The Korean Economic Review
Volume 34, Number 2, Summer 2018, 117-129.

Price Discrimination with Loss Averse and Horizontally Differentiated Consumers*

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This paper considers a monopolist seller facing horizontally differentiated consumers, whose preferences are reference-dependent and loss averse in the frame of Kőszegi and Rabin (2006). Our results on optimal menu suggest that consumer loss aversion does not necessarily limit the benefits of screening under the horizontal demand structure, in contrast to the findings of Hahn, Kim, Kim and Lee (2018) and Herweg and Mierendorff (2013) who consider the case of vertically differentiated preferences.

JEL Classification: D03, D42, D82, D86, L11
Keywords: Price Discrimination, Hotelling Preferences, Reference-dependent Preferences, Loss aversion, Personal Equilibrium, Preferred Personal Equilibrium

I. Introduction

When a firm faces consumers with heterogeneous willingness to pay, it can improve profit by offering multiple product types. Several recent papers, notably by Hahn, Kim, Kim and Lee (2018) − henceforth, HKKL − and Herweg and Mierendorff (2013), have however shown that the effectiveness of the practice of

Received: Jan. 11, 2017. Revised: Jan. 3, 2018. Accepted: Feb. 14, 2018.
* This research was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2014S1A5A2A03065638). Jihong Lee acknowledges financial support from Seoul National University Asia Center (#SNUAC-2017-008).
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price discrimination is limited when the consumers are reference-dependent and loss averse. When screening reference-dependent consumers, some types of consumers would have to purchase bundles that diverge from their reference point and as a result experience an additional utility loss. Such consumers would then find extra incentives to deviate from the bundles designed to screen them, thereby tightening the incentive constraints that the profit-maximizing firm needs to satisfy.

The purpose of this paper is to scrutinize the validity of this observation in an alternative model of consumer preferences. In particular, we consider a monopolist seller who faces consumers with the standard Hotelling preferences over horizontally differentiated goods. The basic model is that of HKKL, who introduce the expectation-based reference point model of Kőszegi and Rabin (2006), henceforth referred to as KR, into a standard screening model with vertically differentiated demands for quality.

We show that, with horizontally differentiated preferences, reference-dependence and loss aversion do not necessarily limit the benefits of screening. The optimal menu turns out to be similar to the profit-maximizing menu under standard preferences, exhibiting maximal product differentiation. With the symmetric Hotelling preferences, consumers do not actually experience any gain or loss from choosing their targeted bundles since the reference points of the two types are essentially identical.

The paper is organized as follows. Section 2 describes the model of HKKL with alternative utility structure and the notion of personal equilibrium due to KR. Section 3 presents the main results, followed by some concluding remarks in Section 4. For an in-depth discussion of related literature, we refer the interested reader to HKKL. There are also several surveys on the emerging literature of contracting with behaviorally motivated agents (e.g. Kőszegi, 2014; Kim and Lee, 2014).

II. Model

2.1. Basic Setup

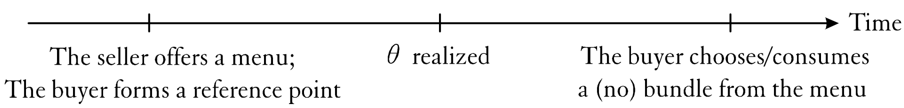
We consider the basic setup introduced by HKKL. A market consists of a monopolistic seller and a buyer. A "bundle" is denoted by $b=(q,t)$, in which the product of characteristic q is sold for the payment of t. Let $\emptyset=(0,0)$ denote the null bundle or outside option.

The buyer's preferences are reference-dependent, consisting of "consumption (or intrinsic) utility" and "gain-loss utility." Type-θ consumer's intrinsic utility for a bundle $b=(q,t)$ is given by

$$m(b;\theta) = \bar{s} - (\theta - q)^2 - t \,, \tag{1}$$

where $q \in [0,1]$ denotes the product position in the Hotelling line.[1] We assume that θ, which represents each type's bliss point, takes binary values, $\theta_L = 0$ and $\theta_H = 1$ with probabilities $p \in (0,1)$ and $1-p$ respectively. We assume that the buyer's utility from the null bundle or outside option is zero, i.e. $m(\emptyset;\theta) = 0$ for each θ. The seller faces the same constant marginal production cost for any product, which is normalized to zero. Additionally, we assume that $\bar{s} > 1$, as this guarantees a positive gross surplus for both types of consumers from any $q \in [0,1]$.

