

## Demand-led Growth and Long-run Convergence in a Neo-Kaleckian Two-sector Model

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*This paper analyzes a two-sector model, with consumption and investment sectors, which incorporates both Kaleckian and Classical views. Starting from a model where investment depends on actual profit rates and rates of capacity utilization, we characterize three regimes with different investment functions and specific adjustment mechanisms to bring about a uniform rate of profit and convergence between the actual and the normal rates of capacity utilization. We find that the paradox of thrift holds in the long run for all regimes. With regards to income distribution, results concerning wage-led growth and the paradox of costs are more ambiguous. The reportioning of the capital stock between the two sectors is also discussed. We conclude that our analysis provides some justification for using simple one-sector Kaleckian models, since the results achieved with the two-sector model are roughly in conformity with those of the one-sector model, depending on the closure being used.*

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### I. Introduction

The Kaleckian growth and distribution model is now at the heart of the empirical work that has been pursued by post-Keynesian economists to assess whether increases in the profit share in national income have favorable or negative effects on economic activity and productivity (Storm and Naastepad, 2012; Onaran and Galanis, 2012). The Kaleckian model of growth and distribution was formally

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suggested and developed in the 1980s, in particular, by Rowthorn (1981), Dutt (1984), Taylor (1985) and Amadeo (1986). The canonical Kaleckian growth model has four crucial features, which are the following: “First, there is the investment function, which may depend on several variables, one of which must be the rate of capacity utilization. Second, prices relative to direct costs are assumed to be given, dependent on conventional forces instead of market forces. Prices are of the cost-plus type. Third, saving out of wages is often assumed to be nil, although it is sufficient to assume that the propensity to save out of wages is smaller than that out of profits, as in the neo-Keynesian growth model. Fourth, the rate of utilization of capacity is assumed to be generally below unity, and labour is assumed not to be a constraint.” (Lavoie, 2014, p. 360).

Besides its use for empirical work, the Kaleckian model of growth and distribution has provided a useful common framework which has allowed heterodox authors of various persuasions to interact with each other on matters of growth and distribution. But there exist unresolved gaps between neo-Kaleckian views on one hand and classical views on the other hand, meaning here neo-Marxian and Sraffian authors. In multi-sector models, disagreements are mainly related to the possible existence of long-run positions and of adjustment mechanisms towards those. The issue, in particular, is whether or not profit rates are equalized among industrial sectors and whether or not there exists convergence between the actual and the normal rates of capacity utilization.

Hein, Lavoie and van Treeck (2011, 2012) have examined the various mechanisms that have been proposed to handle the second issue, that of the possible discrepancy between actual and normal rates of capacity utilization. Arguments have also been offered as to why the equality between these two rates may not necessarily be expected, even in the long run. Although we do not wish to rehearse this debate, it should be pointed out that the debate has occurred within the context of simple one-sector models. Here we wish to examine what can be said within the framework of a multi-sector model. In their extension to a two-sector Kaleckian model, Dutt (1988, 1990) and Lavoie and Ramírez-Gastón (1997) show that the incongruity between the actual and the normal rates still prevails in the long run. Moreover, they insist that a uniform rate of profit could be achieved only under some specific conditions, adding that it is dubious that these conditions would be satisfied, particularly in a modern oligopolistic economy, as described by monopoly power theorists such as Kalecki (1939, 1971), Sweezy (1942), Steindl (1952), Spence (1977) and Cowling (1982).

Most neo-Marxists and some Sraffians would object to these claims, as do Auerbach and Skott (1988), claiming that rational firms would not want to keep undesired excess capacity, and hence would act in such a way that the normal or optimal rate of capacity utilization would be achieved in the long run (or that the

actual rate would gravitate around the normal rate).<sup>1</sup> Furthermore, they argue that profit rates among industries tend to be equalized in the long run, since competition between capitalists would lead to the mobility of (financial) capital from an industrial sector with a low profit rate to one with a high rate (Duménil and Lévy, 1995, 1999; Glick and Campbell, 1995; Semmler, 1984).

The purpose of this paper is to explore whether in a two-sector model there exist qualitative, significant, differences in long-run relationships and traverses towards long-run positions when adjustment mechanisms are incorporated in a simple model.<sup>2</sup> To conduct this task, we characterize regimes with different investment functions and specific adjustment mechanisms, reflecting neo-Kaleckian and classical views. We then examine whether the paradox of thrift, the wage-led regime, and the paradox of costs hold in the long run. While some authors in the past have examined some of the issues and some of the mechanisms that we deal with here, we believe that this is the first paper that does so in a systematic fashion.

The paper is organized as follows. In the next section we develop a dynamic Kaleckian two-sector model with target-return pricing. In the third section, we specify two additional regimes by introducing adjustment mechanisms that intend to reconcile Kaleckian and classical concerns. The last section summarizes and concludes.

## II. A Basic Two-sector Kaleckian Model

In this section, we set a simple two-sector model in an analytical framework, which will then be used to develop more specific investment functions and adjustment mechanisms. Stability conditions and long-run relationships are then analyzed. To do so, in particular, we suppose that a uniform rate of profit depends on whether there exist any barriers to the free mobility of (financial) capital when there are profit rate differentials between industrial sectors,<sup>3</sup> while convergence between the actual and the normal rates depends on whether costing margins and prices change in response to a gap between the actual and the normal rates of

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<sup>1</sup> However, there are a number of Sraffians such as Garegnani (1992), Kurz (1994), Park (1995), Palumbo and Trezzini (2003) and Trezzini (2011) who agree that there could be persistent divergence between the normal and the actual rates of utilization, on average, even in the long run.

<sup>2</sup> The multi-sectoral model is a useful framework in which we can analyze structural economic dynamics that change the production structure and affect economic variables such as the growth rate (Araujo and Lima, 2007). For example, in a multi-sectoral neo-Kaleckian model, Araujo and Teixeira (2011) and Nishi (2014) show that sectoral heterogeneity and reallocation could result in different growth regimes.

<sup>3</sup> Harris (1978, p. 46) points out that ‘a uniform rate of profit is then a consequence of the assumption of competition in this sense [the free mobility of capital] and of the tendencies associated with competition’.

capacity utilization.<sup>4</sup>

## 2.1. Basic Pricing Equations

To keep the model simple, we assume the following: the economy consists of a consumption sector (denoted  $i = 1$ ) and an investment sector (denoted  $i = 2$ ), and there is one firm (or identical firms) in each sector; there are two factors to produce goods, fixed capital and labour; there is no overhead or fixed labour; the investment good is a basic good; capital stocks are non-transferable between both industrial sectors, and have constant efficiency and no depreciation; firms operate plants under constant returns to scale; all wages are consumed and the propensity to save of capitalists is  $s_p$ ; the wage rate is the same in both sectors. Also, let us suppose that firms in both industrial sectors set the prices of their products by following the target-return pricing procedures described by Lanzillotti (1958) and adopted by Lavoie and Ramírez-Gastón (1997). Target-return pricing is a variant of cost-plus pricing, where the price is set by adding a costing margin to some measure of unit cost. Other variants of cost-plus pricing include normal-cost pricing, full-cost pricing and markup pricing. In the case of target-return pricing, the margin depends on a target rate of return, to be achieved when the firm is running at its standard rate of capacity utilization. According to Kaplan et al. (1958), this pricing procedure is the most prevalent one, with Lee (1998) arguing in his review of the pricing literature that it is particularly the case of large firms.

In addition to its realistic feature, an advantage of target-return pricing is that it explicitly takes into account the intersectoral dependence of costing margins among sectors. This pricing procedure will also allow the introduction of explicit adjustment mechanisms that bring about fully-adjusted positions, where productive capacity is utilized at its normal level, with a uniform profit rate.

