget constraint is given by

$$(1-\hat{\imath}_t)c_t^m + \hat{\imath}_t c_t^c + \frac{\alpha\sigma q_t L_{t+1}^b}{P_t} + \frac{\alpha(1-\sigma)L_{t+1}^b}{P_t} + \frac{D_{t+1}}{P_t(1+r_t)} + \frac{M_{t+1}}{P_t}$$
$$= \frac{M_t}{P_t} + \frac{\alpha\sigma L_t^b}{P_t} + \frac{\alpha(1-\sigma)L_t^m}{P_t} + \frac{D_t}{P_t} + y, \tag{6}$$

where M_{t+1} is the demand for money brought over to the next period and y is revenue from sales.

2.2. Banks

A representative bank facilitates financial transactions on behalf of its customer households by participating in the bonds market for exchanging money for one-period government nominal bonds and the interbank payment system for credit settlement. The bank issues within-period credit to its customer households against their income and offers two types of one-period accounts in each period. One is a transaction-account contract (M_{t+1}) that pays no interest and the other is a saving-account contract (D_{t+1}) that pays positive interest. Also, the bank takes cash and bond collateral from the households and deposits them in the government account for credit settlements. Given all deposits received in period t, the bank holds cash collateral as reserves (Z_{t+1}) and invest the rest on one-period government bonds (B_{t+1}). Hence, after the financial market of period t, the bank's reserves are given by

$$\frac{Z_{t+1}}{P_t} = \frac{\alpha(1-\sigma)L_{t+1}^m}{P_t}, \tag{7}$$

and the bank's portfolio is given by

$$\frac{D_{t+1}}{P_t(1+r_t)} + \frac{\alpha\sigma q_t L_{t+1}^b}{P_t} + \frac{\alpha(1-\sigma)L_{t+1}^m}{P_t} = \frac{q_t B_{t+1}}{P_t} + \frac{\alpha\sigma q_t L_{t+1}^b}{P_t} + \frac{Z_{t+1}}{P_t}. \tag{8}$$

The left-hand side of (8) denotes the savings and collateral deposits of the household and the right-hand side denotes bank's assets.

At the end of period t, the bank takes the households' revenues of sales as deposits in the transactions accounts, settles credit payments in the centralized interbank payment system on behalf of customer households, and returns cash collateral to households received from the government. Then, a bank's zero profit condition in period t is determined by

$$\frac{X_t}{P_t}+\frac{D_t}{P_t}+\frac{\alpha\sigma L_t^b}{P_t}+\frac{\alpha(1-\sigma)L_{t+1}^m}{P_t}=\frac{B_t}{P_t}+\frac{\alpha\sigma L_t^b}{P_t}+\frac{Z_{t+1}}{P_t}+\hat{\theta}_t\hat{\iota}_t\hat{c}_t^c-\overline{\theta}_t\overline{\iota}_t\overline{c}_t^c, \tag{9}$$

where $\hat{\theta}_t\hat{\iota}_t\hat{c}_t^c$ is outgoing credit payments settled on a gross basis to other banks through the interbank payment system and $\overline{\theta}_t\overline{\iota}_t\overline{c}_t^c$ is incoming credit payments from other banks that are paid to the customer household. The left side of (9) denotes the bank's expenditure to the households, and the right side denotes bank revenue from investment in government bonds and reserves for credit settlements.

2.3. Government

The government can use two types of policies – monetary policy and collateral policy for a money-stock control. First, monetary policy implemented through open market operations controls the money growth rate and the level of money supply. The government budget constraint is satisfied with

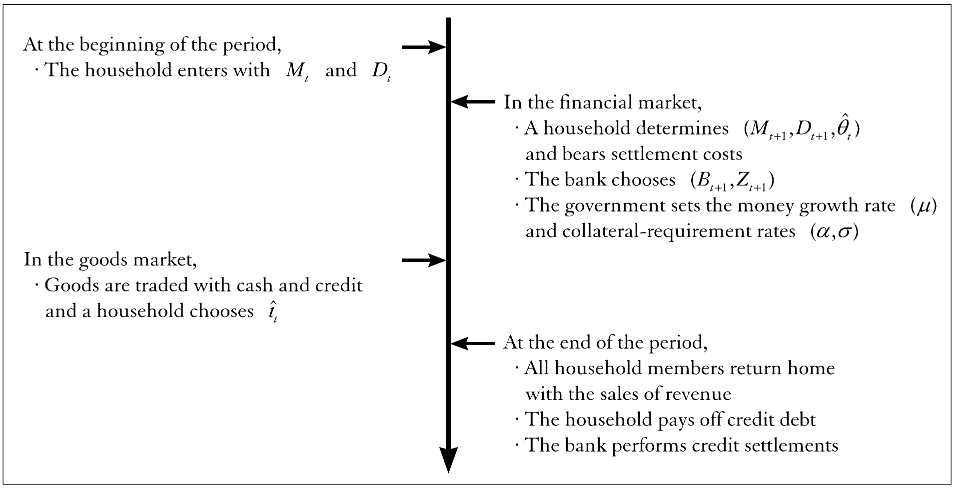
$$B_t^s-q_tB_{t+1}^s=M_{t+1}^s-M_t^s=\mu M_t^s, \tag{10}$$

where B_{t+1}^s represents the amount of the newly issued nominal bond supply, M_{t+1}^s is money supply on closing the financial market, and $\mu>-1$ is the net growth rate of money supply.