To explain the gain-loss utility, consider the timeline. The seller commits to a menu of bundles, M, which is observed by the buyer *before* his type, θ, is known. Thus, θ can be interpreted as a random utility component that becomes known to the buyer only when the uncertainty, which determines the buyer's willingness to pay, gets resolved around the time of consumption. Having observed the menu, the buyer then anticipates his choice of bundle for each possible realization of his type, and this expectation serves as his reference point when actual consumption takes place *after* the uncertainty becomes resolved. We will later require that this expectation be consistent with the buyer's actual choice; that is, the expectation should be *rational*.



To illustrate our model, consider for example an avid sports fan who tracks multiple sports, say, baseball and football. The focus of the consumer's attention nonetheless goes to the sport in which his favorite team performs best. There is a cable TV company offering a specialized premium channel for each sport. Prior to the beginning of the season, such a consumer may form an expectation about which channel to purchase contingent on the relative strength of his two favorite teams, one in baseball and one in football, during pre-season. But, once the regular season starts, and when he actually makes the purchase, the consumer compares his choice to what he *could have* consumed.

As described in the timeline above, our main analysis presents the case in which the buyer's consumption decision takes place *ex post*. Nonetheless, as in HKKL, the scope of our results extends to the case of *ex ante* participation. Some results under

[1] HKKL adopts the standard vertical differentiation model of Mussa and Rosen (1978) where the intrinsic utility is given by $m(b;\theta) = \theta v(q) - t$ where $v(\cdot)$ is an increasing and concave function. q in HKKL denotes the quality of a specific product, whereas in our setup it is the position on the Hotelling line.

this alternative timeline will be reported below.

Formally, let $r_i = (q_i^r, t_i^r)$ denote the bundle that the buyer expects to choose if his type is realized to be θ_i, $i = H, L$. Given $R := \{r_L, r_H\}$, type-θ buyer's gain-loss utility from a bundle $b = (q,t)$ takes the following form:

$$\begin{aligned} n(b;\theta,R) := {} & p[\mu((\theta_L - q_L^r)^2 - (\theta - q)^2) + \mu(t_L^r - t)] \\ & +(1-p)[\mu((\theta_H - q_H^r)^2 - (\theta - q)^2) + \mu(t_H^r - t)], \end{aligned} \tag{2}$$

where μ is an indicator function such that, for any $k_1, k_2 \in \mathbb{R}_+$,

$$\mu(k_1 - k_2) := \begin{cases} k_1 - k_2 & \text{if } k_1 \geq k_2 \\ \lambda(k_1 - k_2), \lambda > 1 & \text{if } k_1 < k_2 \end{cases}.$$

To explain our formulation, consider the term $\mu((\theta_L - q_L^r)^2 - (\theta - q)^2)$ in the RHS of (2), for instance. This captures type θ's gain-loss utility from consuming product q relative to state θ_L, which is then weighted by the probability p with which the buyer had expected θ_L to occur. Note that the consumer in our model experiences gain-loss from the difference in the final *utilities* that he would obtain across two states. The parameter λ measures the degree of loss aversion.

As is standard in the loss aversion literature, we assume that the gain-loss utility is additively separable across the two consumption dimensions, product location and monetary transfer. Given the reference point R (expected choices of bundles), a type-θ buyer's overall utility from $b = (q,t)$ is the sum of consumption and gain-loss utilities:

$$u(b \mid \theta, R) := m(b;\theta) + n(b;\theta,R).$$

2.2. Equilibrium Concepts

We now introduce the notion of *personal equilibrium* proposed by KR and adapted to the price discrimination model by HKKL. This incorporates the idea that the reference point formed by an economic agent should be in accordance with his actual choices.

Definition 1. *Given any menu* $M, R = \{r_i\}_{i=H,L} \subseteq M \cup \{\emptyset\}$ *is a personal equilibrium (PE) if*

$$u(r_i \mid \theta_i, R) \geq u(b \mid \theta_i, R), \forall b \in M \cup \{\emptyset\}, \forall i = H, L. \tag{3}$$

We say that $R=\{r_i\}_{i=L,H}$ *is a truthful personal equilibrium (TPE) if it is a PE given* $M=R$.