A simple mark-up pricing rule can be written as

$$p_i = (1 + \theta_i)w\alpha_i \quad (1)$$

where  $p$  is the price level,  $\theta$  is the costing margin,  $\alpha$  is the labour-output ratio that is assumed to be fixed, and  $w$  is the nominal wage rate.

The actual real outputs  $S$  are defined as follows:

$$S_1 \equiv C \quad (2.1)$$

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<sup>4</sup> Neo-Marxists may disagree with these dichotomic assumptions because competition among capitalists would lead to a change in markups and hence it might be a force to bring about convergence between actual and normal rates of capacity utilization. We believe that Regime 3, to be described below, can be considered as illustrating this case.

$$S_2 \equiv I_1 + I_2 \equiv g_1 K_1 + g_2 K_2 \tag{2.2}$$

where  $C$  is consumption in real terms;  $I_i$  is investment in real terms, that is, sales of machines to the consumption sector and to the machine-producing sector; the  $K_i$  are the capital stocks of the two sectors and  $g_i$  is the rate of accumulation of each sector. From the assumption of target-return pricing, the standard sales  $S^s$  correspond to the standard (or normal) rate of capacity utilization  $u^s$  of each sector; this rate is such that it must provide enough profits to fulfill the target rate of return  $r^s$ . The full-capacity output  $S_{fc}$  and the standard sales are defined respectively as

$$S_{fc,i} \equiv K_i / \sigma_i \tag{3}$$

$$S_i^s \equiv u_i^s S_{fc,i} \tag{4}$$

where  $\sigma$  is the capital to full-capacity output ratio that is assumed to be fixed.

Equating the two equations that define total profits targeted in sector  $i$ ,  $F_{T,i}^s = \theta_i w \alpha_i S_i^s$  and  $F_{T,i}^s = r_i^s p_2 K_i$ , the costing margin of each sector is derived as

$$\theta_1 = \sigma_1 \alpha_2 r_1^s u_2^s / [\alpha_1 u_1^s (u_2^s - \sigma_2 r_2^s)] \tag{5.1}$$

$$\theta_2 = \sigma_2 r_2^s / [u_2^s - \sigma_2 r_2^s] \tag{5.2}$$

The condition ensuring positive costing margins (and hence positive prices) is  $u_2^s > \sigma_2 r_2^s$  for each sector. This condition is necessarily fulfilled by equations (3) and (4) when the standard output of the investment sector is greater than target real profits in that sector ( $S_2^s > r_2^s K_2$ ). Note that the costing margin (and hence the price) in the consumption sector depends on pricing in the investment sector, whereas the costing margin in the investment sector is independent of pricing in the consumption sector as it should be for a basic good. An increase in the price of investment goods leads to a higher costing margin in the consumption sector because firms in the latter sector would try to offset losses due to the increased purchasing price of investment goods by raising their costing margin, for a given target rate of return. Substituting equations (5.1) and (5.2) into (1), the price equation of each sector can be rewritten as:

$$p_1 = \alpha_1 w + A p_2 \tag{1.1}'$$

$$p_2 = B w \tag{1.2}'$$

where  $A = \sigma_1 r_1^s / u_1^s$  and  $B = \alpha_2 u_2^s / (u_2^s - \sigma_2 r_2^s)$ .

The actual rate of capacity utilization  $u$  is defined as

$$u_i \equiv S_i / S_{fc,i} \equiv \sigma_i S_i / K_i \quad (6)$$

and, from equation (1), the actual total profits  $F_T$  are given by

$$F_{T,i} = \theta_i w \alpha_i S_i = m_i p_i S_i \quad (7)$$

where  $m = \theta / (1 + \theta)$  is the gross profit margin, the so-called ‘degree of monopoly’ in Kalecki’s terminology.

Using equations (1.1)’, (1.2)’, and (6), we obtain the actual profit rate, which is defined by  $r = F_T / (p_2 K)$ ,

$$r_i = r_i^s (u_i / u_i^s) \quad (8)$$

With target-return pricing, therefore, the actual profit rate changes proportionally with the actual rate of capacity utilization, for a given target rate of return and a given standard rate of capacity utilization (so do the ratio of the actual to the target rate of profit with the ratio of the actual to the standard rate of utilization). This relationship also implies that the existence of undesired excess capacity ( $u < u^s$ ) results in the actual profit rate being less than the desired rate ( $r < r^s$ ). Also, in the two-sector economy, when  $(u_1 / u_2) = (r_2^s / r_1^s)(u_1^s / u_2^s)$ , the actual profit rates are equalized in both sectors, while when  $r_i = r_i^s$ , the actual rate of capacity utilization is equal to the standard rate.<sup>5,6</sup>

## 2.2. Regime 1: Short-run Solutions with no Adjustment Mechanism<sup>7</sup>

Regime 1 embodies the argument of monopoly power theorists, in line with Dutt

<sup>5</sup> If firms try to operate at full capacity utilization ( $u^f = 1$ ) at least in the long run, that is,  $u^s = 1$ , then the adjustment mechanism to bring about  $r_i^f \rightarrow r_i$  would also result in a uniform rate of profit ( $r_1 = r_2$ ) at the same time, without an additional adjustment mechanism.

<sup>6</sup> With equations (5.1) and (5.2), the gross profit margin (or the net share of profits) of each sector is  $m_i = \sigma_i (r_i^s / u_i^s)$ , and thus the actual profit rate of equation (8) can be rewritten as  $r_i = m_i u_i / \sigma_i$ . This implies that even though the actual profit rates are equalized in both sectors and the actual rates converge toward their standard rates, the net shares of profits in both sectors are not same, because of the structural differences between those sectors.

<sup>7</sup> This subsection is mainly based on Lavoie and Ramírez-Gastón (1997). In the present paper, however, we assume that an investment function depends on both the profit rate and the utilization rate, as in Rowthorn (1981), while in the former model it was a sole function of the rate of capacity utilization. Also, we explicitly derive long-run solutions and mathematically examine whether the paradox of thrift and the paradox of cost hold in the long run, introducing a specific assumption which leads to a different result with respect to how the repositioning of the capital stocks occurs in our model relative to the former model.

(1988) and Lavoie and Ramírez-Gastón (1997). The regime does not incorporate any adjustment mechanism to bring about the equalization between the actual and the standard rates. Hence, in the long run, profit rates do not converge, actual profit rates are not equal to the target rate, and the actual rate of capacity utilization diverges from the standard rate. In other words, costing margins and prices are chosen on the basis of the target rate of return and the standard rate of capacity utilization determined by the ‘rules of thumb’ of entrepreneurs, regardless of the state of effective demand.

We suppose that the investment function depends on the animal spirits of entrepreneurs  $\gamma_{i0}$ ,<sup>8</sup> the actual profit rate and the actual rate of capacity utilization, such that

$$g_i = \gamma_{i0} + \gamma_{i1}r_i + \gamma_{i2}u_i \quad (9)$$

This type of investment function has been used in many Kaleckian models since Rowthorn (1981) and Dutt (1984) adopted it. Here, the rate of capacity utilization reflects the level of effective demand and the rate of profit is an indicator of the actual profitability of firms and hence an indicator of the capacity of firms to obtain funds from banks and financial markets (Lavoie, 1995). Fazzari and Mott (1986-87), Chamberlain and Gordon (1989) and Arestis et al. (2012) all provide empirical evidence showing that capacity utilization (or sales) and profitability (or retained earnings) have a significant positive impact on the investment of firms.

Substituting equation (8) into (9), the investment function can be reduced to a function of the rate of capacity utilization only.

$$g_i = \gamma_{i0} + \gamma_{i3}u_i \quad (9)'$$

where  $\gamma_{i3} = \gamma_{i1}r_i^s / u_i^s + \gamma_{i2}$ . Therefore, an increase [decrease] in the rate of capacity utilization raises [reduces] both the rate of accumulation and the profit rate, that is, these rates move together in the same direction, for a given target rate of return and standard rate of capacity utilization.

