

## AVERTING BEHAVIOR BY VICTIMS IN THE PRESENCE OF PUBLIC GOOD CHARACTERISTICS \*

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*This paper is basically the extension of Oates' model. It is argued that Pigouvian tax alone cannot correct the common forms of externalities with alleviation behaviors in the presence of public good characteristics. In our model, subsidy would be necessary to achieve Pareto-optimality. On the other hand, types of averting behaviors would depend upon alleviation technology, which may classify them as three types: (a) pure private activity; (b) pure public activity; (c) impure public activity. Taking type (b), this paper examines the influence of risk attitudes on the provision of externality with averting behavior and on the proper size of a Pigouvian correction when the externality is a random variable whose distribution is affected by averting behaviors. Then we conclude that an increase in risk aversion could lead to an increase in the level of externality for some plausible conditions. The influence of risk attitudes on the provision of averting behavior will depend on the level of emission (ambient risk) individuals face, which leads to two types of averting behavior either reducing variance of consumption externality (risk-reducing behavior) or increasing variance of consumption externality (risky behavior). The effect of uncertainty on the provision of averting behavior is also examined and it is found that the provision of averting behavior is higher than if individuals were indifferent to risk.*

### I. INTRODUCTION

It is noteworthy that the Pigouvian remedy requires an extra-market inducement solely for the generator of the externality, not for the victims. In this respect, Coase[1960] has argued that decisions concerning the levels of alleviation activity by victims have cost implications for the source of the externality, and these costs do not appear to enter into the victim's decision making calculus.<sup>1</sup>

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<sup>1</sup> For more details, see R. H. Coase(1960, pp. 39-42).

However, the existing literature claims that the explicit introduction of alleviation activity does not invalidate the Pigouvian prescription while it does influence the level of the tax (e.g., Oates[1983]).<sup>2)</sup> We thus seem to have a paradox.

In the presence of bargaining between sources and victims, an efficient outcome can be realized without taxation of either party. As Coase[1960] has argued, depending on the definition of property rights, either the victim will pay the source to engage in the efficient level of abatement activity or the source will pay the victim to allow the discharge of the efficient level of emissions. However, the Coasian outcome depends on the absence both of significant transaction costs and of strategic behavior among the parties. If transaction costs are sufficiently high to prevent voluntary bargains between sources and victims, the victims will not internalize the full range of costs relevant to their decisions on alleviation measures. For this reason, in the absence of bargaining, Coase has recommended a tax on victims that induces the additional averting behavior needed to incorporate the interests of the source of the externality.

In a competitive framework and in the absence of Coasian bargaining, however, the introduction of averting behavior does not alter the basic Pigouvian prescription while it does influence the level of the tax as Oates[1983] has argued. Rationale for these results is quite straightforward. Since alleviation action produces benefits which are external to its source, an inducement in the form of a Pigouvian tax is required to internalize the benefits. In his model averting behavior is a purely private activity: the benefits accrue solely to the particular victim who takes averting behavior. Based on that assumption, he recommends that a Pigouvian tax on the source is all that is required to sustain an efficient pattern of alleviation activity.

It seems to me that if transaction costs are high and one's alleviation behavior affects others' utility levels, neither taxing victims nor freeing them is correct. In our model, subsidy would be necessary to achieve Pareto-optimality. There can be instances where averting behavior has public-good characteristics. Consider the example: a person who picks up trash in a local park benefits other users of the park as well as himself.<sup>3)</sup> In this case alleviation activity might, in general, be suboptimal as a result of the usual public goods and free rider problems. But in most common case alleviation behavior may take the form of a purely private good-individual specific activity. The examples are enormous: clean or repaint exterior of house; install air purifiers or air conditioner in response to air pollution etc.

Another point we can make is that even if uncertainty is involved, we may

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<sup>2)</sup> See Butler and Maher(1981), Mishan(1974), and Oates(1983) for the same conclusion.

<sup>3)</sup> Another example would be that a person who mitigates Gypsy moth infestation might benefit others who own private land etc. See P. M. Jakus(1992) for discussion and details.

obtain similar results to the certainty case.<sup>4</sup> However, uncertainty might affect the level of averting activity and size of Pigouvian taxes. So we may add risk into the provision of averting behavior, thereby examining the counterparts of certainty case for uncertainty case. This will have policy implication for public relief programs and environmental tax schemes.

The present paper will take Oates' model[1983] with the interaction of averting activities. This paper will relax, however, the assumptions that averting behavior is purely private activity, and that there is no risk on the provision of alleviation behavior. It finds that, when the victims take averting actions with affecting one another, the tax scheme is more complex than those general principles suggest, and renders many of the generally accepted propositions on environmental policies applicable only to exceptional cases. It also finds that alleviation behavior with impure public good characteristics should involve its more provision than that with pure public good characteristics, thereby requiring more Pigouvian subsidy for some plausible cases. Moreover, it might be shown that, when the victims have some risk on the provision of averting actions with affecting one another, the Pigouvian correction might go either way in size, depending upon the risk attitudes where uncertainty is involved in one of final level of consumption generated by the provision of alleviation behavior, which is consumed by all the individuals. It can be easily shown that Oates' model is a special case of this.<sup>5</sup> Furthermore when it is involved in uncertainty of externality with averting activity, the introduction of increased risk aversion might not automatically lead to clear-cut conclusions. So we might have some set of possibility of influences that increased risk aversion has on the size of Pigouvian tax/subsidy, not on the direction of that. This paper finds interestingly and positively that an increase in risk aversion could lead to an increase in the provision of averting behavior, thereby requiring more Pigouvian subsidy for some plausible conditions which depend upon the ambient risk level of emissions the individuals face. It also finds that in comparison with certainty cases, the provision of alleviation behavior is higher under risk aversion than if individuals were indifferent to risk.

In section II, this paper will set up a fairly general model based on Oates' model. After specifying the model under certainty, it will find the conditions necessary for optimal alleviation behaviors. In section III, we shall observe consider-

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<sup>4</sup> Uncertainty in this paper is related to risk with given probability. This narrow concept of uncertainty will be used interchangeably with risk. Thus, an increase in uncertainty or risk is defined as a mean preserving spread in the probability distribution over possible outcomes. I wish to thank an anonymous referee in this matter.

<sup>5</sup> As we will see later, this paper will generalize a model of alleviation behavior by using general alleviation production technology, in which Oates' model is appropriate when alleviation technology is purely private. Furthermore this paper introduces externality uncertainty, thereby showing that Oates' model is appropriate only when alleviation technology is purely private and completely certain.

able variances in the optimal conditions according to the type of alleviation technology (or alleviation cost function). In section IV, we will derive some results for the effects of alleviation technologies on the provision of an externality with averting activity in the presence of public good characteristics. In section V, we shall derive some results for the effects of risk attitudes on the provision of the externality with averting activity in the presence of public good characteristics, and discuss reformulation of some propositions concerning averting behavior and risk attitudes (in implementing Pigouvian remedy) based on the findings of this paper in section VI. In section VII, we compare the uncertainty case with the certainty case and it is followed by brief concluding remarks.

## II. THE MODEL UNDER CERTAINTY

The analysis begins with Oates' model in which the activity of producing certain goods generates an external diseconomy on individuals in the system. However, this paper modifies Oates' model at two points: (1) It will consider one good economy consisting of two individuals and one firm for the mathematical convenience and simplicity. Production of a good ( $X$ ) generates a by-product ( $S$ ), the emission of which adversely affects individual  $A$  and  $B$ . This model is easily extended to many individuals, firms, and inputs with no substantive changes in the results. (2) Individual  $A$ 's alleviation action affects individual  $B$ 's utility level through the reduction of public bad. In other words, Individual  $A$ 's such action has benefit (cost) implication on individual  $B$ .