As noted by HKKL, it is straightforward to check the validity of the revelation principle when considering PE menus, and hence, there is no loss of generality in restricting attention to TPE menus. Menu $R=\{r_i\}_{i=L,H}$ is a TPE if and only if the incentive compatibility and individual rationality requirements hold as follows: for each $i=H,L$,

$$u(r_i \mid \theta_i, R) \geq u(r_{-i} \mid \theta_i, R) \qquad (IC_i)$$
$$u(r_i \mid \theta_i, R) \geq u(\emptyset \mid \theta_i, R). \qquad (IR_i)$$

The notion of personal equilibrium, however, invites the possibility of multiple equilibria. From a TPE menu $R=\{b_L, b_H\}$, the buyer might form an alternative reference point $R' \neq R$ and play it as a PE so that the seller fails to achieve the desired outcome.

The notion of *agent-optimal* PE is proposed by KR: a *preferred personal equilibrium* (PPE) is a PE that generates the highest ex ante expected utility to the consumer. Let $\mathcal{P}(M)$ denote the set of all PEs from a menu M; that is, R belongs to $\mathcal{P}(M)$ if $R \subseteq M \cup \{\emptyset\}$ and R satisfies condition (3). Also, given $R=\{(q_L,t_L),(q_H,t_H)\}$, let $U(R)$ denote the buyer's corresponding ex ante expected utility:

$$U(R) := pu(r_L \mid \theta_L, R) + (1-p)u(r_H \mid \theta_H, R).$$

Definition 2. *Given any menu* $M, R=\{r_i\}_{i=H,L} \subseteq M \cup \{\emptyset\}$ *is a preferred personal equilibrium (PPE) if* $R \in \mathcal{P}(M)$ *and* $U(R) \geq U(R')$ *for all* $R' \in \mathcal{P}(M)$. *We say that* $R=\{r_i\}_{i=L,H}$ *is a truthful preferred personal equilibrium (TPPE) if it is a PPE given* $M=R$.

We below characterize the seller's profit-maximizing menu of bundles under both notions of PE and PPE, restricting attention to direct revelation contracts. The analysis of TPPE may however entail loss of generality as in this case the revelation principle no longer holds. For further discussion on this issue, refer to HKKL. While our main equilibrium concept described above involves *ex post* individual rationality, we can also consider *ex ante* participation by requiring $U(R) \geq 0$ instead of the (IR-H) and (IR-L) constraints specified above.

III. Optimal Menu

It can be easily shown that without gain-loss utilities the seller maximizes profit by offering the screening menu $R^m = \{b_L, b_H\} = \{(0,\overline{s}),(1,\overline{s})\}$ with maximal differentiation. The seller fully extracts consumer surplus. Note that with $b_L = \{0,\overline{s}\}$ and $b_H = \{1,\overline{s}\}$ the incentive constraints are non-binding for both types of the consumer and the seller optimally sets the prices at which two participation constraints are binding. Our first result establishes that the optimal TPE under reference dependence and loss aversion is a screening menu similar to R^m with maximal product differentiation.

Proposition 1. *The screening menu* $R^\dagger = \{b_L, b_H\} = \{(0,\frac{(1+\lambda)}{2}\overline{s}),(1,\frac{(1+\lambda)}{2}\overline{s})\}$ *is the optimal TPE for all* p *and* λ.

Proof. First, we prove that the screening menu $R^\dagger = \{(0,\frac{(1+\lambda)}{2}\overline{s}),(1,\frac{(1+\lambda)}{2}\overline{s})\}$ is a TPE by showing that incentive constraints (IC-L), (IC-H) and ex post IR constraints (IR-L), (IR-H) are satisfied. (IR-i):

$$\begin{aligned} u(b_i \mid \theta_i, R^\dagger) &= \overline{s} - t_i \\ &\geq u(\emptyset \mid \theta_i, R^\dagger) = p[t_i - \lambda\overline{s}] + (1-p)[t_i - \lambda\overline{s}] = t_i - \lambda\overline{s}, i = L,H . \end{aligned}$$