In the short run we assume that the distribution of the capital stock between the two sectors is a given and hence that the variable  $k = K_1 / K_2$  is a constant. Assuming that supply adjusts to demand within the period, that is, assuming that output adjusts to the quantities that are being demanded and hence that the rate of capacity utilization is an endogenous variable, we obtain the short-run equilibrium

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<sup>8</sup> The autonomous parameter is also sometimes interpreted as the secular growth rate of sales which is expected by entrepreneurs, but only when investment is a function of the discrepancy between the actual and the normal rates of capacity utilization (Committeri, 1986; Caserta, 1990).

rates of capacity utilization and equilibrium rates of accumulation:<sup>9</sup>

$$u_1^* = \sigma_1 u_1^s (u_2^s - s_p \sigma_2 r_2^s) (\gamma_{10} + \gamma_{20} / k) / \Phi_1 \quad (10.1)$$

$$u_2^* = s_p \sigma_1 \sigma_2 r_1^s u_2^s (\gamma_{10} k + \gamma_{20}) / \Phi_1 \quad (10.2)$$

$$g_1^* = \gamma_{10} + \gamma_{13} \sigma_1 u_1^s (u_2^s - s_p \sigma_2 r_2^s) (\gamma_{10} + \gamma_{20} / k) / \Phi_1 \quad (11.1)$$

$$g_2^* = \gamma_{20} + \gamma_{23} s_p \sigma_1 \sigma_2 r_1^s u_2^s (\gamma_{10} k + \gamma_{20}) / \Phi_1 \quad (11.2)$$

where  $\Phi_1 = s_p \sigma_1 r_1^s u_2^s (1 - \gamma_{23} \sigma_2) - \gamma_{13} \sigma_1 u_1^s (u_2^s - s_p \sigma_2 r_2^s)$  and the local stability condition of short-run equilibrium is  $\Phi_1 > 0$ .

To sum up, here is what happens in the short run. Prices are set by firms, on the basis of their unit wage cost, their target rate of return and the various technical coefficients, such as the capital to capacity ratio and the standard rate of capacity utilization. Prices are thus assumed to be given. The stock of machines in each sector is also assumed to be given (as well as the financial value of the capital in each sector, since the price of machines is a given), so that the new machines being produced in a given period are added to the existing stocks only before the next period starts. Produced output and hence sales in each sector is determined by desired purchases, assuming that there is always enough spare capacity to respond to any increase in demand. Parameters such as the profit shares, which arose from the pricing procedures, and the propensity to save out of profits help determine the actual rates of capacity utilization and the actual rates of profit in each sector, which retroact on the demand for investment goods, and hence on the demand for consumption goods, and hence on the realized rates of profit and of utilization.<sup>10</sup>

### 2.3. Long-run Dynamic Adjustment Mechanism

In the long run,  $k$  is not constant and changes over time. Let us consider the dynamics of  $k$  over time, where  $\dot{k} = dk/dt$ . Substituting equations (11.1) and (11.2) into equation (12) below and differentiating it with respect to  $k$ , we have  $d\hat{k}/dk < 0$  and hence the long-run equilibrium rate of accumulation  $g_1^{**} = g_2^{**} = g^{**}$  is stable.<sup>11</sup> Figure 1 shows the convergence of the two rates of

<sup>9</sup> For the proof, see the Appendix.

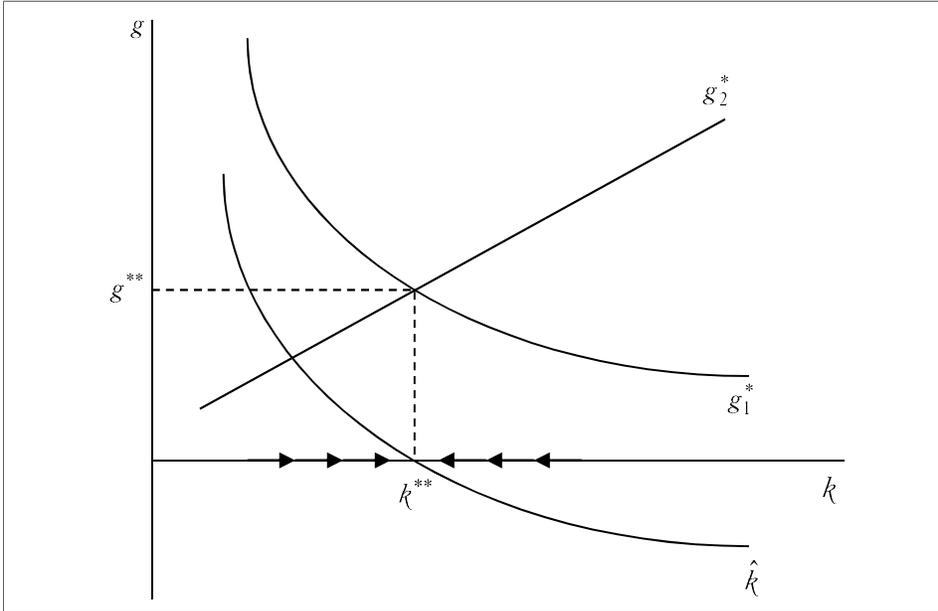
<sup>10</sup> While the profit share in our model is determined by exogenous factors, shown in equation (5.1) and (5.2), such as the monopolistic power of firms and the respective bargaining power of workers and capitalists, in a neo-classical model it is endogenously determined as a result of the profit maximization of firms, that is, by the marginal product of capital. Also, a change in the propensity to save in our model affects the rate of profit through a change in the rate of utilization (thus through the demand side), but in the neo-classical model, it changes the rate of profit through a change in the marginal product of capital under full capacity utilization (through the supply side).

<sup>11</sup> Therefore, in contrast to Park's (1998, p. 285; 1998-1999, p. 304) argument, in a two-sector

accumulation.

$$\hat{k} = \dot{k} / k = g_1^* - g_2^* \tag{12}$$

[Figure 1] Dynamics of rates of accumulation



To examine whether there exist a paradox of thrift and a paradox of cost, we need to find the long-run equilibrium ratio of sectoral capital stocks  $k^{**}$ . For the sake of simplicity, we assume that the entrepreneurs’ animal spirits on investment are the same in both sectors, that is,  $\gamma_{10} = \gamma_{20}$ .<sup>12</sup> Then, by equalizing the two equations (11.1) and (11.2), we obtain the long-run equilibrium ratio of sectoral capital stocks as follows:

$$k^{**} = \gamma_{13} u_1^s (u_2^s - s_p \sigma_2 r_2^s) / (\gamma_{23} s_p \sigma_2 r_1^s u_2^s) \tag{13}$$

Here,  $k^{**}$  is always greater than zero because  $u_2^s > s_p \sigma_2 r_2^s$ . This follows from the assumptions that  $u_2^s > \sigma_2 r_2^s$  and  $s_p < 1$ .

Substituting equation (13) into (10.1) and (10.2), the long-run equilibrium rate of capacity utilization is given by:

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Kaleckian model, a uniform rate of accumulation is achieved and is stable in the long run without an additional equation or condition for a uniform rate of profit.