In addition, it is assumed that there will be no negotiation between individuals and firm. The model is as follows:<sup>6)</sup>

$$(1) \quad U_A = U_A(X_A, Z_A)$$

$$(2) \quad U_B = U_B(X_B, Z_B)$$

$$(3) \quad X = X(L_X, S)$$

$$(4) \quad Z_A = Z_A(S, L_A, L_B)$$

$$(5) \quad Z_B = Z_B(S, L_A, L_B)$$

$$(6) \quad L_O = L_X + L_A + L_B$$

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<sup>6</sup> Throughout the paper, subscripts of letter denote the individuals and the firm in question; superscripts of naught and one denote a pre-change variable and a post-change variable in question, respectively.

$$(7) \quad X = X_A + X_B$$

Equations (1) and (2) are the utility functions for individual *A* and *B*, utility is positively related to the individual consumption of one good, *X*, and negatively related to the individual's exposure to emission, *Z*. The sources of externality appear as a factor input in equation (3), the production function for good *X*; the other factor, a composite factor input, is *L<sub>x</sub>* (call it labor). Abatement activity takes the form of reducing *S* (call it emission) either through the increased use of *L<sub>x</sub>* (input) or lower levels of *X* (output). The extension of Oates' model to incorporate the interactions of averting activities is embodied in Eqs. (4) and (5). *Z<sub>A</sub>* and *Z<sub>B</sub>* are actual level of emissions experienced by *A* and *B*, depending on *S*, the amount of *L* which the other employs to avert public bad(*S*), and the amount of *L* which each individual chooses to employ to mitigate the effects of *S*. Depending on *L<sub>A</sub>* and *L<sub>B</sub>*, *Z<sub>A</sub>* can be greater, equal to or less than *Z<sub>B</sub>*. Eqs. (6) and (7) are adding up constraint (resource constraint).

To determine the Pareto-optimal conditions, we set up the Lagrangian:

$$(8) \quad V = U_A(X_A, Z_A) + \lambda[U_B(X_B, Z_B) - U^0] + \mu[X_A + X_B - X(L_x, S)] \\ + \omega[L_0 - L_x - L_A - L_B]$$

where *U<sup>0</sup>* is *B*'s benchmark level of utility. The maximization yields the following necessary conditions:

$$(9) \quad \frac{\partial X}{\partial S} = -\left( -\frac{\frac{\partial U_A}{\partial Z_A} \frac{\partial Z_A}{\partial S}}{\frac{\partial U_A}{\partial X_A}} + \frac{\frac{\partial U_B}{\partial Z_B} \frac{\partial Z_B}{\partial S}}{\frac{\partial U_B}{\partial X_B}} \right)$$

$$(10) \quad \frac{\partial X}{\partial L_x} = \frac{\frac{\partial U_A}{\partial Z_A} \frac{\partial Z_A}{\partial L_x}}{\frac{\partial U_A}{\partial X_A}} + \frac{\frac{\partial U_B}{\partial Z_B} \frac{\partial Z_B}{\partial L_x}}{\frac{\partial U_B}{\partial X_B}} = \frac{\frac{\partial U_A}{\partial Z_A} \frac{\partial Z_A}{\partial L_x}}{\frac{\partial U_A}{\partial X_A}} + \frac{\frac{\partial U_B}{\partial Z_B} \frac{\partial Z_B}{\partial L_x}}{\frac{\partial U_B}{\partial X_B}}$$

Equation (9) is the familiar result for the efficient level of abatement activity. This states that the marginal product of emissions in production of *X* must be equal to the sum of the marginal rate of substitutions of the individuals. Alternatively, emissions should be at a level such that the marginal social damage of a unit of emissions equals the marginal product of emissions in the production of good *X*.<sup>7</sup> Equation (10) indicates that each individual should engage in the alleviation of emissions to the point where the marginal loss in utility from sacrific-

<sup>7</sup> Note that the marginal product of emissions can be interpreted as marginal abatement cost since the cost at the margin of reducing emissions is simply the value of the forgone output.

ing the output which that unit of labor could have produced just equals the marginal gains in utility of individual *A* and *B* from using another unit of labor for alleviation of individual *A* or *B*. Alternatively, the marginal product of labor should be equal to the sum of *A*'s marginal rate of substitution of *X* for *L* and *B*'s marginal substitution of *X* for *L*.<sup>8)</sup> This is similar to the Samuelsonian condition in the public choice theory of public finance literature. As a result of that, it would make the policy implementation more complicated.

Now consider a competitive market. To find competitive market equilibrium conditions, we can set the Lagrangian for individuals and firm. Individual *A* will maximize utility subject to a budget constraint giving rise to the following Lagrangian:

$$(11) \quad V_A = U_A(X_A, Z_A(S, L_A, L_B)) + \lambda_A[M - X_A - P_L L_A]$$

where  $P_X = 1$  as *X* is a numeraire good, and  $P_L$  is the given price of labor, and  $M$  is the person's given level of income. The maximization yields the following conditions:

$$(12) \quad \frac{\partial V_A}{\partial L_A} = \frac{\partial U_A}{\partial Z_A} \frac{\partial Z_A}{\partial L_A} - \lambda_A P_L = 0$$

$$(13) \quad \frac{\partial V_A}{\partial X_A} = \frac{\partial U_A}{\partial X_A} - \lambda_A = 0$$

Combination of Eqs. (12) and (13) yields:

$$(14) \quad P_L \frac{\partial U_A}{\partial X_A} = \frac{\partial U_A}{\partial Z_A} \frac{\partial Z_A}{\partial L_A}$$

On the other hand, firm maximizes profit subject to a technology constraint giving the following Lagrangian:

$$(15) \quad \Pi = X(S, L_X) - P_L L_X$$

Finding the stationary value of (15) yields:

$$(16) \quad \frac{\partial X}{\partial L_X} = P_L$$

From (14) and (16), we obtain:

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<sup>8)</sup> The marginal product of labor can be thought of as the marginal cost of alleviation or the marginal social defense cost. For the similar argument, see footnote (7).

$$(17) \quad \frac{\partial U_A}{\partial X_A} \frac{\partial X}{\partial L_X} = \frac{\partial U_A}{\partial Z_A} \frac{\partial Z_A}{\partial L_A}$$

Equation (17) states that the value in production of  $L_X$  is equal to the private value in averting behavior. This condition for a competitive market equilibrium does not coincide with Pareto-optimal condition (10) for efficient behavior. It is easy to see from (10) and (17) that a subsidy schedule of  $(\partial U_B/\partial Z_B)(\partial Z_B/\partial L_A)/(\partial U_B/\partial X_B)$  on each unit of labor for alleviation of individual A induces individual A to achieve Pareto-optimal condition (10). On the other hand, an emission tax equal to the marginal social damage of a unit of emissions ( $=\partial X/\partial S$ ) induces the firm to achieve Pareto-optimal condition (9).

The rationale for this result is quite simple. Since abatement activity produces benefits which are external to the firm, an inducement in the form of a Pigouvian tax is required to internalize the benefits. Similarly, alleviation activity by the victim who undertakes to reduce the amount of emissions benefits the other, which is external to the one. Consequently, the utility-maximizing individual will not extend alleviation activity to the point where marginal benefits equal marginal cost from society's viewpoint. A Pigouvian tax on the firm alone is, therefore, an insufficient instrument to sustain an efficient pattern of both abatement and alleviation activity contrary to Oates' model.