Note that the price $\frac{(1+\lambda)}{2}\overline{s}$ is in fact derived from the binding IR constraints. (IC-i):

$$\begin{aligned} u(b_i \mid \theta_i, R^\dagger) &= \overline{s} - \frac{(1+\lambda)}{2}\overline{s} = \frac{(1-\lambda)}{2}\overline{s} \\ &> u(b_{-i} \mid \theta_i, R^\dagger) = \overline{s} - 1 - \frac{(1+\lambda)}{2}\overline{s} - \lambda(1-p)(1) - \lambda p(1) \\ &= \frac{(1-\lambda)}{2}\overline{s} - (1+\lambda), i = L,H . \end{aligned}$$

Hence, the menu $R^\dagger$ is a TPE for all p and λ. Next, we prove that the screening menu $R^\dagger$ is the optimal TPE maximizing the seller's expected profit. Note that the seller's expected profit under $R^\dagger$ is

$$E\pi(R^\dagger) = pt_L + (1-p)t_H = p\frac{(1+\lambda)}{2}\overline{s} + (1-p)\frac{(1+\lambda)}{2}\overline{s} = \frac{(1+\lambda)}{2}\overline{s} .$$

For the optimality it will suffice to show that the expected profit is smaller than $\frac{(1+\lambda)}{2}\overline{s}$ for all other menus, regardless of whether or not they constitute a PE. First, consider a class of screening menus $R^S = \{b_L, b_H\} = \{(q_L, t_L),(q_H, t_H)\}$ with

$0 < q_L < q_H < 1$. Assume that $q_L < 1 - q_H$ without loss of generality. Then it must be that $t_L > t_H$ in the screening equilibrium since b_H is targeted to the consumer who is farther away from the bliss point. The ex post IR constraints require that

$$\begin{aligned} u(b_L \mid \theta_L, R^S) &= \overline{s} - (q_L)^2 - t_L + (1-p)[(1-q_H)^2 - (q_L)^2 - \lambda(t_L - t_H)] \\ &\geq u(\emptyset \mid \theta_L, R^S) = p[t_L - \lambda(\overline{s} - (q_L)^2)] + (1-p)[t_H - \lambda(\overline{s} - (1-q_H)^2)] \end{aligned} \tag{4}$$

and

$$\begin{aligned} u(b_H \mid \theta_H, R^S) &= \overline{s} - (1-q_H)^2 - t_H + p[t_L - t_H - \lambda((1-q_H)^2 - (q_L)^2)] \\ &\geq u(\emptyset \mid \theta_H, R^S) = p[t_L - \lambda(\overline{s} - (q_L)^2)] + (1-p)[t_H - \lambda(\overline{s} - (1-q_H)^2)] \end{aligned} \tag{5}$$

Multiplying each inequality by p and $1-p$ respectively and adding up two inequalities we obtain

$$\begin{aligned} E\pi(R^S) = pt_L + (1-p)t_H &= \frac{1+\lambda}{2}\overline{s} - \frac{1+\lambda}{2}p(q_L)^2 - \frac{1+\lambda}{2}(1-p)(1-q_H)^2 \\ &\quad - \frac{(\lambda-1)p(1-p)}{2}[(1-q_H)^2 - (q_L)^2 + (t_L - t_H)] \\ &< \frac{(1+\lambda)}{2}\overline{s} = E\pi(R^\dagger) \end{aligned}$$

Therefore, the expected profit under R^S is strictly less than $E\pi(R^\dagger)$. Second, consider a class of pooling menus $R^P = \{(q,t)\}$. For $q \leq \frac{1}{2}$, the ex post participation constraints require that

$$\begin{aligned} u(b \mid \theta_L, R^P) &= \overline{s} - q^2 - t + (1-p)[(1-q)^2 - (q)^2] \\ &\geq u(\emptyset \mid \theta_L, R^P) = p[t - \lambda(\overline{s} - q^2)] + (1-p)[t - \lambda(\overline{s} - (1-q)^2)] \end{aligned}$$

and

$$\begin{aligned} u(b \mid \theta_H, R^P) &= \overline{s} - (1-q)^2 - t + p\lambda[(1-q)^2 - (q)^2] \\ &\geq u(\emptyset \mid \theta_H, R^P) = p[t - \lambda(\overline{s} - q^2)] + (1-p)[t - \lambda(\overline{s} - (1-q)^2)]. \end{aligned}$$