Next, collateral policy implemented through the management of the collateral-requirement rates ($\alpha\in[0,1]$ and $\sigma\in[0,1]$) for the gross settlement not only keeps the stability of the payment system but also controls the stock of money in circulation.[6] Hence, collateral policy affects both the bonds and settlement market whereas monetary policy only does the bonds market. Figure 1 summarizes the itinerary of events of the household in period t.

[6] Note that α stays between zero and one because the government does not need collateral deposits exceeding the amount of credit payments for the gross settlement. Moreover, $\alpha\in[0,1]$ would not trigger credit default since the government could confiscate the household's other nominal assets and income.

[Figure 1] Timing of Events in Period t



III. Equilibrium Dynamics

Definition: A symmetric competitive equilibrium is given by the sequences $\{c_t^m, c_t^c, \hat{\imath}_t, \hat{\theta}_t, X_t, D_{t+1}, M_{t+1}, B_{t+1}, L_{t+1}^b, L_{t+1}^m, Z_{t+1}, M_t^s, P_t, q_t\}_{t=0}^{\infty}$ and $\{\alpha, \sigma, \mu, \gamma\}$:

1. The household maximizes (1) with (2) subject to (5) and (6) and the nonnegativity constraints, $M_{t+1} \geq 0$

2. The bank is satisfied with (7) - (9).

3. The government budget constraint is satisfied with (10).

4. The markets clear in each of the following periods.
   (a) Goods market: a) $c_t^j(i) = c_t^j$ for $j \in \{m, c\}$ at every market i to avoid arbitrage opportunities, b) Resource constraint: $(1-\hat{\imath}_t)c_t^m + \hat{\imath}_t c_t^c = y$.
   (b) Money market: $M_{t+1} = M_{t+1}^s$ and $M_{t+1}^s = (1+\mu)M_t^s$.
   (c) Bonds market: $B_t - q_t B_{t+1} + \alpha\sigma L_t^b - \alpha\sigma q_t L_{t+1}^b = B_t^s - q_t B_{t+1}^s = \mu M_t$, $B_{t+1} = D_{t+1}$, and $L_{t+1}^b / P_t = \hat{\theta}_t \hat{\imath}_t c_t^c$.
   (d) Settlement market: $c_t^c = \overline{c}_t^c$, $\hat{\theta}_t = \overline{\theta}_t$, $\hat{\imath}_t = \overline{\imath}_t$, and $L_{t+1}^b / P_t = L_{t+1}^m / P_t = \hat{\theta}_t \hat{\imath}_t c_t^c$ symmetry.

Hereinafter we focus on a steady-state equilibrium in which for all t, $d = d_{t+1} = D_{t+1} / P_t$, $m = m_{t+1} = M_{t+1} / P_t$, $b = b_{t+1} = B_{t+1} / P_t$, $l^m = l_{t+1}^m = L_{t+1}^m / P_t$ and $l^b = l_{t+1}^b = L_{t+1}^b / P_t$, $z = z_{t+1} = Z_{t+1} / P_t$, $\hat{\imath} = \hat{\imath}_t$, $\hat{\theta} = \hat{\theta}_t$, $c^m = c_t^m$, $c^c = c_t^c$, $q = q_t$,

$r = r_t$, and $P_t / P_{t-1} = 1 + \mu$. Assume that λ_1 and λ_2 denote the Lagrange multipliers associated with the consolidated liquidity-in-advance constraint in (5) and the budget constraint in (6), respectively.

In the steady-state equilibrium, the first-order-conditions with respect to d and b imply that the return on one-period savings is equal to that on one-period government bonds:

$$\frac{1}{1-r} = q = \frac{\beta}{1+\mu}, \tag{11}$$

and the first-order-condition with respect to m is given by

$$\frac{c^c}{c^m} = \frac{1+\mu}{\beta}. \tag{12}$$

From (11) and (12), the price of one-period government nominal bonds (q) and the ratio of consumption with credit (c^c) to consumption with cash (c^m) are only dependent on the money growth rate (μ) controlled by monetary policy. Hence, collateral policy – a change in (α, σ) – has no effect on q and c^c / c^m .