### III. TYPE OF AVERTING BEHAVIORS DEPENDING ON ALLEVIATION PRODUCTION TECHNOLOGY

In this section, we shall classify various averting behaviors depending on alleviation production technology. The fact that averting activity is possible means that the quantity of emissions ( $S$ ) entering into the utility (or cost) function, denoted by  $Z_A$ , can be made smaller than the quantity that would have entered if the victims had remained completely passive, denoted by  $S$ . In addition, the fact that averting activity will benefit others implies that the quantity of emissions removed by one victim ( $A$ ) can have cost (benefit) implication on the production of  $Z_B$  by the other victim (say,  $B$ ).

We now examine the implication of a production relationship between averting activity ( $L_A$ ,  $L_B$ ) and the level of effective emissions ( $Z_A$ ,  $Z_B$ ). Thus far we have not assumed that production technology of emissions has some functional form. So the production technology has taken a fairly general form:

$$(18) \quad Z_A = Z_A(S, L_A, L_B)$$

If  $S$  is assumed as the quantity of emission when the victims remain completely passive and  $Z_A$  is the level of  $S$  after removing some amount of emissions throu-

gh alleviation activity, we might rewrite Eq. (18) as:<sup>9)</sup>

$$(19) \quad Z_A = S - K(S, L_A, L_B)L_A$$

where  $K(S, L_A, L_B)$  might be interpreted as a variable unit cost in real terms depending upon  $S$ ,  $L_A$ , and  $L_B$ . The  $K(\cdot)$  may be called congestion function as in the Public Finance literature. However, in this context it would rather be called alleviation function because  $S$  will be removed effectively as  $K(\cdot)$  becomes higher. The production technology,  $Z_A$ , may be classified into several types according to its alleviation functions: those whose alleviation function depends upon (a) the quantity of emissions; or (b) the scale of alleviation activity; or, most common, (c) both.

$$(Type 1) \quad K(S, L_A, L_B)L_A = KL_A$$

where  $K$  is a constant. This type is the simplest polar case. Practical examples of cases yielding this type are probably many, for the averting activity in this case takes the form of pure private good. In this type, the optimality condition will be:

$$(20) \quad \frac{\partial X}{\partial L_x} = - \frac{\frac{\partial U_A}{\partial Z_A} K}{\frac{\partial U_A}{\partial X_A}} = - \frac{\frac{\partial U_B}{\partial Z_B} K}{\frac{\partial U_B}{\partial X_B}}$$

Since alleviation activity in this type is a purely private activity, the utility-maximizing individual will extend alleviation activity to the point where marginal benefits equal marginal cost from society's vantage point. A Pigouvian tax on the firm is, therefore, all that is required for Pareto-optimality while alleviation behavior does influence the level of the tax, as Oates has argued. Another interesting point is that we may use interchangeably "averting activity", "environmental quality", and "expenditure" in this case, since we can express the above as  $Q_A = S - KL_A$  where  $Q$  indicates the quality of environment given  $S$ .<sup>10)</sup>

<sup>9</sup> In usual thought, the cost of alleviation activity could be written as follows:  $L_A = E(S - Z_A)$  where  $E$  is real expenditure of alleviation activity. Noting that  $Z_A$  depends upon  $S$ ,  $L_A$ , and  $L_B$  and inverting the above expression, we obtain:  $Z_A = S - E^{-1}(L_A)$ . Then letting  $E^{-1}$  to  $K$  and considering  $K$  dependent on  $S$ ,  $L_A$ , and  $L_B$ , we obtain the final form same as Eq. (19).

<sup>10</sup> In this case, we might consider the quality as a purchased good. As averting activity is increasing, the quality of environment  $Q$  will increase. In fact,  $Q$  is an effectively removed public bad  $S$  so that it is implicitly assumed one to one correspondence between  $Q$  and  $S$  (actually  $Q = -Z$ , but I will not care about it).  $Q$  and  $L$  are assumed to have one to one correspondence in this as well. In section IV, we will take this interpretation in order to simplify the problem when uncertainty is introduced. For details, see P. M. Jakus(1992) in the application to pest control.



(Type 2)  $K(S, L_A, L_B)L_A = K(S)[L_A + L_B]$ <sup>11</sup>

In contrast to the type (1) function, type (2) introduces possibilities of use of either the alleviation activity of individual *A* or the alleviation activity of individual *B* and *K* depending on *S*. In this type, individuals do not care about whether *A* undertakes averting activity or *B* does it. So, averting activity is a pure public good in this type. The optimality condition will be:

$$(21) \quad \frac{\partial X}{\partial L_x} = -K \left( \frac{\frac{\partial U_A}{\partial Z_A}}{\frac{\partial U_A}{\partial X_A}} + \frac{\frac{\partial U_B}{\partial Z_B}}{\frac{\partial U_B}{\partial X_B}} \right).$$

This condition states that the utility-maximizing individual will not extend averting activity to the Pareto-optimal point, so that the competitive market solution is suboptimal as usual in the public good provisions. In the quality interpretation, utility function might be written as  $U_A(X_A, Q_A + Q_B)$  since  $Z_A = S - K(L_A + L_B)$  and  $S - KL_A = Q_A$  &  $S - KL_B = Q_B$ . Another similar type in this class could be that  $Z_A = C(S, L_A + L_B)$  where  $C(\cdot)$  is nonlinear. This type has the same optimality condition as above except that  $\partial C/\partial L$  is placed instead of *K*.

(Type 3)  $K(S, L_A, L_B)L_A = K(S, L_A + L_B)L_A$

In the alleviation function (*K*), it does not matter whether  $L_A$  is employed or  $L_B$  is employed. However, in the production technology ( $Z_A$ ),  $L_A$  does matter. Thus this type seems to fit the impure public good (mixed, joint good). The practical examples are not scarce. Gypsy moth infestation control might be an appropriate example as Jakus [1992] takes. In quality interpretation, noting that given  $L_A, Z_A$  depends upon  $L_A + L_B$  and given  $L_A + L_B, Z_A$  depends upon  $L_A$ , we might write utility function as  $U(X_A, Q_A, Q_A + Q_B)$  in this type.

Consider first the characterization of an optimal allocation of resources. The optimal condition equivalent to (20) and (21) will be:

$$(22) \quad \frac{\partial X}{\partial L_x} = - \left( \frac{\frac{\partial U_A}{\partial Z_A} \frac{\partial K}{\partial L}}{\frac{\partial U_A}{\partial X_A} \frac{\partial L}}{\partial X_A}} L_A + \frac{\frac{\partial U_B}{\partial Z_B} \frac{\partial K}{\partial L}}{\frac{\partial U_B}{\partial X_B} \frac{\partial L}}{\partial X_B}} L_B + K \frac{\frac{\partial U_A}{\partial Z_A}}{\frac{\partial U_A}{\partial X_A}} \right)$$

$$= - \left( \frac{\frac{\partial U_A}{\partial Z_A} \frac{\partial K}{\partial L}}{\frac{\partial U_A}{\partial X_A} \frac{\partial L}}{\partial X_A}} L_A + \frac{\frac{\partial U_B}{\partial Z_B} \frac{\partial K}{\partial L}}{\frac{\partial U_B}{\partial X_B} \frac{\partial L}}{\partial X_B}} L_B + K \frac{\frac{\partial U_B}{\partial Z_B}}{\frac{\partial U_B}{\partial X_B}} \right)$$

<sup>11</sup> At first sight, this seems to be strange. However, this type of technology can be derived in the following way:  $K(S, L_A, L_B)L_A = K(S)[(L_A + L_B)/L_A]L_A$ , which give rise to the form of type (2).