Multiplying each inequality by p and $1-p$ respectively and adding up two inequalities we obtain

$$E\pi(R^P) = t \le \frac{(1+\lambda)}{2}\overline{s} - \frac{(1+\lambda)}{2}pq^2 - \frac{(1+\lambda)}{2}(1-p)(1-q)^2$$
$$-\frac{(\lambda-1)p(1-p)}{2}[(1-q)^2-(q)^2]$$
$$< \frac{(1+\lambda)}{2}\overline{s} = E\pi(R^\dagger)$$

Therefore, the expected profit under R^P is strictly less than $E\pi(R^\dagger)$. It can be easily verified that the same holds for $q \ge \frac{1}{2}$. Finally, for a class of *reverse-screening* menus $R^R = \{b_L, b_H\} = \{(q_L, t_L), (q_H, t_H)\}$ with $0 < q_H < q_L < 1$ we can easily show that $E\pi(R^R) < \frac{(1+\lambda)}{2}\overline{s} = E\pi(R^\dagger)$, similarly to the case of screening menus R^S. This proves that the screening menu $R^\dagger$ is the optimal TPE for all p and λ. □

This proposition shows that the optimal contract for loss averse consumers differs from that for loss neutral ones only in price. In other words, the seller does not have an incentive to distort the positions of the products, which contrasts with the result of HKKL. In HKKL where products are vertically differentiated, as an effort to reduce the potential feeling of consumer loss, the monopolist makes less use of price discrimination or no discrimination at all. Here, the monopolist does not have such an incentive to distort the products because matching the consumers' intrinsic ideal specification maximizes the intrinsic surplus and minimizes corresponding consumer loss at the same time.

Note that the prices increase in λ, which is due to the binding participation constraints. As consumers become more loss averse, the loss from not buying the product becomes larger when the reference is buying the product. In TPE where the consumers truthfully reveal their types, thus most advantageous to the seller, the seller exploits the feeling of loss by setting a high price. This would not however be the case if consumers could form their reference in the way to maximize their *ex ante* utility. We next explore optimal contract under the notion of TPPE as well as the ex ante participation constraint.

Our next result shows that a screening TPPE menu exists for all p and λ. This contrasts with the corresponding results from the vertical differentiation model where a screening PPE menu fails to exist for a certain parameter range with large λ.[2]

Proposition 2. *There exists a screening TPPE menu* $R^{\dagger\dagger} = \{b_L, b_H\} = \{(0,\tilde{t}), (1,\tilde{t})\}$, *where* $\tilde{t} \in [\overline{s}, \frac{(1+\lambda)}{2}\overline{s}]$, *for all p and* λ.

[2] Refer to Proposition 5 in HKKL.

Proof. It suffices to show that there exists $\tilde{t} \in [\overline{s}, \frac{(1+\lambda)}{2}\overline{s}]$ such that $R^{\dagger\dagger} = \{(0,\tilde{t}),(1,\tilde{t})\}$ constitutes a PE and yields an ex ante expected utility larger than any other feasible reference R'. Note that $R^{\dagger\dagger}$ satisfies (IC-L), (IC-H), (IR-L), and (IR-H) for $\tilde{t} \in [\tilde{s}, \frac{(1+\lambda)}{2}\overline{s}]$, and so it is a PE: (IR-i):

$$\begin{aligned} u(b_i \mid \theta_i, R^{\dagger\dagger}) &= \overline{s} - \tilde{t} \\ &\geq u(\emptyset \mid \theta_i, R^{\dagger\dagger}) = p[\tilde{t} - \lambda\overline{s}] + (1-p)[\tilde{t} - \lambda\overline{s}] = \tilde{t} - \lambda\overline{s}, i = L, H \end{aligned}$$