<sup>12</sup> For the analysis of a general case, through the use of graphs, see Lavoie and Ramírez-Gastón (1997).

$$u_i^{**} = (\Omega / \Phi_1) / \gamma_{i3} \quad (14)$$

where  $\Omega = \sigma_1[\gamma_{10}\gamma_{13}u_1^s(u_2^s - s_p\sigma_2r_2^s) + \gamma_{20}\gamma_{23}s_p\sigma_2r_1^s u_2^s]$  Hence, we find a long-run relationship between the rates of capacity utilization of the two sectors

$$u_1^{**} = (\gamma_{13} / \gamma_{23})u_2^{**} \quad (15)$$

which implies that the rate of capacity utilization of one sector is associated positively to that of the other sector. Hence, with a given target rate of return and standard rate of capacity utilization, a positive shock on the rate of capacity utilization in the consumption sector will bring about an increase in the rate of accumulation and the profit rate of that sector through equations (8) and (9)', as well as an increase in the rate of accumulation and the profit rate of the investment sector through equation (15), and *vice versa*.

Putting equations (8), (9)' and (15) together, the long-run profit rate and the long-run rate of accumulation are given respectively as follows:<sup>13</sup>

$$r_i^{**} = [r_i^s / (\gamma_{i1}r_i^s + \gamma_{i2}u_i^s)](\Omega / \Phi_1) \quad (16)$$

$$g^{**} = g_i^{**} = \gamma_{i0} + \Omega / \Phi_1 \quad (17)$$

Differentiating equations (14), (16) and (17) with respect to the propensity to save and the target rate of return,<sup>14</sup> we eventually obtain Table 1, along with the first derivatives of equation (13). Therefore, the paradox of thrift holds in the long run, that is, a higher propensity to save leads to a lower rate of accumulation. The model is still wage-led, that is, an increase in real wages has a favourable impact on the rate of accumulation and the rate of capacity utilization (or an increase in the target rate of return has a negative impact on economic variables). However, a change in the target rate of return in a sector has an uncertain impact on the profit

<sup>13</sup> Note that in the present model a change in labour productivity does not have any effect on the rate of capacity utilization, the profit rate and the rate of accumulation, both in the short run and in the long run. This result arises because a change in real wage rates is offset by a change in the share of wages. For instance, a rise in labour productivity of the consumption sector (a decrease in  $\alpha_1$ ) decreases the price of consumption goods in equation (1.1)', with the constant price of investment goods. This causes the real wage rate to increase, for a given nominal wage rate. But, a rise in labour productivity reduces the share of wages because the costing margin of the consumption sector increases in equation (5.1). The two opposite forces are offset by each other as shown in equations (A2.1) and (A3.1) in the Appendix, so that a change in labour productivity has no real effect.

<sup>14</sup> In our model, income distribution refers to the distribution between workers and capitalists. An increase in the target rate of return ( $r_i^s$ ) leads to an increase in the costing margins of equations (5.1) and (5.2), as well as to an increase in the profit shares (or the gross profit margins) as shown in equation (7).

rate of that sector (as indicated by the ? sign).

[Table 1] Long-run effects in an economy with no adjustment mechanism

	$s_p$	$r_1^s$	$r_2^s$
$u_i^{**}$	–	–	–
$r_1^{**}$	–	?	–
$r_2^{**}$	–	–	?
$g^{**}$	–	–	–
$k^{**}$	–	–	–

As can be seen from the last row of Table 1, the ratio of sectoral capital stocks is associated negatively with an increase in costing margins, i.e., a higher costing margin induces a rise in the proportion of the capital stock located in the investment sector. This implies that the case of  $dk^{**} / dr_2^s > 0$  is ruled out, when assuming that the entrepreneurs’ animal spirits about investment are the same in both sectors ( $\gamma_{10} = \gamma_{20}$ ), whereas this was possible under specific conditions in Lavoie and Ramírez-Gastón (1997, pp. 159-160). In other words, unexpectedly (when having in mind the stable case of the Hicksian traverse (Hicks, 1965)), a higher rate of accumulation is associated with a lower proportion of the capital stock being located in the investment good industry.

### III. Alternative Regimes with Adjustment Mechanisms

#### 3.1. Regime 2: A Uniform Rate of Profit

Next, we specify Regime 2 as a case where there is an intersectoral mobility of (financial) capital so that profit rates in both industrial sectors are equalized. In this regime, we modify the investment function by assuming that investment decisions in the consumption sector follow equation (9), whereas investment decisions in the investment sector depend on the rate of accumulation of the consumption sector, modulated however by the difference between actual profit rates (Lavoie and Ramírez-Gastón, 1993).<sup>15</sup> This assumption is reasonably realistic since the

<sup>15</sup> This idea originates from Dutt (1988, p. 154, fn. 31), but he suggests equation (18.1) for the investment sector and equation (18.2) for the consumption sector. If we were to adopt Dutt’s choice in Regime 2, the main results that we obtain, such as the paradox of thrift and the paradox of costs, would still hold: in particular, when  $r_1^s = r_2^s$  and  $u_1^s = u_2^s$ , the two sets (ours and Dutt’s) of investment functions yield exactly the same results. This implies that whether growth regimes are wage-led or profit-led does not depend on whether the consumption sector or the investment sector is assumed to

investment sector essentially responds to the demand for investment goods arising from the consumption sector, which in turn depends on its own rate of accumulation.<sup>16</sup> In this case, we can replace the investment function (9) by:

$$g_1 = \gamma_0 + \gamma_1 r_1 + \gamma_2 u_1 \quad (18.1)$$

$$g_2 = g_1 + \beta(r_2 - r_1) \quad (18.2)$$

where  $\beta > 0$  is a reaction coefficient which measures the speed of adjustment to profit rate differentials.<sup>17</sup> With target-return pricing, as shown in equation (8), the equalization between the two actual profit rates takes place through a change in the actual rates of capacity utilization (that is, through quantity adjustment processes, but not price adjustment processes), for given standard rates. In this regime, since there is no impact on costing margins and prices, i.e., since the monopoly power to control prices is kept constant in each sector, firms have undesired excess capacity even in the long run.

We find the short-run equilibrium rate of profit as follows:<sup>18</sup>

$$r_1^* = \sigma_1 \gamma_0 r_1^s u_1^s u_2^s (u_2^s - s_p \sigma_2 r_2^s) (1 + 1/k) / \Phi_2 \quad (19.1)$$

$$r_2^* = s_p \sigma_1 \sigma_2 \gamma_0 r_1^s r_2^s u_1^s u_2^s (1 + k) / \Phi_2 \quad (19.2)$$

where  $\Phi_2 = s_p \sigma_1 r_1^s u_1^s u_2^s (u_2^s - \beta \sigma_2 r_2^s) - \sigma_1 u_1^s u_2^s (u_2^s - s_p \sigma_2 r_2^s) [(\gamma_1 r_1^s + \gamma_2 u_1^s)(1 + 1/k) - \beta r_1^s / k]$  and the short-run stability condition is  $\Phi_2 > 0$ .

In the long run, the two sectoral rates of profit are equalized, when the following

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base its investment decisions on equation (18.2) of our model. Rather, it is more likely to be related to the specification of the investment function itself. For instance, Bhaduri and Marglin (1990) show that different potential regimes could exist in a one-sector model when the profit rate in the investment function is replaced by the profit share. On the other hand, Dutt (1995; 1997) assumes that (identical) firms in both sectors choose their total rate of accumulation dependent on the average (or generalized) rate of capacity utilization and the average rate of profit, in order to induce a uniform rate of profit. Meanwhile, Duménil and Lévy (1999) suppose that a centralized capitalist agency controls the total amount of capital and distributes it to each industry according to the difference in profit rates. In both cases, equalization between profit rates is ensured by adopting equation (18.2).

<sup>16</sup> The assumption is also reminiscent of Pasinetti's (1981) hyper-vertically integrated sectors.