From this condition, we can see that Mr. *A*'s averting activity has three effects: (a) direct effect on  $Z_A$ ; (b) indirect effect on  $Z_A$  through  $K(\cdot)$ ; (c) indirect effect on  $Z_B$  through  $K(\cdot)$ . We can also know that indirect effect of  $L_A$  on  $Z_A$  is different from that of  $L_A$  on  $Z_B$ . So individuals' averting behaviors are not perfect substitutes even in the indirect effects in this type (3). The simpler case of this type could be that  $Z_A = C(S, L_A, L_A + L_B)$  where  $C(\cdot)$  might be nonlinear. In this type, the optimal condition will be :

$$(23) \quad \frac{\partial X}{\partial L_A} = - \left( \frac{\partial U_A / \partial L}{\partial U_A / \partial X_A} + \frac{\partial U_B / \partial L}{\partial U_B / \partial X_B} + \frac{\partial U_A / \partial L_A}{\partial U_A / \partial X_A} \right) \\ = - \left( \frac{\partial U_A / \partial L}{\partial U_A / \partial X_A} + \frac{\partial U_B / \partial L}{\partial U_B / \partial X_B} + \frac{\partial U_B / \partial L_B}{\partial U_B / \partial X_B} \right)$$

This condition is the familiar one in the club good theory of the Public Finance literature.<sup>12</sup> The first order condition above includes the private good effect of  $L_A$  in addition to the public good effect as  $L_A$  contributes to  $L (= L_A + L_B)$  like Eq. (22).

#### IV. PROVISION OF AVERTING BEHAVIOR AND PIGOUVIAN REMEDY UNDER CERTAINTY

The purpose of this section is to examine the influence that averting behavior with public good characteristics has upon the level of an external economy. Consider our competitive market economy again. For averting behavior with a pure public good characteristic, it is assumed that alleviation technology of  $Z_A$  has the form of type (2) in section III, i.e.,  $Z_A = K(S)[L_A + L_B]$ . Individual *A* maximizes Eq. (11) and the maximization yields the condition:

$$(24) \quad \frac{\partial X}{\partial L_A} = P_t = - \frac{\frac{\partial U_A}{\partial Z_A} K}{\partial U_A / \partial X_A}$$

which is the same as the case of type (1) technology.

For averting behavior with an impure public good characteristic, it is assumed that alleviation technology of  $Z_A$  has the form of type (3) in section III, i.e.,  $Z_A = K(S, L_A + L_B)L_A$ . The individual *A* maximizes Eq. (11) and the maximization yields the condition :

$$(25) \quad \frac{\partial X}{\partial L_A} = P_t = - \left( \frac{\frac{\partial U_A}{\partial Z_A} \frac{\partial K}{\partial L}}{\partial U_A / \partial X_A} L_A + K \frac{\frac{\partial U_A}{\partial Z_A}}{\partial U_A / \partial X_A} \right).$$

<sup>12</sup> See Cornes and Sandler(1986) for the similar conditions.

By using comparative static analysis, we derive the effect of impure public good characteristics on the provision of averting behavior. To do this, we first take the FOC for the case of type (2) and then evaluate it at the solution value to the case of type (3). Using this technique, we plug the solution value for  $L_A$  to type (3) technology into (24) and, after rearranging, derive

$$(26) \quad \left( \frac{\frac{\partial U_A}{\partial Z_A} \frac{\partial K}{\partial L}}{\frac{\partial U_A}{\partial X_A} \frac{\partial L}{\partial X_A}} L_A \right) \leq 0$$

since  $\partial K/\partial L \geq 0$  and  $\partial U_B/\partial Z_B < 0$ . It follows from (26) that the case of type (3) technology should involve the provision of more averting behavior as compared with the case of type (2) technology, even with that of type (1) technology. This result explains partly that free riding is not pervasive even if averting behavior has some public good characteristics because it is not a complete public good.

In the absence of administrative costs, Pigouvian subsidies/taxes is calculated by finding both the competitive market and the Pareto-efficient conditions and then equating the subsidies/taxes to those terms in the Pareto condition which are missing in the competitive market solution. In our model with public good characteristics, a Pigouvian subsidy corrects for suboptimality by subsidizing external benefits on other parties.

For type (2) technology, the Pigouvian subsidy needed to achieve Pareto optimality would be:

$$(27) \quad S_2 = -K \left( \frac{\frac{\partial U_B}{\partial Z_B}}{\frac{\partial U_B}{\partial X_B}} \right).$$

Similarly, for type (3) technology, the Pigouvian subsidy needed to achieve Pareto optimality would be:

$$(28) \quad S_3 = - \left( \frac{\frac{\partial U_B}{\partial Z_B} \frac{\partial K}{\partial L}}{\frac{\partial U_B}{\partial X_B} \frac{\partial L}{\partial X_B}} L_B \right).$$

The difference between the subsidy in (27) and that in (28) gives us an expression to determine the effect that public good characteristics in averting behavior have on the relative sizes of the subsidies. The resulting difference is:

$$(29) \quad S_1 - S_2 = - \left( \frac{\frac{\partial U_B}{\partial Z_B}}{\frac{\partial U_B}{\partial X_B} \frac{\partial L}{\partial X_B}} \left( \frac{\partial K}{\partial L} (S, L) \cdot L_B - K(S) \right) \right)$$

where  $L$  is the sum of  $L_A$  and  $L_B$ . From (29), we see that the sign is not determined unambiguously. Thus, we may need more structure on  $K(S, L_A + L_B)L_A$  in order to determine the sign of (29). If we assume that  $K$  is a type of the Cobb-Douglas function, i.e.,  $K(S, L_A + L_B)L_A = K(S)[L_A + L_B]L_A$ , then we have that  $S_3 - S_2 > 0$  since  $\delta K/\delta L = K(S)$  and averting behavior by individual  $B$  is symmetric to that of individual  $A$  (exerting at least one unit of averting activity by individual  $B$ ). It follows that more subsidy may be needed for averting behavior in impure public good characteristics. The reason for this result may be that since the provision of averting behavior by  $A$  is greater in the presence of impure public good characteristics than in the presence of pure public good characteristics and individual  $B$  wants more of  $L_A$ , it is desirable to subsidize averting behavior with impure public good characteristics more than to do that with pure public good characteristics. However, the sign of (29) seems to be ambiguous in general.

As seen above, the provision of alleviation behavior is really different depending upon the alleviation production technology that individuals use. Next we turn our attention to the uncertainty economy, by which we mean that the provision of averting behavior affects the distribution of the relevant externality,  $Z$ , where  $i = A$  and  $B$ . In this context, the degree of uncertainty is exactly related to the risk attitude towards averting behavior.

## V. PROVISION OF AVERTING BEHAVIOR UNDER UNCERTAINTY

In our model, we have two externalities where the one comes from  $S$  (production externality) and the other comes from  $L_A$  and  $L_B$  (consumption externality). Now we introduce externality uncertainty to ascertain how risk attitudes affect the expected level of externality and the size of Pigouvian subsidy. Like the earlier sections, we pay attention to consumption externality uncertainty where the provision of alleviation activity  $L_A$  by individual  $A$  affects the probability distribution function of final consumption characteristic  $Z_A$  and the externality would arise because  $L_B$ , a choice variable of individual  $B$ , influences the distribution of  $Z_A$ . Individuals can observe one another's activity of  $L$ <sup>13</sup>; however, the corresponding values of  $Z_A$  are not known until after the individual decides his own choice of  $L$ . So individual  $A$  perceives his choice of  $L_A$  as affecting the distribution of  $Z_A$  and  $Z_B$  without changing  $L_B$ .<sup>14</sup>

<sup>13</sup> Recall that  $Z_A = Z_A(S, L_A, L_B) = S - K(S, L_A, L_B)L_A$ .