Next, a cash-credit cutoff value $[\hat{\imath} \in (0,1)]$ is determined where the benefit from a credit purchase at market i for the given settlement costs is equal to that from a cash purchase $[\ln(c^c) - (1-\hat{\theta})\gamma\Gamma(\hat{\imath}) + \lambda_2 c^c = \ln(c^m) - (\lambda_1 + \lambda_2)c^m]$ so that

$$(1-\hat{\theta})\gamma\Gamma(\hat{\imath}) = \ln\left(\frac{c^c}{c^m}\right), \tag{13}$$

where, given the constraints of (5) and (6), $\lambda_1 + \lambda_2 = 1/c^m$ from the first-order condition with respect to c^m and $\lambda_2 = 1/c^c$ from the first-order condition with respect to c^c . At a cutoff share between the gross and net settlement $[\hat{\theta} \in (0,1)]$ is determined by

$$\alpha(1-\sigma) = \frac{(1-\hat{\theta})\int_0^{\hat{\imath}} \gamma\Gamma(i)di}{(1-\hat{\theta})\hat{\imath}c^c}. \tag{14}$$

The left-hand side of (14) denotes the foregone interest from the per-unit cash collateral $[\alpha(1-\sigma)]$ of the gross settlement and the right-hand side denotes the per-unit transactions cost $[(1-\hat{\theta})\int_0^{\hat{\imath}} \gamma\Gamma(i)di / (1-\hat{\theta})\hat{\imath}c^c]$ of the net settlement. From (13) and (14), collateral policy has an effect on the decision on $\hat{\theta}$ and $\hat{\imath}$ on the contrary to the decision on q and c^c / c^m in (11) and (12).

Then, from (5) - (12) with the clearing conditions for the bonds and goods market, the consolidated liquidity-in-advance constraint, the budget constraint, bank's reserves (z) can be expressed by

$$(1-\hat{\imath})c^m + \alpha(1-\sigma)\hat{\theta}\hat{\imath}c^c = m, \tag{15}$$

$$(1-\hat{\imath})c^m + \hat{\imath}c^c = y, \tag{16}$$

$$z = \alpha(1-\sigma)\hat{\theta}\hat{\imath}c^c. \tag{17}$$

From (15) and (17), the household's demand for money $[m=(1-\hat{\imath})c^m + z]$ consists of cash in circulation as the means of payments and cash collateral in the form of reserves as the means of settlements. Moreover, from (15) and (16), the household's demand for money can be expressed by $y-m=[1-\alpha(1-\sigma)\hat{\theta}]\hat{\imath}c^c$ for $\alpha \in [0,1]$ and $\sigma \in [0,1]$, where the gross-settlement share ($\hat{\theta}$) should be satisfied with $m \le y$ such that

$$\hat{\theta} \le \frac{1}{\alpha(1-\sigma)}. \tag{18}$$

Finally, from (12), (14), and (16), there exists a unique $\hat{\imath}(\gamma,\alpha,\sigma,\mu)$ in terms of $\gamma \in (0,\alpha(1-\sigma)y]$, $\alpha \in [0,1]$, $\sigma \in [0,1]$, and $\mu > -1$:[7]

$$\left(\frac{1}{\hat{\imath}} + \frac{1+\mu-\beta}{\beta}\right)\int_0^{\hat{\imath}} \Gamma(i)di = \left(\frac{1+\mu}{\beta}\right)\frac{\alpha(1-\sigma)y}{\gamma}. \tag{19}$$

Given (11), the cash-credit choice $\hat{\imath}$ in the left-hand side of (19) must be equal to the gross return on the ratio of cash collateral of the gross settlement $[\alpha(1-\sigma)]$ to the transactions-cost level of the net settlement (γ) in terms of endowments y in the right-hand side. The cash-credit choice determined in (19) may provide a government-policy transmission channel that would have real effects on equilibrium outcomes. From (12) - (18), the remaining variables $\hat{\theta}(\gamma,\alpha,\sigma,\mu)$, $c^m(\gamma,\alpha,\sigma,\mu)$, $c^c(\gamma,\alpha,\sigma,\mu)$, and $z(\gamma,\alpha,\sigma,\mu)$ can be determined.

IV. Collateral Policy under the Friedman Rule

This section will discuss the role of collateral policy in an economy under the Friedman rule in which the net nominal interest rate is zero. Assume that $\Gamma(i) = 2i$

[7] See appendix A for the derivation and the proof of the existence of unique $\hat{\imath}$.

in order to make our analysis on policy effects more tractable. Then, from (19), the cash-credit choice $(\hat{\imath})$ is determined by

$$\left(\frac{1+\mu-\beta}{\beta}\right)\hat{\imath}^2+\hat{\imath}-\left(\frac{1+\mu}{\beta}\right)\frac{\alpha(1-\sigma)y}{\gamma}=0\,, \tag{20}$$

where $\int_0^{\hat{\imath}}\Gamma(i)di=\hat{\imath}^2$.