<sup>17</sup> Park (1997B, 1998, 1998-1999), even though investment functions equations (18.1) and (18.2) ensure a uniform rate of profit, claims that two questions remain to be answered: why is it that the investment function in the consumption sector fails to reflect differences in profit rates among industries; and why do capitalists of the investment sector ignore their own growth rates when making their investment decisions? Although his critique may be valid, it is not realistic to assume that firms take investment decisions regardless of the overall level of economic activity. Entrepreneurs should incorporate economy-wide factors in their investment decisions because they provide information about 'future profitability' and about whether the performance of individual firms results from 'chance events' (Dutt, 1990, p. 223, fn. 51).

<sup>18</sup> For the proof, see the Appendix.

condition is fulfilled:

$$\hat{k}^{**} = (u_2^s / s_p \sigma_2 r_2^s) - 1 \tag{20}$$

Note that the long-run ratio of sectoral capital stocks depends only on the propensity to save and the parameters of the investment sector. This remarkable result indicates that the evolution of the sectoral capital stocks is associated closely to the pricing decision of the investment sector.<sup>19</sup> Intuitively, it comes from the specific investment function of the investment sector that is exactly the same as that of the consumption sector in the long run. With this specification, the impact of the investment behaviour of each sector on the ratio of sectoral capital stocks will offset each other in the long run, so that this ratio will be related only to the parameters of the investment sector which produces basic goods.

Let us consider the stability condition. By using equation (18.2), the rate of change in the ratio of sectoral capital stocks can be represented as a function of profit rates as follows:

$$\hat{k} = g_1^* - g_2^* = \beta(r_1^* - r_2^*) \tag{21}$$

Using equations (19.1) and (19.2), we have  $d\hat{k} / dk < 0$ , provided the following condition is satisfied

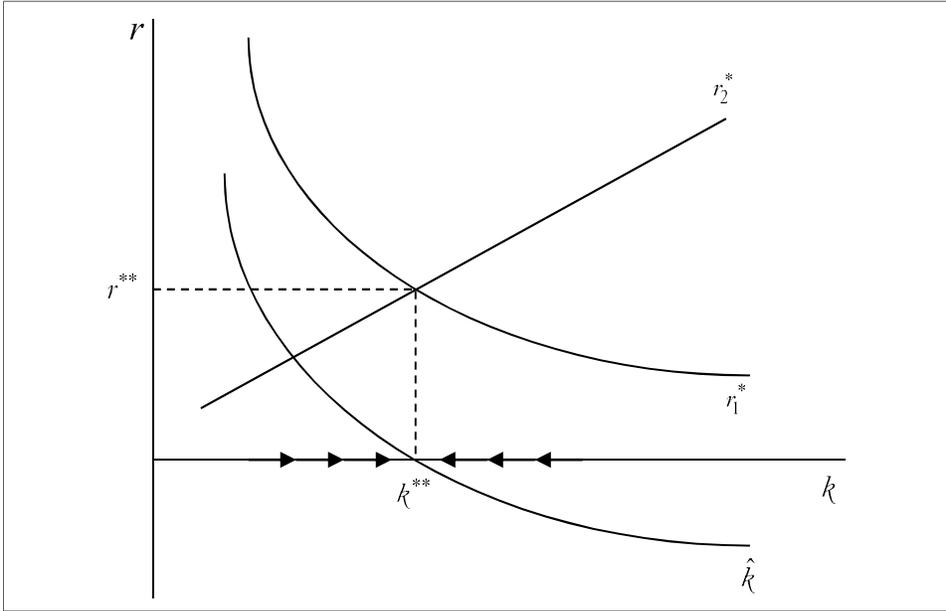
$$(\gamma_2 u_1^s / r_1^s) + \gamma_1 < \beta < s_p \tag{22}$$

That is, a condition for long-run stability is that the speed of adjustment of profit rate differentials must be smaller than the propensity to save.<sup>20</sup> Figure 2 shows the convergence of the two profit rates. Assuming that the economy is in the steady state, if  $k > k^{**}$  [ $k < k^{**}$ ] in a given period, then  $r_2 > r_1$  [ $r_2 < r_1$ ] by equations (19.1) and (19.2), and hence  $g_2 > g_1$  [ $g_2 < g_1$ ] by equation (18.2). Since it leads to  $\hat{k} < 0$  [ $\hat{k} > 0$ ] until  $k = k^{**}$ , the economy will converge towards the steady state over time. Therefore, profit rates (and hence the rates of accumulation) in both sectors converge towards a locally stable equilibrium rate.

<sup>19</sup> This is compatible with the result obtained in Lavoie and Ramírez-Gastón (1997, pp. 159-161). However, while they prove it in Regime 1 as specified in this paper, we find it in Regime 2, not in Regime 1.

<sup>20</sup> This condition is consistent with the standard condition for stability in one-sector Kaleckian growth models, ‘where the parameters have to be such that the savings function is more sensitive to changes than the investment function’ (Lavoie and Ramírez-Gastón, 1997, p. 153).

[Figure 2] Dynamics of profit rates



Substituting equations (20) into (19.1) and (19.2), we obtain a uniform rate of profit as follows:

$$r^{**} = r_1^{**} = r_2^{**} = \gamma_0 r_1^s / [(s_p - \gamma_1)r_1^s - \gamma_2 u_1^s] \tag{23}$$

where  $s_p > (\gamma_2 u_1^s / r_1^s) + \gamma_1$  is satisfied by the stability condition. This result shows, somewhat surprisingly, that in this simple model the long-run equilibrium (uniform) rate of profit does not depend on the variables of the investment sector. Furthermore, the long-run equilibrium rate of profit is exactly identical to the one obtained in a one-sector Kaleckian model.<sup>21</sup> As mentioned above, this is because in Regime 2 the long-run investment function of the investment sector gets reduced to that of the consumption sector, so that the latter plays a crucial role in the determination of long-run positions.

In turn, substituting equation (23) into (8) and (18.1), the long-run rate of capacity utilization and the long-run rate of accumulation are obtained

$$u_1^{**} = \gamma_0 u_1^s / [(s_p - \gamma_1)r_1^s - \gamma_2 u_1^s] \tag{24.1}$$

$$u_2^{**} = \gamma_0 r_1^s u_2^s / \{r_2^s [(s_p - \gamma_1)r_1^s - \gamma_2 u_1^s]\} \tag{24.2}$$

<sup>21</sup> For instance, see a one-sector model with target-return pricing in Lavoie (2003).

$$g^{**} = g_1^{**} = g_2^{**} = s_p \gamma_0 r_1^s / [(s_p - \gamma_1) r_1^s - \gamma_2 u_1^s] \tag{25}$$

The first derivatives of equations (23) - (25) with respect to the propensity to save and the target rate of return are given in Table 2, along with the first derivatives of equation (20). An increase in the propensity to save lowers both the rate of accumulation and the profit rate, and hence the paradox of thrift still holds, even when we modify the investment function in order to bring about a uniform profit rate in a simple two-sector model. Also, an increase in the target rate of return of the consumption sector lowers the rate of accumulation, the rate of utilization and the rate of profit. The economy is thus wage-led and tied to the paradox of costs when considering increases in the costing margin of the consumption sector. By contrast, the target rate of return of the investment sector has no impact on the rate of accumulation and the profit rate, despite its negative impact on the rate of utilization of that sector.

[Table 2] Long-run effects in an economy with a uniform rate of profit

	$s_p$	$r_1^s$	$r_2^s$
$u_1^{**}$	–	–	0
$u_2^{**}$	–	–	–
$r^{**}$	–	–	0
$g^{**}$	–	–	0
$k^{**}$	–	0	–

In the last row of Table 2, we see that an increase in the costing margins of the consumption sector does not modify the ratio of sectoral capital stocks in the long run, while this ratio has an inverse relationship with the costing margins of the investment sector. The latter result is compatible with what was obtained in Regime 1.