<sup>14</sup> This quantity-constrained level of the externality-generating activity indicates that a Nash zero conjectural variation is imposed. The introduction of nonzero conjectures would not change the nature of the conclusions, but would surely complicate the results since additional interactions would have to be included. So it would not be attempted in this paper. See Sandler and Sterbenz(1988) for this discussion.

For now, we assume that alleviation technology is taken as type (2).<sup>15</sup> If we take the quality interpretation so that  $L$  enters into utility function implicitly.<sup>16</sup> So the provision of  $L_A$  affects the probability distribution function (p.d.f.) of the final consumption characteristic  $Q = Q_A + Q_B$ , which is consumed by both individuals  $A$  and  $B$ .<sup>17</sup> From Mr.  $B$ 's viewpoint, increase in  $Q$  by  $L_A$  is a consumption externality. Thus the individual  $A$ 's concave, twice continuously differentiable utility function can be written as

$$U_A = U_A(X_A, Q_A + Q_B) = U_A(X_A, Q) \text{ for Mr. } A$$

where  $Q$  is a random variable, whose distribution is dependent upon  $S$  and  $L_i$  where  $i = A, B$ . This sum represents the aggregate contributions of individuals toward an activity which promotes a pure public good,  $Q$ . If we keep the notation  $Z_A = Z_B = Z$ , then  $Z$  stands for public bad,  $-Q$ .

Each individual determines his optimal levels,  $X$  and  $L$  so as to maximize expected utility subject to the quantity-constrained level of the externality generating activity (e.g.,  $L_B$  for individual  $A$ ), and subject to his budget constraint in the similar way to our model with certainty,

$$M_A = X_A + p_L L_A$$

where  $M$  is income,  $p_L$  is the price of a unit of  $L$ , and the numeraire's price ( $p_x$ ) is one.

We assume the following technology of the externality in which increased provisions of  $L_A$  improve the distribution of  $Z = -[Q_A + Q_B]$ : if  $L_A^1 > L_A^0$ , and  $L_B^1 = L_B^0$  then  $Z^1 = Z^0 - e$ , where  $e$  is a nonnegative random variable with positive probability of being strictly greater than zero. This assumption implies that increased provision of  $L_A$  improves the distribution of  $Z$  and also implies that if  $L_A^1 > L_A^0$ , and  $L_B^1 = L_B^0$  then  $G^1(Z)$  "FSD"  $G^0(Z)$  where  $G^1(Z)$  is the c.d.f for  $Z$  when  $L_A^1$  is used, while  $G^0(Z)$  is the c.d.f for  $Z$  when  $L_A^0$  is used.<sup>18</sup> For individual  $B$ , we can assume the similar technology since reciprocal externality is assumed. In our technology assumption, an important parameters ( $L_A, L_B$ ) of the

<sup>15</sup> Even though we might take type (3), we would not gain much intuition from that. In that case we are involved with more complicated mathematical calculations as in the certainty case. For type (1), we do not need any Pigouvian subsidy which is not interesting to us.

<sup>16</sup> For this point see footnote (10). Averting activity generates good environmental quality which is consumed by both individuals in this model.

<sup>17</sup> In the previous section, we have  $Z_A$  for Mr.  $A$ . Now we have  $Q$  since  $Q$  is sort of a public good generated by  $L_A$  and  $L_B$ . So  $Z_A = Z_B = -Q$  where  $Q$  is equal to  $Q_A + Q_B$  from which  $S - KL_A = -Q_A$  and  $S - KL_B = -Q_B$ .

<sup>18</sup> FSD means "First Degree Stochastically Dominates." By definition of FSD, we have that  $\int U(\cdot)g^1(Q)dZ > \int U(\cdot)g^0(Q)dZ$  where  $g(\cdot)$  denotes the p.d.f of  $Q$ .

distribution of the random variable ( $Z$ ) is being optimally chosen and this choice leads to a distribution ( $G(Z)$ ) that first degree stochastically dominates another distribution ( $G'(Z)$ ). Thus the mean of the random variable's distribution is endogenous to the problem.

Without affecting the result, we might assume that utility function has the following quasi-linear form to eliminate income effect:<sup>19</sup>

$$(30) \quad u_A(X_A, Z) = X_A + V_A(Z) \quad \text{for Mr. A}$$

The individual maximizes his expected utility subject to his budget constraint and to the given or observed value of  $L_B$ . That is, individual  $A$  solves the following problem:

$$(31) \quad \max_{L_A} E[M_A - p_L L_A + V_A(Z)]$$

where  $E(\cdot)$  represents the operation of  $\int_0^1 (\cdot) g(Z; L_A, L_B) dZ$  and  $g(\cdot)$  represents the p.d.f of  $Z = -[Q_A + Q_B]$ .<sup>20</sup> This p.d.f has parameters  $L_A$  and  $L_B$  in which  $L_A$  is endogenous and  $L_B$  is observed and taken as given. The FOC for this case (call it uncertainty case 1) is:

$$(32) \quad -p_L - E[KV'_A(\cdot)] + E_{L_A}[X_A + V_A(Z)] = -p_L - E[KV'_A(\cdot)] \\ + E_{L_A}[V_A(Z)] = 0$$

where  $K$  denotes the coefficient of  $L_A$  in the alleviation function,  $E_{L_A} = \int_0^1 [\cdot] g_{L_A}(Z; L_A, L_B) dZ$  and  $g_{L_A} = \frac{\partial g(\cdot)}{\partial L_A}$ . The equality in (32) follows from that  $\int g(\cdot) dZ = 1$  and  $\int g_{L_A}(\cdot) dZ = 0$ . This FOC implies that individual  $A$  chooses  $L_A$  so as to equate the per unit price of  $L$  with the marginal impact on his expected utility that results from the purchase of  $L_A$ ; the second term is the direct utility effect of the level of emission by increased averting activity and the third term is the indirect utility effect of a stochastically dominating change in the distribution of emission by increased averting activity. An analogous FOC holds for individual  $B$ . We further assume that increasing  $L_A$  increases the expectation

<sup>19</sup> See Sandmo(1980) for more discussion.

<sup>20</sup> If we assume that joint p.d.f of  $Q_A$  and  $Q_B$  is  $f(\cdot)$ , we can derive p.d.f of  $Q = Q_A + Q_B$  in terms of  $f(\cdot)$  by using transformation technique as following: for  $Q = Q_A + Q_B$ , let  $Q_B = W = S_1(W)$  and then  $Q_A = Q - Q_B = Q - W = S_2(Q, W)$ . So we can derive Jacobian matrix  $|J| = |\partial(s_1, s_2)/\partial(Q, W)|$ . Using this, we can obtain the p.d.f of  $Q = Q_A + Q_B$ :  $g(Q) = \int f(Q - W, W) dW$ . Thus we may note that  $g(Z; L_A, L_B)$  could be taken as the form of  $h(L_A, L_B)g(Z)$  for example.

of utility, but at a decreasing rate. In other words, the SOC (second order condition) is met when  $\int V_A(Z)g_{L_A L_A}(\cdot)dZ < 0$ . We also assume that  $\int V_B(Z)g_{L_B L_B}(\cdot)dZ < 0$  and  $\int V_B(Z)g_{L_A L_B}(\cdot)dZ < 0$ . This assumption implies that increases in  $L_B$  (or  $L_A$ ) decreases the expected marginal effect of  $L_A$  (or  $L_B$ ) on  $B$ 's utility.