Suppose the government manages the money growth rate in a decreasing fashion such that the gross rate of money growth is equal to the discount factor $(\mu=\beta-1)$. Then, the net nominal interest rate is zero and the economy is under the Friedman rule. From (20) given $\gamma\in(0,\alpha(1-\sigma)y]$, $\hat{\imath}$ is determined by

$$\hat{\imath}=\frac{\alpha(1-\sigma)y}{\gamma}\geq 1\,. \tag{21}$$

The Friedman rule reduces the opportunity cost of holding cash to zero. From (21), the household spends on credit for all market $i(\hat{\imath}=1)$ and uses money solely as the means of settlements. From (13), the household would demand cash only for settling credit debt on a gross basis $(\hat{\theta}=1)$. In other words, no household settles credit debt on a net basis in order to avoid bearing the transactions costs.

Further, from (14), (16), and (21), consumption with credit is equal to y. The demand for reserves from (17) is given by $z=\alpha(1-\sigma)y$. Therefore, in a steady-state equilibrium, if the Friedman rule holds, then credit is the sole medium of exchange similar to the economy of Choi (2021) and the equilibrium allocation is efficient. Collateral policy, that is, a change in α or σ, may affect the bank's reserves z, but have no effects on other equilibrium outcomes. It is worth noting that in the standard cash-in-advance model with credit, but without the gross settlement, for example, Ireland (1994), Freeman and Kydland (2000), and Lucas and Nicolini (2015), money does not serve as the means of settlements and the Friedman rule would derive credit out of the economy in optimum.

V. Collateral Policy with Positive Interest Rate

This section will explore the real effects of collateral policies on equilibrium outcomes when the nominal interest rate is positive $(\mu>\beta-1)$ and compare the results with those under the Friedman rule. If the nominal interest rate is positive, then the gross settlement involves foregone interest arising from holding cash collateral. Also, assume $\Gamma(i)=2i$ that satisfies (20).

5.1. Effects of Collateral Requirements (α)

Suppose the government manages the rate of collateral requirements (α) holding the weight of bond collateral (σ) fixed. Then, under a positive interest rate, a change in α may distort the opportunity cost of holding cash as the means of payments and using costly credit and result in real effects on equilibrium outcomes as in the Proposition 1.

Proposition 1 *From (11) - (18) and (20), the effects of* α *on* $(\hat{\imath}, \hat{\theta}, c^m, z)$ *are given by (1)* $\partial\hat{\imath}/\partial\alpha > 0$*; (2)* $\partial\hat{\theta}/\partial\alpha > 0$*; (3)* $\partial c^m/\partial\alpha < 0$ *and* $\partial(1-\hat{\imath})c^m/\partial\alpha < 0$*; (4)* $\partial c^c/\partial\alpha < 0$ *and* $\partial\hat{\imath}c^c/\partial\alpha > 0$*; and (5)* $\partial z/\partial\alpha = \partial\alpha(1-\sigma)\hat{\theta}\hat{\imath}c^c/\partial\alpha > 0$.

Proof. See appendix B. ■

For example, an increase in α increases both cash- and bond-collateral requirements. It increases bank's reserves and bank's investment in government nominal bonds. A greater amount of cash serves as the means of settlements in the payment system. This policy action results in two opposite effects. One is a negative effect from having less cash as the means of payments. Consumption with cash decreases. The other is a positive effect from substituting the means of payments from cash into credit. Given the positive interest rate of nominal bonds, an increase in bank's investment in response to a greater use of credit increases the overall return. Hence, the gross-settlement share increases and consumption with credit, too, increases. Note that the collateral policy does not have any effects on the nominal interest rate and the inflation rate.

Balancing out the tradeoff between the marginal benefit and cost of holding cash as the means of payments arising from the management of α is optimal for the government. Suppose social welfare is defined and expressed by using (12) and (13):

$$W = (1-\hat{\imath})\ln(c^m) + \hat{\imath}\ln(c^c) - (1-\hat{\theta})\hat{\imath}^2 = \ln(c^m) + \frac{\hat{\imath}}{2}\left(\frac{1-\mu}{\beta}\right). \tag{22}$$

Then, under the condition of (18), the effect of α on social welfare in (22) is not monotonic and determined by

$$\frac{\partial W}{\partial\alpha} = 0 = \frac{1}{c^m}\frac{\partial c^m}{\partial\alpha} + \frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\frac{\partial\hat{\imath}}{\partial\alpha} \tag{23}$$

and the optimal rate of collateral requirements that maximizes social welfare can be characterized by the Proposition 2.

Proposition 2 *Under the positive nominal interest rate* $(\mu > \beta - 1)$ *with (23), the optimal rate of aggregate collateral* (α^{*}) *is determined by*

$$\alpha^{*} = \frac{\gamma}{(1-\sigma)y}\left(\frac{\beta}{1+\mu}\right)\left\{\frac{\frac{1+\mu-\beta}{\beta} - \ln(\frac{1+\mu}{\beta})}{[\ln(\frac{1+\mu}{\beta})]^{2}}\right\}. \tag{24}$$

Proof. See appendix C. ■

The economy under the positive nominal interest rate gives some leeway for the government's implementation of collateral policy on the contrary to the economy under the Friedman rule in Section 4. Collateral policy that reduces the burden of overall collateral deposits for the gross settlement can make the payment system less distortionary to some extent and improve social welfare by smoothing out consumption with cash and credit. From (11) and (24), the optimal rate of collateral requirements (α^{*}) is mainly subject to the opportunity cost of holding cash as the means of payments, that is, the nominal interest rate $[1+r=(1+\mu)/\beta>1]$.