It should further be noted that actual profit rates get equalized in this regime despite the fact that target rates of return remain exogenously set. The profit rate equalization obtained in this regime is thus unlikely to be the one described by Sraffian authors, who most likely assume the realization of equalized target rates of return in the pricing equations, and thus an additional mechanism is required to fully reconcile the Kaleckian and Sraffian views.

### 3.2. Regime 3: A Fully-Adjusted Economy

Regime 3 presents a classical model, that is, a fully-adjusted economy ‘in which a

uniform rate of profit prevails, and the productive capacity installed in each industry is exactly sufficient to produce the quantities that the market absorbs when commodities are sold at their natural prices' (Vianello, 1985, p. 71). Following Lavoie (1995; 1996; 2003), we assume that the economy arrives at fully-adjusted positions in the long run through an adjustment of target rates of return towards actual profit rates, i.e., through the endogenous target rates. Reading from equation (8), we know that when the actual rate of capacity utilization is above the standard rate, the actual profit rate is higher than the target rate. In that case, firms will slowly raise the target rate of return until the actual rate of capacity utilization arrives at the standard rate with a decrease in effective demand, and *vice versa*. Eventually, an economy achieves fully-adjusted positions as the actual rate of capacity utilization converges towards the standard rate.

We specify the adjustment mechanism as follows:

$$\dot{r}_i^s = \pi_i (r_i - r_i^s) \quad (26)$$

where  $\pi > 0$  is a reaction coefficient. Thus, the target rate of return of an industrial sector is influenced by the profit rate realized in that sector. This kind of adjustment process, associated to a normal-cost pricing formula, has close similarities to the full-cost pricing model presented by Boggio (1986).

In this regime, changes in effective demand lead only to changes in the *size* of productive capacity, not to changes in the *rate* of capacity utilization in the long run (Vianello, 1985, pp. 72-73).<sup>22</sup> With the fully-adjusted target rate of return, the fluctuation of actual profit rates leads to changes in costing margins and in prices of goods: for instance, if the actual rate of capacity utilization is above the standard rate, then the price of a product will increase due to rising costing margins.<sup>23</sup> In this case, relative prices depend on the given values of the parameters. Using equations (1), (5.1) and (5.2), the long-run equilibrium relative price can be written as

$$(p_1 / p_2)^{**} = \alpha_1 / \alpha_2 + [(\sigma_1 \alpha_2 u_2^s - \sigma_2 \alpha_1 u_1^s) / (\alpha_2 u_1^s u_2^s)] r^{**}$$

Hence, if  $(\sigma_1 / \alpha_1) / (\sigma_2 / \alpha_2) > u_1^s / u_2^s$ , then  $\partial(p_1 / p_2)^{**} / \partial r^{**} > 0$ , and *vice versa*. In this regime, therefore, relative prices depend on the income distribution

<sup>22</sup> This is also pointed out by Garegnani (1983, p. 75); 'a satisfactory long-period theory of output does not require much more than (a) an analysis of how investment determines saving through changes in the level of *productive capacity* (and not only through changes in the *level of utilization* of productive capacity)...

<sup>23</sup> It is compatible with a price equation suggested by Duménil and Lévy (1995; 1999). They suggest that in a neo-Kaleckian model it can be presented as  $p_i = p_{i(-1)} + \phi(u_{i(-1)} - u_{i(-1)}^s)$  (in the present model, through equation (26)). Therefore, higher price levels correspond to actual rates of capacity utilization being higher than standard rates.

between workers and capitalists, but the direction of the change in relative prices depends on the chosen technology of the economy and the standard rates of capacity utilization set by firms.

Here, it is not easy to show mathematically the stability of this regime because the substitution of equation (23) into (26) leads to a non-linear form. Instead of finding a stability condition, we show that there exists a locally stable long-run position by using a phase diagram. Since  $\dot{r}^s = 0$  in the steady state,

$$r^{**} = r_i^{**} = r_i^s \tag{27}$$

To draw the  $\dot{r}_1^s = 0$  curve, replacing  $r_1$  in the left hand side of equation (23) with  $r_1^s$  and solving for  $r_1^s$ , we have

$$r_1^s = (\gamma_0 + \gamma_2 u_1^s) / (s_p - \gamma_1) \tag{28}$$

Since the  $\dot{r}_1^s = 0$  curve is independent of  $r_2^s$ , it is drawn as a vertical line on a plane with axes given by  $r_1^s$  and  $r_2^s$ . If the target rate of return in the consumption sector is larger than the actual profit rate of that sector, i.e., on the right hand side of the  $\dot{r}_1^s = 0$  curve in Figure 3, the target rate of return will decrease over time ( $\dot{r}_1^s < 0$ ) by equation (26), and *vice versa*.

Let us consider the  $\dot{r}_2^s = 0$  curve. Replacing  $r_2$  in the left hand side of equation (23) with  $r_2^s$ , we obtain

$$r_2^s = \gamma_0 r_1^s / [(s_p - \gamma_1) r_1^s - \gamma_2 u_1^s] \tag{29}$$

Since  $dr_2^s / dr_1^s < 0$  and  $d^2 r_2^s / (dr_1^s)^2 > 0$ , the  $\dot{r}_2^s = 0$  curve is convex to zero. If the target rate of return in the investment sector is greater than the actual profit rate of that sector, it will increase over time ( $\dot{r}_1^s < 0$ ) by equation (26), and *vice versa*.

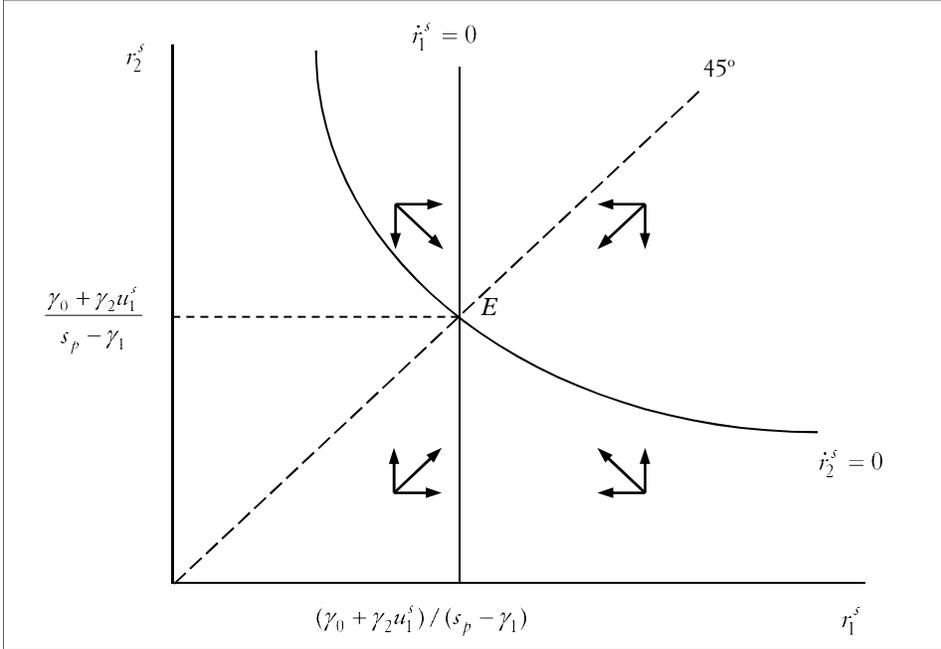
As shown in Figure 3, the target rates of return converge towards point E where the two curves intersect, and hence a uniform target rate of return is locally stable at that point.<sup>24</sup> In other words, the long-run target rate of return converges towards a uniform rate of profit over time, and it results in the fully-adjusted economy in the

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<sup>24</sup> Strictly speaking, we must simultaneously show that the ratio of sectoral capital stocks also converge towards the (locally) stable long-run position when there exists a deviation from the initial steady-state level. Intuitively, in the short period, a shock that pushes the economy away from its initial steady state will immediately change the actual profit rate and the ratio of sectoral capital stocks without a change in the target rate of return. As the adjustment mechanism works over time, this ratio would converge towards the new steady state as shown in Regime 2, with the adjustment of the target rate of return. Kim (2007) shows that there are locally stable long-run positions through simulation of Regime 3 within a stock-flow consistent framework.

long run.