Now we examine the effect that an increase in risk aversion in the Arrow-Pratt sense has on the level of an external economy. To represent the increase in risk aversion, we consider the following concave transformation of Eq. (30).<sup>21</sup>

$$(33) \quad H_A = H[X_A + V_A(Z)] \text{ for Mr. A} \\ = H[u_A]$$

This case refers to an increase in risk aversion and individual  $A$  maximizes the following:

$$(34) \quad \max_{L_A} E[H[M_A - p_A L_A + V_A(Z)]]$$

The FOC for this case [call it uncertainty case 2] is

$$(35) \quad -p_A E[H'(u_A)] - E[H'(u_A)V'_A(Z)K] + E_{L_A}[H(u_A)] = 0$$

The interpretation of Eq.(35) is identical to its counterpart for uncertainty case 1, given in Eq.(32). We also assume that the SOC is satisfied in this case.

Now by using comparative static analysis, we derive the effect of an increase in risk aversion on the provision of averting behavior: We take the FOC for uncertainty case 1 (given in Eq.32) and then take the solution value for  $L_A$  corresponding to the more risk averse case's FOC, given in Eq.(35), and plug it into uncertainty case 1's FOC. In other words, uncertainty case 1's FOC is evaluated at the solution value to uncertainty case 2's (more risk aversion) case. If we could determine the sign of the FOC for uncertainty case 1, we might draw useful results for the effect of the increase in risk aversion. Following this technique, we take the FOC for uncertainty case 1, given in Eq.(32) and rescale the FOC by  $E[H'(\cdot)]$  which is a constant evaluated at case 2's FOC:

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<sup>21</sup> It is not difficult to see why this is the case: using Arrow-Pratt measure of risk aversion, we might show that this transformation function is more risk averse because

$$r_u = -\frac{u''}{u'} \text{ for } u_A \text{ and } r_H = -\left[\frac{H''u' + H'u''}{H'u'}\right] = -\frac{u''}{u'} - \frac{H''}{H'} > r_u \text{ since } H'' < 0$$

where  $r_u$  and  $r_H$  are the Arrow-Pratt measures of risk aversion for  $u_A$  and  $H(u_A)$ , respectively. This will give the justification for our transformation.

$$(35) \quad -p_L E[H'(u_A)] - E[H'(u_A)]E[KV'_A(\cdot)] + E[H'(u_A)]E_{L_A}[V_A(Z)] = 0$$

We then take the solution value for  $L_A$  to uncertainty case 2's FOC, given in Eq. (35), and plug it into Eq.(35'). After rearranging, we can derive the following equation:<sup>22)</sup>

$$(36) \quad E_{L_A}[-H(u_A) + E[H'(u_A)]u_A(Z, X_A)]$$

To determine the sign of Eq.(36), we should know whether  $-H(\cdot) + E[H'(\cdot)]u_A(\cdot)$  is a decreasing function of  $u_A$  or not (call this  $-H(\cdot) + E[H'(\cdot)]u_A(\cdot)$  as  $K(u_A)$ ). Since  $E[H'(\cdot)]$  is the rescaling constant evaluated at uncertainty case 2's solution, differentiation of  $K(u_A)$  with respect to  $u_A$  yields

$$\partial[-H(\cdot) + E[H'(\cdot)]u_A(\cdot)] / \partial u_A = \partial K(u_A) / \partial u_A = -H'(\cdot) + E[H'(\cdot)].$$

If  $H'(\cdot) > E[H'(\cdot)]$ , then  $K(u_A)$  is a decreasing function of  $u_A$  and vice versa. In other words, if the entire improvement in the distribution of  $Z$  which results from increasing  $L_A$  occurs where  $H'(\cdot) > E[H'(\cdot)]$ , then  $K(u_A)$  is a decreasing function of  $u_A$ . Since increasing  $L_A$  leads to a new distribution of  $u_A(\cdot)$  that first degree stochastically dominates the old distribution, we have that:

$$E_{L_A}[-H(u_A) + E[H'(u_A)]u_A(Z, X_A)] < 0.$$

In this situation we can conclude that the more risk averse solution must involve provision of more  $L_A$  as compared with the less risk averse solution. That is simply because more provision of averting activity gives higher  $u_A$  by FSD and  $K(\cdot)$  is a decreasing function of  $u_A$ .

The intuition behind this result involves the way in which changes in  $L_A$  affect the degree of uncertainty. In the situation where  $H'(\cdot) > E[H'(\cdot)]$ , all of the improvement in the distribution concern high points of  $Z$  (low points of utility) on the distribution which are moved to the low points by improvement.<sup>23)</sup> Such an improvement reduces the variance and the degree of uncertainty. The increase in  $L_A$  may be called risk-reducing averting behavior in this case. Since this situation involves high points of  $Z$ , it is appropriate for the case of the severe emission, i.e., the case that a level of exogenous ambient risk is high.

<sup>22</sup> From uncertainty case 2's FOC, we know that  $-pE[H'(\cdot)] - E[H'V'_A K] = -E_{L_A}[H(u_A)]$ . Then replacing  $-pE[H'(\cdot)] - E[H'V'_A K]$  by  $-E_{L_A}[H(u_A)]$  in uncertainty case 1's FOC will give the desired result.

<sup>23</sup> In this case we have similar lines of reasoning in production theory. If we think of  $H'(\cdot)$  as  $MP$  and  $E[H'(\cdot)]$  as  $AP$ , a firm would increase production in the case that  $MP > AP$ .



If the entire improvement in the distribution of  $Z$  occurs where  $H'(\cdot) < E[H'(\cdot)]$ , then  $I(u_A)$  is an increasing function of  $u_A$  as seen above. In this case, we have that:

$$E_{L_A}[-H(u_A) + E[H(u_A)]u_A(Z, X_A)] > 0.$$

Thus, we can conclude that the more risk averse solution should involve provision of less  $L_A$  as compared with the less risk averse solution because the sign of uncertainty case 1 (less risk aversion case)'s FOC is positive when evaluated at uncertainty case 2's solution. The intuition behind this result is that all of the improvement in the distribution involve low points of  $Z$  (high points of utility) on the distribution which are moved to the still lower points by increase in  $L_A$ .<sup>21</sup> Such an improvement increases the variance and the degree of uncertainty. When risk aversion is then increased, there is less provision of  $L_A$ . In this case the increase in  $L_A$  may be called risky averting behavior. Since this situation involves low points of  $Z$ , it is appropriate for the case of the less severe emission or benign emission, i.e., the case that a level of exogenous ambient risk an individual faces is low.

However, in general an increase in risk aversion leads to no unambiguous results on the provision of averting activity because the sign of  $I(\cdot)$  cannot be determined unambiguously without knowing specific distributional form of  $Z$  and the level of exogenous ambient risk ( $S$ ).

## VI. PIGOUVIAN SUBSIDY UNDER UNCERTAINTY

As shown in section II and III, it needs the Pigouvian subsidy to achieve Pareto optimality even with uncertainty when there are public good characteristics in averting behavior. We now examine the effect of increased risk aversion on the proper level of a Pigouvian subsidy, following the assumption and setting in section V.

We first find the subsidies for uncertainty case 1 and uncertainty case 2. An identical procedure to section II and III can be used for finding the counterparts of the certainty case. To determine the Pareto optimal conditions, we set up the Lagrangian:

$$(37) \quad E[V_A(Z) + X_A] + E[V_B(Z) + X_B] + \mu[X_A + X_B - X(L_A, S)] + \omega[L_O - L_X - L_A - L_B]$$

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<sup>21</sup> This case does not have similar lines of reasoning in production theory because  $MP < AP$  and they are increasing.