5.2. Effects of Bond-Collateral Requirements (σ)

Now, suppose the government manages the weight of bond-collateral deposits (σ) over aggregate collateral deposits holding the rate of collateral requirements (α) fixed. Then, under a positive interest rate $(\mu > \beta - 1)$, the effects of σ on equilibrium outcomes are determined by the Proposition 3.

Proposition 3 *From (11) - (18) and (20), the effects of* σ *on* $(\hat{\imath}, \hat{\theta}, c^{m}, c^{c}, z)$ *are given by (1)* $\partial \hat{\imath} / \partial \sigma < 0$*; (2)* $\partial \hat{\theta} / \partial \sigma < 0$*; (3)* $\partial c^{m} / \partial \sigma > 0$ *and* $\partial (1-\hat{\imath}) c^{m} / \partial \sigma > 0$*; (4)* $\partial c^{c} / \partial \sigma > 0$ *and* $\partial \hat{\imath} c^{c} / \partial \sigma < 0$*; and (5)* $\partial z / \partial \sigma = \partial \alpha (1-\sigma) \hat{\theta} \hat{\imath} c^{c} / \partial \sigma < 0$.

Proof. See appendix D. ■

For example, an increase in the weight of bond collateral would make the household, who spends on credit associated with the gross settlement, economize cash too much. The household would secure some cash for consumption-good purchases. Hence, a reduction of the weight of cash collateral from an increase in σ may decrease both bank's reserves and investment in government nominal bonds. Unlike Ireland (1994) and Choi (2011), this collateral policy action not only adds the cost of the gross settlement but also increases currency in circulation, which results in two opposite effects. One is a positive effect from having a larger amount of currency in circulation as the means of payments. Consumption with cash increases. The other is a negative effect from substituting the means of payments from credit into cash that would distort the marginal rate of substitution between consumption with cash and credit in (12). Cash is used for purchasing a greater variety of consumption good across markets i. Consumption with credit from markets with the use of credit would increase to satisfy (12). Further, given the positive interest rate of nominal bonds, a decrease in bank's investment reduces the overall return. Hence, both aggregate consumption with credit $(\hat{\imath} c^c)$ and the gross-settlement share decrease.

Hence, it is optimal for the government to balance out the tradeoff between the marginal benefit and cost of holding cash as the means of payments arising from the management of σ. Under the condition of (18), the effect of σ on social welfare in (22) is given by

$$\frac{\partial W}{\partial \sigma} = 0 = \frac{1}{c^m}\frac{\partial c^m}{\partial \sigma} + \frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\frac{\partial \hat{\imath}}{\partial \sigma} \tag{25}$$

and the optimal rate of bond collateral that maximizes social welfare can be characterized by the Proposition 4.

Proposition 4 *Under the positive nominal interest rate* $(\mu > \beta - 1)$ *with (25), the optimal rate of bond collateral* (σ^*) *is determined by*

$$\sigma^* = 1 - \frac{\gamma}{\alpha y}\left(\frac{\beta}{1+\mu}\right)\left\{\frac{\frac{1+\mu-\beta}{\beta} - \ln(\frac{1+\mu}{\beta})}{[\ln(\frac{1+\mu}{\beta})]^2}\right\}. \tag{26}$$

Proof. See appendix E. ■

While holding the collateral-requirement rate (α) fixed, collateral policy that cuts down the proportion of cash-collateral deposits for the gross settlement allows more cash to be circulated as the means of payments in goods trades. It can smooth out consumption with cash and credit and be welfare improving to some extent. From (11) and (26), the optimal rate of bond collateral (σ^*) is also subject to the nominal interest rate similar to the optimal rate of aggregate collateral (α^*).

VI. Concluding Remarks

We constructs a simple monetary model to shed light on the real effects of multiple types of collateral for the interbank payment system on the choice of cash and credit, banking activity, the choice of the gross and net settlement, and consumption. The gross settlement involves the burden of two types of collateral - cash and one-period government nominal bonds - and the net settlement incurs a system-management cost instead. Money serves not only as the means of payments but as the means of settlements.

In a steady-state equilibrium, with the positive nominal interest rate, a reduction of cash-collateral requirements may increase the amount of cash as the means of payments and consumption with cash. However, it adds up the cost of foregone interest. The optimal collateral policy is to balance out the marginal benefit and cost of holding cash.

Some amount of leeway is given for the extension of the model. For example, in this paper, one-period government debt is the only type of an interest-bearing asset that serves as collateral in the payment system. However, in practice, other financial assets with longer maturities can serve as collateral and have a different degree of pledgeability. An interesting extension of the model would be to incorporate various kinds of nominal assets, for example, short-term government debt and long-term government debt, with different degrees of pledgeability as in Williamson (2016) and to explore the real effects of limited pledgeability on the stability of the payment system, banking activity, and the optimal collateral policy.