[Figure 3] Dynamics of target rates of return



Therefore, the long-run ratio of sectoral capital stocks and the long-run uniform rate of profit are respectively

$$k^{**} = u_2^s (s_p - \gamma_1) / [s_p \sigma_2 (\gamma_0 + \gamma_2 u_1^s)] - 1 \tag{30}$$

$$r^{**} = r_1^{**} = r_2^{**} = r_1^{s**} = r_2^{s**} = (\gamma_0 + \gamma_2 u_1^s) / (s_p - \gamma_1) \tag{31}$$

Thus, whereas the long-run ratio of sectoral capital stocks is not independent of the standard rate of capacity utilization of the consumption sector, the long-run (uniform) rate of profit is still independent of the variables of the investment sector and its value is the same as that in a one-sector model (due to the reason presented in Regime 2).

The long-run actual rate of capacity utilization converges towards the standard rate,

$$u_i^{**} = u_i^s \tag{32}$$

and substituting equation (31) into (25), the long-run rate of accumulation is obtained

$$g^{**} = g_1^* = g_2^{**} = (\gamma_0 + \gamma_2 u_1^s) / [1 - (\gamma_1 / s_p)] \quad (33)$$

In a fully adjusted economy, the long-run actual rate of accumulation therefore depends only on the propensity to save and the parameters of the consumption sector, as was already the case in Regime 2, with its value being also the same as that found in a one-sector model.

Differentiating equations (31) and (33) with respect to the propensity to save, we have:

$$dr^{**} / ds_p < 0; \quad dg^{**} / ds_p < 0$$

An increase in the propensity to save will shift both the  $\dot{r}_1^s = 0$  curve and the  $\dot{r}_2^s = 0$  curve to the left by equations (28) and (29), and as shown in Figure 4, profit rates in both sectors will go down over time. Intuitively, we can see that this occurs because in the initial period an increase in the propensity to save induces lower actual rates of capacity utilization in the consumption sector first, and then lower rates in the investment sector with a time lag, which bring about lower actual profit rates. Since the target rates of return are higher than actual profit rates in the short run, the target rates will decrease towards the actual rates by the adjustment mechanism, and eventually the long-run profit rate in the new steady state will be lower than in the initial steady state. Therefore, an increase in the propensity to save leads to a lower long-run profit rate even in a fully-adjusted economy. A lower long-run profit rate lowers the long-run rate of accumulation and hence the ‘paradox of thrift’ still holds in this regime, although there is no long-run impact on capacity utilization rates. The ‘paradox of thrift’ is sustained even in this ‘classical’ regime because of the presence of hysteresis effects: there is hysteresis in growth due to the flexible target rate of return.

The first derivative of equation (30) yields

$$dk^{**} / ds_p > 0$$

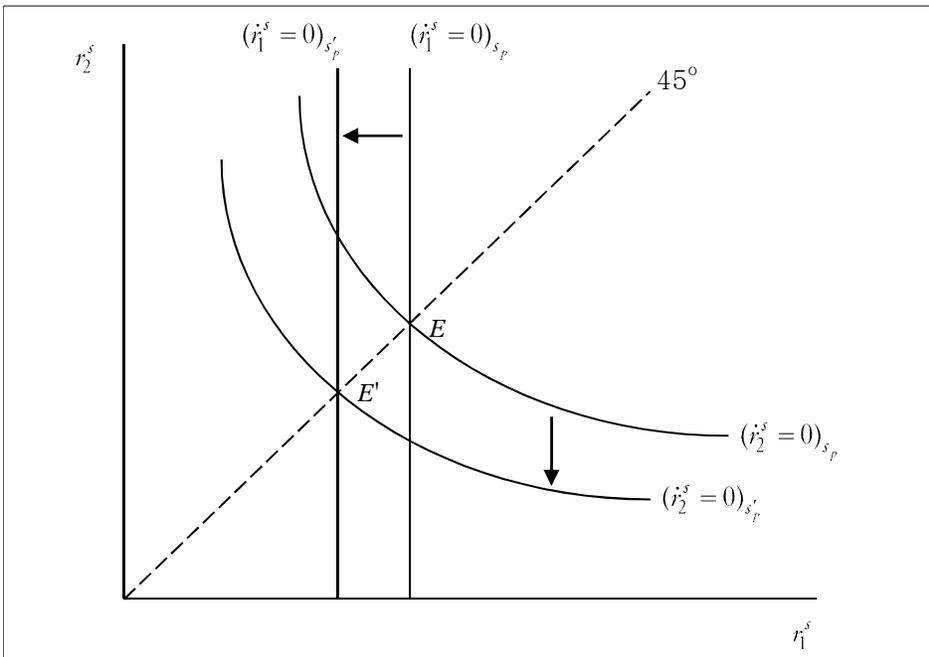
and hence, unlike Regime 1 and Regime 2, the higher propensity to save leads to a rise in the ratio of sectoral capital stocks. This is what one would normally expect: a lower rate of accumulation is associated with a higher proportion of the capital stock being allocated to the consumption sector.<sup>25</sup>

We can further note, checking equations (31) and (33), that changes in income distribution have no more long-run effects on the accumulation rate and the profit

<sup>25</sup> Indeed it cannot be otherwise if the rate of utilization is exogenously given in the long run, as it is here, because, as Lavoie and Ramírez-Gastón (1997, p. 161) recall, by definition  $g = u_i / \sigma_i (1 - k)$ .

rate. The accumulation rate only depends on the coefficients of the investment function of the consumption sector, on its standard rate of utilization, and on the propensity to save. However, a temporary increase in real wages will have positive effects on economic activity in the short run. Hence, while the assumed adjustment mechanisms will bring the economy to a steady state with no undesired excess capacity, where changes in income distribution have no effect on accumulation and utilization rates, their average values over the transition period will be higher than their steady-state values. This is a major point, made repeatedly by Sraffians (Garegnani, 1992; Serrano, 1995; Park, 1997A; Cesaratto, 2015; Freitas and Serrano, 2015), and recently reasserted and proven within the context of a standard Kaleckian model with the help of a different converging mechanism (Allain, 2015; Lavoic, 2016).<sup>26</sup>

[Figure 4] Dynamics of the higher propensity to save



### IV. Conclusion

In this paper, we have analyzed three regimes specified by different investment functions and specific adjustment mechanisms, incorporating the arguments of both

<sup>26</sup> This mechanism relies on a non-capacity creating autonomous component of aggregate demand combined with the addition of a weak Harrodian component in the investment function.

neo-Kaleckians and neo-Marxists/Sraffians into a two-sector model.

We find that the ‘paradox of thrift’ holds for all regimes in the long run, as summarized in Table 3, whether there exists or not an adjustment mechanism. It is compatible with the result obtained in canonical neo-Kaleckian models, so that economic growth is ‘demand-led’ or ‘consumption-led’ in the short run as well as in the long run.

[Table 3] Long-run effects of the higher propensity to save

	Regime 1	Regime 2	Regime 3
$g^{**}$	–	–	–
$u_i^{**}$	–	–	0
$r_i^{**}$	–	–	–

As to the impact of changes in income distribution, things are more ambiguous. In the regime deprived of a classical adjustment mechanism, the economy remains wage-led, but the paradox of costs may or may not apply, meaning that a sector that increases its target rate of return may indeed achieve a higher realized profit rate in the long run. With a mechanism designed to equalize realized profit rates, the economy remains wage-led only when costing margins of the consumption sector are modified; when changes in real wages arise as a consequence of a change in the costing margin of the investment sector, this has no long-run consequences on the realized profit rate and the rate of accumulation. When adding a mechanism adjusting target rates of return to realized profit rates, changes in the parameters determining income distribution have no effect on long-run profit rates and growth rates.