After eliminating the Lagrangian multiplier, we derive the following FOC for the provision of  $L$ :

$$(38) \quad \frac{\partial X}{\partial L_A} = \hat{p}_L = -E[V'_A(Z)K] + E_{L_A}[u_A] - E[V'_B(Z)K] + E_{L_A}[u_B]$$

which is its exact counterpart of the certainty case, given in Eq.(10).<sup>25)</sup>

Comparing (37) with its counterpart of individuals' problem, given by (32), we can know that the required subsidy in this case is

$$(39) \quad S^* = -E[V'_B(Z(L_A^*, L_B^*))K] + E_{L_A}[u_B(X_B^*, Z(L_A^*, L_B^*))]$$

where  $[X_B^*, L_A^*, L_B^*]$  denote the Pareto optimal levels for the respective variable in this uncertainty case 1. As seen in section II under the certainty world, the Pigouvian subsidy is equal to the marginal effect that individual  $A$ 's choice of  $L_A$  has on the expected utility level of individual  $B$ . By FSD,  $S^*$  is positive for an external economy.

To see the effects of an increase in risk aversion, we consider the following concave transformation of the respective individuals' utility functions  $[H(\cdot)]$  for individual  $A$  and  $[G(\cdot)]$  for individual  $B$ . In order to derive Pareto optimality condition for this increased risk aversion case, we choose  $X_A$ ,  $X_B$ ,  $L_A$ , and  $L_B$  so as to maximize

$$(40) \quad E[H[V_A(Z) + X_A]] + E[G[V_B(Z) + X_B]] + \mu[X_A + X_B - X(L_A, S)] \\ + \omega[L_0 - L_A - L_B].$$

After eliminating the Lagrangian multipliers, we derive the following FOC:

$$(41) \quad \frac{\partial X}{\partial L_A} = \hat{p}_L = -E[H'(\cdot)]^{-1}E[H'(\cdot)V'_A(Z)K] + E[H'(\cdot)]^{-1}E_{L_A}[H(u_A)] \\ - E[G'(\cdot)]^{-1}E[G'(\cdot)V'_B(Z)K] + E[G'(\cdot)]^{-1}E_{L_A}[G(u_B)]$$

for individual  $A$  and an analogous FOC holds for individual  $B$ .

Comparing Eq.(41) and Eq.(35), we can deduce that the Pigouvian subsidy with  $L_A$  in more risk-averse case is

$$(42) \quad S^{**} = -E[G'(\cdot)]^{-1}E[G'(\cdot)V'_B(Z(L_A^{**}, L_B^{**}))K] \\ + E[G'(\cdot)]^{-1}E_{L_A}[G(u_B(X_B^{**}, Z(L_A^{**}, L_B^{**})))]$$

<sup>25)</sup> Note that, however, we have taken quality interpretation in section II's model. So we actually added more structure but the intuition behind those is the same.

which again reflects the marginal effect of  $L_A$  on individual  $B$ 's expected utility and  $[X_B^{**}, L_A^{**}, L_B^{**}]$  denote the Pareto optimal levels for the respective variable in this more risk averse case. The Pigouvian subsidy is equal to the marginal effect that individual  $A$ 's choice of  $L_A$  has on the expected utility level of individual  $B$ . By FSD,  $S^{**}$  is positive for an external economy as in  $S^*$  since  $G(\cdot)$  is an increasing function of  $L$ .

Comparing Eq.(39) and Eq.(42) by comparative static, we now examine the effect of increased risk aversion on the relative sizes of the subsidies. We take the difference between the subsidy in Eq.(42) ( $S^{**}$ ) and that in Eq.(39) ( $S^*$ ) as follows:

$$(43) \quad S^{**} - S^* = -E[G'(\cdot)]^{-1} E[G'(\cdot) V'_B(Z(L_A^{**}, L_B^{**})) K] \\ + E[G'(\cdot)]^{-1} E_{L_A} [G(u_B(X_B^{**}, Z(L_A^{**}, L_B^{**}))) \\ - E[V'_B(Z(L_A^*, L_B^*)) K] + E_{L_A} [u_B(X_B^*, Z(L_A^*, L_B^*))]$$

If  $L_A^{**} \leq L_A^*$  and  $L_B^{**} \leq L_B^*$ , then we would have the following result:

$$S^{**}(X_B^{**}, L_A^{**}, L_B^{**}) - S^*(X_B^*, L_A^*, L_B^*) \geq S^{**}(X_B^{**}, L_A^*, L_B^{**}) - S^*(X_B^{**}, L_A^*, L_B^*)$$

because  $S^*$  is a decreasing function of  $L_A$  and  $L_B$  by the SOC given in section V between Eqs.(32) and (33), and  $(\cdot)$  indicates the point to be evaluated. Rearranging (43) to yield

$$(44) \quad E[G'(V_B(Z^{**}) + X_B^{**})](S^{**} - S^*) \\ = E_{L_A} [G[V_B(Z^{**}) + X_B^{**}] - G'(V_B(Z^{**}) + X_B^{**})[V_B(Z^{**}) + X_B^{**}]] \\ = E_{L_A} [G(\cdot) - G'(\cdot)u_B] |_{(X_B^*, L_A^*, L_B^*)}$$

where all in the right hand side (RHS) are evaluated at  $[X_B^{**}, L_A^{**}, L_B^{**}]$ . So we can determine the sign of  $(S^{**} - S^*)$  if we can decide the sign of the RHS.<sup>26)</sup> By the same line of logic as in section V [recall the cases for  $I(\cdot) = -H + E[H'(\cdot)]u_A(\cdot)$  and call  $J(u_B)$  as  $G(\cdot) - E[G'(\cdot)]u_B(\cdot)$ ], we can derive the following:

$$\partial\{G(\cdot) - E[G'(\cdot)]u_B(\cdot)\} / \partial u_B = \partial J(u_B) / \partial u_B = G'(\cdot) - E[G'(\cdot)].$$

If  $G'(\cdot) > E[G'(\cdot)]$ , then  $J(u_B)$  is an increasing function of  $u_B$  and vice versa. Using this result, we may determine the sign of the RHS in Eq.(44) by the same line of reasoning as in section V. If the entire improvement in the distribution

<sup>26)</sup> Note that  $E[G'(\cdot)] > 0$  and thus it is sufficient for  $S^{**} - S^* > 0$  that RHS  $> 0$ .

of  $Z$  due to an increase in  $L_A$  involves the value of  $Z$  where  $G'(\cdot) > E[G'(\cdot)]$ , then  $J(u_B)$  is an increasing function of  $u_B$  as seen above. Since  $u_B$  is decreasing in  $Z$ , an increase in  $L_A$  leads to a new distribution of  $u_B$  that first degree stochastically dominates the old distribution. Hence we have that:

$$E[G'(\cdot)](S^{**} - S^*) = E_{L_A}[J(u_B)] > 0.$$

From this we can know that increased risk aversion needs a greater subsidy. In this case the variance is reduced by reasoning in the similar way to the situation as in section V. The economic interpretation behind this would be that by increased risk aversion on individual  $B$ , he wants more of  $L_A$  provided, thus having greater expected marginal effects of the externality and requiring a larger subsidy. However, this result holds only for the case that:

$$(45) \quad L_A^{**} \leq L_A^*, L_B^{**} \leq L_B^*, \text{ and } G'(\cdot) > E[G'(\cdot)].$$

So the scenario to meet this condition may be that if risk averse  $A$  and  $B$  take risky averting activity, thereby providing low level of  $L_A$  and  $L_B$ , and the improvement by  $A$  occurs at high points of  $Z$  on  $B$ , then individual  $B$  will have greater expected marginal effect of the externality, thereby requiring a larger subsidy. If the condition works in other direction, the reverse result holds as in section V. Thus, an increase in risk aversion could lead to an increase in the level of provision and Pigouvian subsidy for such plausible condition as above.