Appendix

A. Existence and Uniqueness of $\hat{\imath}$

A.1. Derivation of (19)

Suppose $\hat{\imath}c^c = \phi y$ for $\phi \in (0,1)$. Then, from (16), $(1-\hat{\imath})c^m = (1-\phi)y$ and the following holds

$$\frac{(1-\hat{\imath})c^m}{\hat{\imath}c^c} = \frac{1-\phi}{\phi}. \tag{27}$$

Now, from (12) and (14) $\hat{\imath}c^c = \phi y$,

$$\phi = \frac{\int_0^{\hat{\imath}} \Gamma(i)di}{\alpha(1-\sigma)y}. \tag{28}$$

Insert (12) and (28) into (27),

$$\left(\frac{1}{\hat{\imath}} - 1\right)\frac{\beta}{1+\mu} = \frac{\alpha(1-\sigma)y}{\int_0^{\hat{\imath}} \gamma\Gamma(i)di} - 1, \tag{29}$$

and we get (19) by rearranging terms in (29).

A.2. Existence and Uniqueness of $\hat{\imath}$

From (19), the right-hand side is constant over $\hat{\imath}$. For the left-hand side, first, suppose $\hat{\imath}$ approaches zero. Then, by using the L'Hôpital's rule and the Leibniz integral rule with $\Gamma(0) = 0$, the limit value is zero:

$$\lim_{\hat{\imath}\to 0}\left[\frac{\int_0^{\hat{\imath}} \Gamma(i)di}{\hat{\imath}} + \left(\frac{1+\mu-\beta}{\beta}\right)\int_0^{\hat{\imath}} \Gamma(i)di\right] = \lim_{\hat{\imath}\to 0}\frac{\int_0^{\hat{\imath}} \Gamma(i)di}{\hat{\imath}} = \lim_{\hat{\imath}\to 0}\frac{\Gamma(\hat{\imath})}{1} = 0. \tag{30}$$

Hence, if $\hat{\imath}$ approaches zero, then (30) implies that the left-hand side of (19) is always smaller than the right-hand side. Second, suppose $\hat{\imath}$ approaches one. Then, the limit value is given by

$$\lim_{\hat{\imath}\to 1}\left[\frac{\int_0^{\hat{\imath}} \Gamma(i)di}{\hat{\imath}} + \left(\frac{1+\mu-\beta}{\beta}\right)\int_0^{\hat{\imath}} \Gamma(i)di\right] = \left(\frac{1+\mu}{\beta}\right)\int_0^{1} \Gamma(i)di. \tag{31}$$

By assumption of $\int_0^1 \Gamma(i)di \geq \alpha(1-\sigma)y/\gamma$, the left-hand side of (19) is always not smaller than the right-hand side.

Finally, suppose $X(\hat{\imath}) = \int_0^{\hat{\imath}} \Gamma(i)di$. Then, $X(\hat{\imath})$ is increasing on $\hat{\imath}$ and $X(0) = 0$. By the Leibniz integral rule, $X'(\hat{\imath}) = \Gamma(\hat{\imath}) > 0$ is increasing as well. By the mean value theorem,

$$\frac{\partial[\frac{X(\hat{\imath})}{\hat{\imath}}]}{\partial \hat{\imath}} > 0, \tag{32}$$

and, hence, the left-hand side of (19) is an increasing function on $\hat{\imath}$,

$$\frac{\partial[\frac{\int_0^{\hat{\imath}} \Gamma(i)di}{\hat{\imath}}]}{\partial \hat{\imath}} + \left(\frac{1+\mu-\beta}{\beta}\right)\frac{\partial \int_0^{\hat{\imath}} \Gamma(i)di}{\partial \hat{\imath}} = \frac{\Gamma(\hat{\imath})\hat{\imath} - \int_0^{\hat{\imath}} \Gamma(i)di}{\hat{\imath}^2} + \left(\frac{1+\mu-\beta}{\beta}\right)\Gamma(\hat{\imath}) > 0. \tag{33}$$

Therefore, from (30) - (33), there exists a unique $\hat{\imath} \in (0,1)$ from (19).