(IC-i):

$$\begin{aligned} u(b_i \mid \theta_i, R^{\dagger\dagger}) &= \overline{s} - \tilde{t} \\ &> u(b_{-i} \mid \theta_i, R^{\dagger\dagger}) = \overline{s} - 1 - \tilde{t} - \lambda(1-p)(1) - \lambda p(1) \\ &= \overline{s} - \tilde{t} - (1+\lambda), i = L, H \end{aligned}$$

for all $\tilde{t} \in [\overline{s}, \frac{(1+\lambda)}{2}\overline{s}]$. Note that

$$U(R^{\dagger\dagger}) = pu(b_L \mid \theta_L, R^{\dagger\dagger}) + (1-p)u(b_H \mid \theta_H, R^{\dagger\dagger}) - p(1-p)(0) - (1-p)p(0) = \overline{s} - \tilde{t}\ .$$

Therefore, it will suffice to show that there exists $\tilde{t} \in [\overline{s}, \frac{(1+\lambda)}{2}\overline{s}]$ such that ex ante expected utilities under alternative references are all smaller than $\overline{s} - \tilde{t}$, irrespective of whether or not they constitute a PE. For alternative references $R^H = \{b_H, b_H\} = \{(1,\tilde{t}),(1,\tilde{t})\}$, $R^L = \{b_L, b_L\} = \{(0,\tilde{t}),(0,\tilde{t})\}$, $R^R = \{b_H, b_L\} = \{(1,\tilde{t}),(0,\tilde{t})\}$, $R^{H\emptyset} = \{b_H, \emptyset\} = \{(1,\tilde{t}),\emptyset\}$, $R^{L\emptyset} = \{b_L, \emptyset\} = \{(0,\tilde{t}),\emptyset\}$, $R^{\emptyset H} = \{\emptyset, b_H\} = \{\emptyset,(1,\tilde{t})\}$ and $R^{\emptyset L} = \{\emptyset, b_L\} = \{\emptyset,(0,\tilde{t})\}$ respectively, we respectively obtain:

$$\begin{aligned} U(R^H) &= p(\overline{s} - 1 - \tilde{t}) + (1-p)(\overline{s} - \tilde{t}) + \lambda p(1-p)(-1) + (1-p)p(1) \\ &= \overline{s} - \tilde{t} - p - (\lambda - 1)p(1-p) < u(R^{\dagger\dagger}) = \overline{s} - \tilde{t}\ , \end{aligned}$$

$$\begin{aligned} U(R^L) &= p(\overline{s} - \tilde{t}) + (1-p)(\overline{s} - 1 - \tilde{t}) + p(1-p)(1) + \lambda(1-p)p(-1) \\ &= \overline{s} - \tilde{t} - (1-p) - (\lambda - 1)p(1-p) < u(R^{\dagger\dagger}) = \overline{s} - \tilde{t}\ , \end{aligned}$$

$$U(R^R) = p(\overline{s} - 1 - \tilde{t}) + (1-p)(\overline{s} - 1 - \tilde{t}) = \overline{s} - \tilde{t} - 1 < u(R^{\dagger\dagger}) = \overline{s} - \tilde{t}\ ,$$

$$\begin{aligned} U(R^{H\emptyset}) &= p(\overline{s} - 1 - \tilde{t}) + p(1-p)(\overline{s} - 1 - \lambda\tilde{t}) + (1-p)p(\tilde{t} - \lambda(\overline{s} - 1)) \\ &= p(\overline{s} - \tilde{t}) - p - (\lambda - 1)p(1-p)(\overline{s} - 1 + \tilde{t}) < u(R^{\dagger\dagger}) = \overline{s} - \tilde{t}\ , \end{aligned}$$

$$\begin{aligned} U(R^{L\emptyset}) &= p(\bar{s}-\tilde{t}) + p(1-p)(\bar{s}-\lambda\tilde{t}) + (1-p)p(\tilde{t}-\lambda\bar{s}) \\ &= p(\bar{s}-\tilde{t}) - (\lambda-1)p(1-p)(\bar{s}+\tilde{t}) < u(R^{\dagger\dagger}) = \bar{s}-\tilde{t}\ , \end{aligned}$$

$$\begin{aligned} U(R^{\emptyset H}) &= (1-p)(\bar{s}-\tilde{t}) + p(1-p)(\tilde{t}-\lambda\bar{s}) + (1-p)p(\bar{s}-\lambda\tilde{t}) \\ &= (1-p)(\bar{s}-\tilde{t}) - (\lambda-1)p(1-p)(\bar{s}+\tilde{t}) < u(R^{\dagger\dagger}) = \bar{s}-\tilde{t}\ , \end{aligned}$$