Our analysis also shows that in regimes with a uniform rate of profit, where firms in the investment sector adjust to investment decisions in the consumption sector, the long-run profit rate and the long-run rate of accumulation depend only on parameters of the consumption sector and the propensity to consume, which are exactly the same as those obtained in a one-sector neo-Kaleckian model. In addition, for non-fully adjusted regimes, the ratio of sectoral capital stocks is associated negatively with an increase in the target rate of return, that is, the proportion of the capital stock located in the investment sector is related inversely to the long-run rate of accumulation. However, the reportioning that occurs in the dynamically-stable two-sector Cambridgean or Hicksian model with full capacity is restored in the fully-adjusted regime described here.

Therefore, although there still exist debates about long-run positions and the existence of adjustment mechanisms, our analysis suggests that aggregate demand plays a crucial role in the determination of the growth path, both in the short and in

the long run, and hence that economic growth could be consumption-led and wage-led. Our analysis also offers some justification for using simple one-sector neo-Kaleckian growth models, since most of the key results of these models are sustained in more sophisticated two-sector models with various closures; this however needed to be demonstrated, and this is what we have done here.

Finally, we have explored long-run relationships in a simple two-sector model, but growth regimes in multi-sector models could be sensitive to the model specification and the chosen values of parameters because of complicated dynamics due to sectoral heterogeneity. This means that we need to examine multi-sector models including various economic sectors such as financial and overseas sectors,<sup>27</sup> and to investigate characteristics of growth regimes through empirical analysis, using multi-sectoral data. These topics are left for future research.

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<sup>27</sup> For example, Kim and Lavoie (2016) explore the dynamics towards long-run positions as well as the impact of changes in various parameters within a multi-sector Kaleckian growth model that incorporates endogenous labour-saving technical progress and price inflation arising from the conflicting claims of workers and firms over income distribution.

## Appendix. Short-run equilibria in Regime 1 and 2

### (1) Regime 1

From pricing equation (1), the national accounts yield

$$p_i \mathcal{S}_i = w \alpha_i \mathcal{S}_i + r_i p_2 K_i \tag{A1}$$

Assuming that supply adjusts to demand within the period, we can rewrite the actual real output equations (2.1) and (2.2) as follows:<sup>28</sup>

$$\mathcal{S}_1 = (w / p_1)(\alpha_1 \mathcal{S}_1 + \alpha_2 \mathcal{S}_2) + (1 - s_p)(p_2 / p_1)(r_1 K_1 + r_2 K_2) \tag{A2.1}$$

$$\mathcal{S}_2 = g_1 K_1 + g_2 K_2 \tag{A2.2}$$

Using equations (1.1)', (1.2)' and (A1), equation (A2.1) can be rewritten as

$$(s_p \sigma_1 r_1^s u_2^s) \mathcal{S}_1 - u_1^s (u_2^s - s_p \sigma_2 r_2^s) \mathcal{S}_2 = 0 \tag{A3.1}$$

and substituting equations (6) and (9)' into (A2.2),

$$-\gamma_{13} \sigma_1 \mathcal{S}_1 + (1 - \gamma_{23} \sigma_2) \mathcal{S}_2 = (\gamma_{10} K_1 + \gamma_{20} K_2) \tag{A3.2}$$

Hence, by putting equations (A3.1) and (A3.2) together,

$$\begin{bmatrix} s_p \sigma_1 r_1^s u_2^s & -u_1^s (u_2^s - s_p \sigma_2 r_2^s) \\ -\gamma_{13} \sigma_1 & (1 - \gamma_{23} \sigma_2) \end{bmatrix} \begin{bmatrix} \mathcal{S}_1 \\ \mathcal{S}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ (\gamma_{10} K_1 + \gamma_{20} K_2) \end{bmatrix} \tag{A4}$$

and by solving equation (A4) on  $\mathcal{S}_1$  and  $\mathcal{S}_2$ , we obtain

$$\mathcal{S}_1 = u_1^s (u_2^s - s_p \sigma_2 r_2^s) (\gamma_{10} K_1 + \gamma_{20} K_2) / \Phi_1 \tag{A5.1}$$

$$\mathcal{S}_2 = s_p \sigma_1 r_1^s u_2^s (\gamma_{10} K_1 + \gamma_{20} K_2) / \Phi_1 \tag{A5.2}$$

where  $\Phi_1$  is the determinant of the first matrix on the left hand side of equation (A4), given by:

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<sup>28</sup> Equation (A2.1) is equivalent to saying that investment must be equal to saving, implying that:  $p_2 \mathcal{S}_2 = s_p (r_1 p_2 K_1 + r_2 p_2 K_2)$ .

$$\Phi_1 = s_p \sigma_1 r_1^s u_2^s (1 - \gamma_{23} \sigma_2) - \gamma_{13} \sigma_1 u_1^s (u_2^s - s_p \sigma_2 r_2^s)$$

and the short-run stability condition is  $\Phi_1 > 0$ , while the numerator of equation (A5.1) is positive at all times since we already assumed  $u_2^s > \sigma_2 r_2^s$  for outputs to be positive.

Substituting equations (A5.1) and (A5.2) into (6), we obtain equations (10.1) and (10.2), and then equations (11.1) and (11.2).

## (2) Regime 2

Substituting equations (6), (8), (18.1) and (18.2) into (A2.2), we obtain

$$-\sigma_1 u_2^s [(\gamma_1 r_1^s + \gamma_2 u_1^s)(1 + 1/k) - \beta r_1^s / k] S_1 + u_1^s (u_2^s - \beta \sigma_2 r_2^s) S_2 = \gamma_0 u_1^s u_2^s (K_1 + K_2) \quad (\text{A6})$$

and with equation (A3.1),

$$\begin{bmatrix} s_p \sigma_1 r_1^s u_2^s & -u_1^s (u_2^s - s_p \sigma_2 r_2^s) \\ -\sigma_1 u_2^s [(\gamma_1 r_1^s + \gamma_2 u_1^s)(1 + 1/k) - \beta r_1^s / k] & u_1^s (u_2^s - \beta \sigma_2 r_2^s) \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \gamma_0 u_1^s u_2^s (K_1 + K_2) \end{bmatrix} \quad (\text{A7})$$

Hence, by solving equation (A7), we have

$$S_1 = \gamma_0 (u_1^s)^2 u_2^s (u_2^s - s_p \sigma_2 r_2^s) (K_1 + K_2) / \Phi_2 \quad (\text{A8.1})$$

$$S_2 = s_p \sigma_1 \gamma_0 r_1^s u_1^s (u_2^s)^2 (K_1 + K_2) / \Phi_2 \quad (\text{A8.2})$$

where  $\Phi_2$  is the determinant of the first matrix on the left hand side of equation (A7), given by:

$$\begin{aligned} \Phi_2 = & s_p \sigma_1 r_1^s u_1^s u_2^s (u_2^s - \beta \sigma_2 r_2^s) - \sigma_1 u_1^s u_2^s (u_2^s - s_p \sigma_2 r_2^s) [(\gamma_1 r_1^s \\ & + \gamma_2 u_1^s)(1 + 1/k) - \beta r_1^s / k] \end{aligned}$$

and the short-run stability condition is  $\Phi_2 > 0$ .

Using equations (6), (8), (A8.1) and (A8.2), we obtain equations (19.1) and (19.2).

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