B. Proof of Proposition 1

From (20) with $\Gamma(i) = 2i$, the choice of cash and credit $[\hat{\imath} \in (0,1)]$ is determined by

$$\hat{\imath} = \frac{1}{2}\left(\frac{\beta}{1+\mu-\beta}\right)\left\{-1+\left[1+4\left(\frac{1+\mu-\beta}{\beta}\right)\left(\frac{1+\mu}{\beta}\right)\frac{\alpha(1-\sigma)y}{\gamma}\right]^{\frac{1}{2}}\right\}, \tag{34}$$

and the effect of α on $\hat{\imath}$ is positive,

$$\frac{\partial \hat{\imath}}{\partial \alpha} > 0. \tag{35}$$

From (13) and (35), the effect on $\hat{\theta}$ is also positive,

$$\frac{\partial \hat{\theta}}{\partial \alpha} = \left(\frac{1-\hat{\theta}}{\hat{\imath}}\right)\frac{\partial \hat{\imath}}{\partial \alpha} > 0. \tag{36}$$

Next, from (12) and (16),

$$\frac{y}{c^m} = 1 - \hat{\imath} + \hat{\imath}\left(\frac{c^c}{c^m}\right) = 1 - \left(\frac{1+\mu-\beta}{\beta}\right)\hat{\imath}. \tag{37}$$

and the effect of α on c^m given (35) is negative,

$$\frac{\partial c^m}{\partial \alpha} = -\frac{(c^m)^2}{y}\left(\frac{1+\mu-\beta}{\beta}\right)\frac{\partial \hat{\imath}}{\partial \alpha} < 0 , \tag{38}$$

where $\mu > \beta - 1$. Moreover, from (12) and (16),

$$\frac{y}{c^c} = (1-\hat{\imath})\frac{c^m}{c^c} + \hat{\imath} = \frac{\beta}{1+\mu} + \left(\frac{1+\mu-\beta}{1+\mu}\right)\hat{\imath} .$$

and the effect on c^c given (35) is also negative,

$$\frac{\partial c^c}{\partial \alpha} = -\frac{(c^c)^2}{y}\left(\frac{1+\mu-\beta}{1+\mu}\right)\frac{\partial \hat{\imath}}{\partial \alpha} < 0 . \tag{39}$$

Therefore, from (35) and (38) with (16), the effects on $(1-\hat{\imath})c^m$ and $\hat{\imath}c^c$ are given by

$$\frac{\partial (1-\hat{\imath})c^m}{\partial \alpha} < 0 . \tag{40}$$

$$\frac{\partial \hat{\imath} c^c}{\partial \alpha} = -\frac{\partial (1-\hat{\imath})c^m}{\partial \alpha} > 0 . \tag{41}$$

Finally, from (36) and (41), the effect of α on z is positive:

$$\frac{\partial z}{\partial \alpha} = \frac{\partial \alpha (1-\sigma)\hat{\theta}\hat{\imath}c^c}{\partial \alpha} > 0 .$$

C. Proof of Proposition 2

Given the effect of a change in α on social welfare from (23) with (35) and (37), the following holds

$$\begin{aligned} 0 &= \frac{1}{c^m}\frac{\partial c^m}{\partial \alpha} + \frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\frac{\partial \hat{\imath}}{\partial \alpha} \\ &= \left[-\frac{c^m}{y}\left(\frac{1+\mu-\beta}{\beta}\right) - \frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\right]\frac{\partial \hat{\imath}}{\partial \alpha} \end{aligned}$$

$$=\left\{-\frac{1}{[1+(\frac{1+\mu-\beta}{\beta})\hat{\iota}]}\left(\frac{1+\mu-\beta}{\beta}\right)+\frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\right\}\frac{\partial\hat{\iota}}{\partial\alpha} \tag{42}$$

and in optimum, the following must hold

$$\frac{1+\mu-\beta}{\beta}=\frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\left[1+\left(\frac{1+\mu-\beta}{\beta}\right)\hat{\iota}\right]. \tag{43}$$

From (43), multiply $\hat{\iota}$ on both sides and rearrange terms given (20):

$$\begin{aligned}\frac{2\hat{\iota}(\frac{1+\mu-\beta}{\beta})}{\ln(\frac{1+\mu}{\beta})}&=\hat{\iota}+\left(\frac{1+\mu-\beta}{\beta}\right)\hat{\iota}^2\\&=\left(\frac{1+\mu}{\beta}\right)\frac{\alpha(1-\sigma)y}{\gamma}.\end{aligned} \tag{44}$$

Therefore, from (34) and (44) given σ, the optimal α^* is determined by

$$\alpha^*=\frac{\gamma}{(1-\sigma)y}\left(\frac{\beta}{1+\mu}\right)\left\{\frac{\frac{1+\mu-\beta}{\beta}-\ln(\frac{1+\mu}{\beta})}{[\ln(\frac{1+\mu)}{\beta})]^2}\right\}.$$

D. Proof of Proposition 3

Given $\Gamma(i)=2i$, from (34), the effect of σ on $\hat{\iota}$ is negative,

$$\frac{\partial\hat{\iota}}{\partial\sigma}<0. \tag{45}$$

From (13) and (35), the effect on $\hat{\theta}$ is also negative,

$$\frac{\partial\hat{\theta}}{\partial\sigma}=\left(\frac{1-\hat{\theta}}{\hat{\iota}}\right)\frac{\partial\hat{\iota}}{\partial\sigma}<0. \tag{46}$$