$$\begin{aligned} U(R^{\emptyset L}) &= (1-p)(\bar{s}-1-\tilde{t}) + p(1-p)(\tilde{t}-\lambda(\bar{s}-1)) + (1-p)p(\bar{s}-1-\lambda\tilde{t}) \\ &= (1-p)(\bar{s}-\tilde{t}) - (1-p) - (\lambda-1)p(1-p)(\bar{s}-1+\tilde{t}) < u(R^{\dagger\dagger}) = \bar{s}-\tilde{t} \end{aligned}$$

for all $\tilde{t} \in [\bar{s}, \frac{(1+\lambda)}{2}\bar{s}]$. Note that $U(R^{\emptyset\emptyset}) = 0$. This implies that the minimum value of $\tilde{t}$ sustaining $R^{\dagger\dagger}$ as PPE is $\bar{s}$, since $R^{\emptyset\emptyset}$ may not be a PE. So, there must exist $\tilde{t} \in [\bar{s}, \frac{(1+\lambda)}{2}\bar{s}]$ allowing for $R^{\dagger\dagger}$ to be a PPE. If $R^{\emptyset\emptyset}$ fails to be a PE, then it is possible to set $\tilde{t} > \bar{s}$. □

Since the way to maximize the intrinsic surplus coincides with that to minimize the loss, the monopolist does not have any reason to deviate from $(q_L, q_H) = (0,1)$ to incorporate consumer loss aversion. On the other hand, the capability for the monopolist to extract the surplus from consumers depends on how the reference forms. As the concept of TPPE posits the situation that is most advantageous to consumers, the prices cannot be larger than those in TPE contract. This point can be most clearly observed when the participation decision is made ex ante.

With the ex ante participation constraint, i.e. $U(\cdot) \geq 0$, the screening menu $R^m = \{b_L, b_H\} = \{(0,\bar{s}),(1,\bar{s})\}$ with maximal differentiation turns out to be the optimal TPPE, as stated in the following proposition.

Proposition 3. *With ex ante participation, the screening menu* $R^m = \{b_L, b_H\} = \{(0,\bar{s}),(1,\bar{s})\}$ *is the optimal TPPE for all* p *and* λ.

Proof. First, we prove that the screening menu R^m is a PE by showing that (IC-L), (IC-H) and ex ante IR constraints are satisfied: (IC-L):

$$\begin{aligned} u(b_L \mid \theta_L, R^m) &= \bar{s}-\bar{s} = 0 \\ &\geq u(b_H \mid \theta_L, R^m) = \bar{s}-1-\bar{s}-\lambda(1-p)(1)-\lambda p(1) = -(1+\lambda) < 0\ . \end{aligned}$$

(IC-H):

$$\begin{aligned} u(b_H \mid \theta_H, R^m) &= \bar{s}-\bar{s} = 0 \\ &\geq u(b_L \mid \theta_H, R^m) = \bar{s}-1-\bar{s}-\lambda p(1)-\lambda(1-p)(1) = -(1+\lambda) < 0\ . \end{aligned}$$

(IR):

$$U(R^m) = pu(b_L \mid \theta_L, R^m) + (1-p)u(b_H \mid \theta_H, R^m) - p(1-p)(0) - (1-p)p(0) = 0 .$$

Hence, the menu R^m is a PE for all p and λ. Next, we prove that R^m is a PPE by showing that R^m yields an ex ante expected utility larger than any other alternative references. Note that $U(R^m) = 0$. So, it will suffice to show that ex ante expected utilities under alternative references are all smaller than zero. For $R^H = \{b_H, b_H\} = \{(1, \overline{s}), (1, \overline{s})\}$, $R^L = (b_L, b_L\} = \{(0, \overline{s}), (0, \overline{s})\}$ and $R^R = (b_H, b_L\} = \{(1, \overline{s}), (0, \overline{s})\}$, we obtain

$$\begin{aligned} U(R^H) &= p(\overline{s} - 1 - \overline{s}) + (1-p)(\overline{s} - \overline{s}) + \lambda p(1-p)(-1) + (1-p)p(1) \\ &= -p - (\lambda - 1)p(1-p) < 0 , \end{aligned}$$