## VI. COMPARISON WITH THE CERTAINTY CASE OF PROVISION OF AVERTING BEHAVIOR

In this section, we examine how the optimal provision of averting activity compares with the solution under certainty. Under certainty, the solution is characterized by equality between price of averting activity and marginal effect of averting activity on utility. Although there is no obvious way to make comparison, we compare the optimal provision of averting activity under uncertainty with the situation where the level of  $Z$  is known to be equal to the expected value of the original distribution.<sup>27</sup> In this paper, the latter level of provision of averting activity is called the certainty provision of averting activity.

The first order condition (24), assuming quasi-linear utility form as in section V, can be written as:

<sup>27</sup> Note that for example, the spraying of insecticides ( $L$ ) affects the distribution of mosquitoes ( $S$ ), thereby reducing number of mosquitoes ( $Z$ ). Randomness of  $Z$  comes from that of  $S$ . Thus  $EZ = ES - K(ES)[L_A + L_B]$ .

$$(46) \quad -p_L - [KV'_A(Z)] = 0.$$

Evaluating (32) at  $E[Z]$ , we get

$$(47) \quad p_L = -[K(ES) \cdot V'_A(EZ)].$$

Recall that the first order condition (32) under uncertainty is

$$(48) \quad -p_L - E[K(\cdot) V'_A(\cdot)] + E_{L_A}[V_A(Z)] = 0.$$

Evaluating (48) at  $E[Z]$  and the solution of (47), we then obtain:

$$(49) \quad K(ES)V'_A(EZ) - E[K(ES)V'_A(EZ)] + E_{L_A}[V_A(EZ)] = E_{L_A}[V_A(EZ)].$$

Since the indirect utility effect of a stochastically dominating change in the distribution of  $Z$  is positive by FSD, Eq.(49) is positive. When the level of  $Z$  is known to be equal to the expected value of the original distribution, the first-order condition (32) under uncertainty evaluated at the optimal provision of averting activity under certainty becomes positive. This implies that the provision of averting behavior under certainty is smaller than that of averting behavior under uncertainty. In other words, if there is uncertainty involved in consumption externality, then the provision of averting behavior is more than that of the certainty case. Thus this result shows that the effect of uncertainty on the decision of averting behavior.

## VIII. CONCLUDING REMARKS

This paper has argued that Pigouvian taxes alone cannot be expected to correct the common forms of externalities with alleviation behaviors. Pigouvian taxes alone are incapable of providing the individuals with an incentive to undertake averting activity having public characteristics as in our model. Laissez-faire policy on the individuals never be the optimal policy in the presence of public good characteristics of averting behavior, unlike prediction of Oates' model. Likewise, Coasian policy to tax victims in order to induce optimal level of averting behaviors would not be optimal, either. In the case of our model, subsidy would be necessary to achieve Pareto optimality.

Type of averting behavior would depend on the alleviation technology, which may classify averting behaviors as three types: (a) purely private averting activity; (b) pure public averting behavior; (c) impure public averting activity. Under certainty, our model shows that impure public good characteristics should plausibly involve more provision of averting activity than pure public good characteristi-

cs and that relative sizes between them are not determined in general except for the Cobb-Douglas alleviation function. Taking the simple form of type (2) class, we have derived in what situations an increase in risk aversion would lead to the victims to provide more or less of the activity giving rise to the externality ( $Z$ ). It might be classified as 2 possible situations: (a) more provision of  $L_A$  in the more risk aversion case, (b) less provision of  $L_A$  in the more risk aversion case. Since the situation (a) involves high points of  $Z$  (low points of utility) on the distribution which are moved to the low points by improvement, it is most likely to occur for the case of the severe emission, i.e., the case that a level of exogenous ambient risk is high. Such an improvement reduces the variance and the degree of uncertainty. The increase in  $L_A$  may be called risk-reducing averting behavior in this case. Since the situation (b) involves low points of  $Z$  (high points of utility) on the distribution which are moved to the still lower points by increase in  $L_A$ , it is most likely to occur for the case of the less severe emission or benign emission, i.e., the case that a level of exogenous ambient risk an individual faces is low. Such an improvement increases the variance and the degree of uncertainty. When risk aversion is then increased, there is less provision of  $L_A$ .

Then we have considered the effect that increased risk aversion has on the size of Pigouvian subsidies. Similarly we have found 2 possible cases, depending upon how changes in Mr.  $A$ 's averting behavior affected the distribution of the externality ( $Z$ ) on Mr.  $B$ : (a) a greater subsidy in the more risk aversion case, (b) a lower subsidy in the more risk aversion case. The case (a) is likely to take place if risk averse  $A$  and  $B$  take risky averting activity, thereby providing low level of  $L_A$  and  $L_B$ , and the improvement by  $A$  occurs at high points of  $Z$  on  $B$ , the individual  $B$  will have greater expected marginal effect of the externality, thereby requiring a larger subsidy. Thus, we might say that unlike the standard result in the theory of the firm, increases in risk aversion would lead to increase in the level of provision and Pigouvian subsidy under plausible conditions.

Once uncertainty has been introduced, the first stochastic dominant property of the provision of averting behavior becomes important in addition to the attitudes toward risk of the individual. Thus the provision of averting activity is higher under uncertainty than under certainty. The more provision of averting behavior is caused by sort of risk premium, i.e., the indirect utility effect of a stochastically dominating change in the distribution of  $Z$  by averting behavior.

Although the simplicity of our model and our usage of the expected utility hypothesis requires our results to be viewed as explanatory, it may give some useful insight to studying the impact of different policy instruments and suggest at least three areas for further research. First, the importance of the technology of averting behavior suggests a need for an empirical examination of the relationship between the provision of averting behavior and the severity of emission. Second, this paper have focused just on consumption externality, assuming away production externality. So this paper did not go into more general model where

there are interactions between consumption externality ( $Q_a$ ,  $Q_b$ ) and production externality ( $X_a$ ,  $X_b$ ). Also this paper did not cover the model where there are interactions among price uncertainty, externality uncertainty and output uncertainty, leaving these topics for the future research.

## REFERENCES

- Bartik, T. J. "Evaluating the Benefits of Non-marginal Reductions in Pollution Using Information on Defensive Expenditures." *Journal of Environmental Economics and Management* 15, 1988, 111-127.
- Butler R. and Maher M. "The Control of Externalities: Abatement Vs. Damage Prevention." Working Draft, 1981.
- Coase, R. H. "The Problem of Social Cost." *The Journal of Law and Economics*, 1960, 1-44.
- Cornes, R., and T. Sandler. *The Theory of Externalities, Public Goods, and Club Goods*. London: Cambridge University Press, 1986.
- Cornes, R., and T. Sandler. "Externalities, Expectation, and Pigouvian Taxes." *Journal of Environmental Economics and Management* 12, 1985, 1-13.
- Jakus, P. M. "Valuing the Private and Public Dimensions of a Mixed Good: an Application to Pest Control." Ph.D Dissertation at NCSU, 1992.
- Mishan. E. J. "What is the Optimal Level of Pollution?" *Journal of Political Economy* 82, 1974, 1278-99.
- Oates, W. "The Regulation of Externalities: Efficient Behavior by Sources and Victims." *Public Finance*, 1983, 362-375.
- Sandler, T and F. P. Sterbenz. "Externalities, Pigouvian Corrections, and Risk Attitudes." *Journal of Environmental Economics and Management* 15, 1988, 488-504.
- Sandmo, A. "Anomaly and Stability in the Theory of Externalities." *Quarterly Journal of Economics* 94, 1980, 799-807.
- Shibata, H., and J. S. Winrich. "Control of Pollution when the Offended Defend Themselves." *Economica* 50, 1983, 425-437.