Next, from (12) and (16),

$$\frac{y}{c^m}=1-\hat{\iota}+\hat{\iota}\left(\frac{c^c}{c^m}\right)=1+\left(\frac{1+\mu-\beta}{\beta}\right)\hat{\iota}. \tag{47}$$

and the effect of σ on c^m given (45) is positive,

$$\frac{\partial c^m}{\partial \sigma} = -\frac{(c^m)^2}{y}\left(\frac{1+\mu-\beta}{\beta}\right)\frac{\partial \hat{\imath}}{\partial \sigma} > 0\,, \tag{48}$$

where $\mu > \beta - 1$. Moreover, from (12) and (16),

$$\frac{y}{c^c} = (1-\hat{\imath})\frac{c^m}{c^c} + \hat{\imath} = \frac{\beta}{1+\mu} + \left(\frac{1+\mu-\beta}{1+\mu}\right)\hat{\imath}\,.$$

and the effect on c^c given (45) is also positive,

$$\frac{\partial c^c}{\partial \sigma} = -\frac{(c^c)^2}{y}\left(\frac{1+\mu-\beta}{1+\mu}\right)\frac{\partial \hat{\imath}}{\partial \sigma} > 0\,. \tag{49}$$

Therefore, from (45) and (48) with (16), the effects on $(1-\hat{\imath})c^m$ and $\hat{\imath}c^c$ are given by

$$\frac{\partial (1-\hat{\imath})c^m}{\partial \sigma} > 0\,. \tag{50}$$

$$\frac{\partial \hat{\imath}c^c}{\partial \sigma} = -\frac{\partial (1-\hat{\imath})c^m}{\partial \sigma} < 0\,. \tag{51}$$

Finally, from (46) and (51), the effect of σ on z is negative:

$$\frac{\partial z}{\partial \sigma} = \frac{\alpha(1-\sigma)\partial\hat{\theta}\hat{\imath}c^c}{\partial \sigma} < 0\,.$$

E. Proof of Proposition 4

The effect of a change in σ on social welfare from (23) with (45) and (47), the following holds

$$\begin{aligned} 0 &= \frac{1}{c^m}\frac{\partial c^m}{\partial \sigma} + \frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\frac{\partial \hat{\imath}}{\partial \sigma} \\ &= \left[-\frac{c^m}{y}\left(\frac{1+\mu-\beta}{\beta}\right) - \frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\right]\frac{\partial \hat{\imath}}{\partial \sigma} \end{aligned}$$

$$=\left\{-\frac{1}{[1+(\frac{1+\mu-\beta}{\beta})\hat{\imath}]}\left(\frac{1+\mu-\beta}{\beta}\right)+\frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\right\}\frac{\partial\hat{\imath}}{\partial\sigma} \tag{52}$$

and in optimum, the following must hold

$$\frac{1+\mu-\beta}{\beta}=\frac{1}{2}\ln\left(\frac{1+\mu}{\beta}\right)\left[1+\left(\frac{1+\mu-\beta}{\beta}\right)\hat{\imath}\right]. \tag{53}$$

From (53), multiply $\hat{\imath}$ on both sides and rearrange terms given (20):

$$\frac{2\hat{\imath}(\frac{1+\mu-\beta}{\beta})}{\ln(\frac{1+\mu}{\beta})}=\hat{\imath}+\left(\frac{1+\mu-\beta}{\beta}\right)\hat{\imath}^2$$
$$=\left(\frac{1+\mu}{\beta}\right)\frac{\alpha(1-\sigma)y}{\gamma}. \tag{54}$$

Therefore, from (34) and (54) given α, the optimal σ^* is determined by

$$\sigma^*=1-\frac{\gamma}{\alpha y}\left(\frac{\beta}{1+\mu}\right)\left\{\frac{\frac{1+\mu-\beta}{\beta}-\ln(\frac{1+\mu}{\beta})}{[\ln(\frac{1+\mu)}{\beta})]^2}\right\}.$$

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지급결제시스템, 담보의 종류, 은행, 그리고 담보정책*

최 형 선**

초 록 본 논문은 화폐모형을 통하여 결제시스템의 안정성을 위한 다양한 담보준비기준이 소비자의 지급결제수단의 선택, 은행활동 및 소비에 대한 실질효과를 분석하고 있다. 화폐는 소비재 구매를 위한 교환의 매개 뿐만 아니라 결제의 수단으로서의 기능을 경제에 제공하고 있다. 정상균형에서 명목이자율이 양의 값을 가질 때 담보정책은 사회후생을 향상시킬 수 있다. 예를 들어 지급준비율로 불리는 현금담보율이 감소할 경우 교환의 매개로서의 화폐량이 증가함으로써 현금소비를 촉진할 수 있다. 그러나 양의 명목이자율에 따라 추가적인 현금보유로 기회비용이 발생하게 된다. 따라서 최적담보정책은 현금보유의 한계편익 및 비용이 균형을 이루는 수준에서 결정된다.

핵심 주제어: 지급결제시스템, 담보, 지급준비금, 은행, 담보정책
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** 경희대학교 경제학과 교수, e-mail: hyungsunchoi@khu.ac.kr