$$\begin{aligned} U(R^L) &= p(\overline{s} - \overline{s}) + (1-p)(\overline{s} - 1 - \overline{s}) + p(1-p)(1) + \lambda(1-p)p(-1) \\ &= -(1-p) - (\lambda - 1)p(1-p) < 0 , \end{aligned}$$

and

$$U(R^R) = p(\overline{s} - 1 - \overline{s}) + (1-p)(\overline{s} - 1 - \overline{s}) = -1 < 0$$

respectively.

Finally, we prove that the screening menu R^m is optimal (maximizing the seller's expected profit). Note that the seller's expected profit under R^m is $E\pi(R^m) = pt_L + (1+p)t_H = p(\overline{s}) + (1-p)(\overline{s}) = \overline{s}$. For the optimality it will suffice to show that the seller's expected profit is smaller than $\overline{s}$ for all other menus, regardless of whether or not they constitute a PE. First, consider a class of screening menus $R^S = \{b_L, b_H\} = \{(q_L, t_L), (q_H, t_H)\}$ with $0 < q_L < q_H < 1$. Assume that $q_L < 1 - q_H$ without loss of generality. The ex ante participation constraint requires that

$$\begin{aligned} U(R^S) = {} & p(\overline{s} - (q_L)^2 - t_L) + (1-p)(\overline{s} - (1-q_H)^2 - t_H) \\ & -(\lambda - 1)p(1-p)[(1-q_H)^2 - (q_L)^2 + t_L - t_H] \geq 0 . \end{aligned}$$

Rearranging the inequality, we have

$$\begin{aligned} E\pi(R^S) = pt_L + (1-p)t_H \leq {} & p(\overline{s} - (q_L)^2 + (1-p)(\overline{s} - (1-q_H)^2) \\ & -(\lambda - 1)p(1-p)[(1-q_H)^2 - (q_L)^2 + t_L - t_H] < \overline{s} \end{aligned}$$

So, the seller's expected profit under R^S is strictly less than $\overline{s}$. Second, consider a class of pooling menus $R^P = \{(q,t)\}$. For $q \le \frac{1}{2}$, the ex ante participation constraint implies that

$$\begin{aligned} U(R^P) &= p(\overline{s} - q^2 - t) + (1-p)(\overline{s} - (1-q)^2 - t) - (\lambda - 1)p(1-p)(1-2q) \\ &= \overline{s} - q^2 - t - [1 + (\lambda - 1)p](1-p)(1-2q) \ge 0 . \end{aligned}$$

Rearranging the inequality leads to

$$E\pi(R^P) = t \le \overline{s} - q^2 - [1 + (\lambda - 1)p](1-p)(1-2q) < \overline{s} .$$

Therefore, the expected profit under R^P must be strictly less than $\overline{s}$. It can be easily verified that the same holds for $q \ge \frac{1}{2}$. Finally, for a class of reverse-screening menus $R^R = (b_L, b_H\} = \{(q_L, t_L), (q_H, t_H)\}$ with $0 < q_H < q_L < 1$ we can easily show that $E\pi(R^P) < \overline{s}$, similarly to the case of screening menus R^S. □

Note that the buyer ex post may end up with a negative utility of $\frac{(1-\lambda)}{2}\overline{s}$. This happens because the seller extracts the entire ex ante surplus.

IV. Conclusion

In this paper, we have made an attempt to scrutinize some recent results on firm behavior that suggest limits to complex, stochastic pricing strategy when faced with loss averse consumers. In the price discrimination setup with a monopoly seller and heterogeneous consumers, our results demonstrate that the effect of consumer loss aversion depends on the demand structure: with horizontally differentiated demand, consumer loss aversion does not have the same bite as in the vertically differentiated case.

However, this observation may be sensitive to how gains and losses are defined. In our analysis, gain-loss utility is defined with respect to the expected utility level for each possible event. However, if it is defined in comparison with, for example, the average specification $E(q)$, the monopolist may want to distort q_L and q_H towards the center so as to reduce the distances between the reference point and the chosen q's. A formal analysis of this case is left for a possible future research.

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