Eco-Technology Licensing under Emission Tax: Royalty vs. Fixed-Fee

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This article considers the licensing strategies of eco-technology when an innovator provides pollution abatement goods to oligopolistic polluting firms that produce consumption goods and emit environmental pollutants. In the presence of emission tax, two types of licensing contracts, royalty and fixed-fee, are examined to analyze market equilibrium and to compare their welfare consequences. We show that an eco-innovator provides a non-exclusive license under a royalty contract while it might exclude polluting firms under the fixed-fee licensing contract. However, when mixed licensing contract where royalty and fixed-fee contracts are combined together is available, we show that eco-innovator provides non-exclusive license. We also show that, compared to royalty licensing, exclusive fixed-fee contract will increase the welfare but its welfare effect depends on the level of emission tax. Finally, we derive the optimal emission tax and show that an appropriate emission tax combined with non-exclusion policy or profit-cap regulation can increase the welfare.

JEL Classification: L13, D45, Q55, Q58
Keywords: Eco-Technology, Fixed-Fee Licensing, Royalty Licensing, Emission Tax

I. Introduction

In most of the environmental and resource economics literature, the R&D activity of pollution abatement is assumed to be determined only by polluters. However,
recently the provision of goods and services to abate pollution or to manage environmental resources has become the core business of specialized private firms in the eco-industry. Tighter environmental regulations have also contributed to the emergence of this eco-industry. The importance of the eco-industry has been recognized by numerous reports from national and international institutions such as OECD (1996), Berg et. al. (1998), Ecotech Research and Consulting Ltd. (2002), and Kennett and Steenblik (2005).1,2

On the other hand, most eco-technologies are likely to be patented and thus the eco-industry for abatement equipment can be recently characterized by a monopolistic situation. Figure 1 shows the recent trend of environmental technology. Figure 1(a) presents the ratio of environmental technology in total number of patents and indicates that the number of environmental technology is increasing and significant among total patents. Figure 1(b) provides the number of patents under PTC3 in general environmental management technologies.4 It shows that these technologies invented in five countries constitute over 60% of total patents worldwide.

[Figure 1] Trend of environmental technology patents

Source: Author’s calculations using data from OECD patent databases.

1 According to the Environmental Business International (2012), the global market size of the eco-industry was approaching approximately US$899 billion in 2012 and is expected to reach US$992 billion by 2017.

2 The economic framework of the eco-industry was introduced by Feess and Muehlheusser (2002) and David and Sinclair-Desgagne (2005). Canton et al. (2008, 2012) examined the effect of emission tax on the activity of polluting oligopolistic firms toward eco-industry. Lee and Park (2011) also incorporated the subsidy on abatement goods produced in the eco-industry under free entry market structure.

3 The Patent Cooperation Treaty is abbreviated to PTC. The PTC came into existence in 1978, and currently lists 145 countries as contracting signatories.

4 General environmental management consists of air pollution abatement, water pollution abatement and waste management technologies.
Taking the monopolistic characteristics of eco-patents in the eco-industry into consideration, we can raise some economic questions. For example, how does the patent licensing strategy of eco-technology affect the incentives of licensees who emit environmental pollutants? How do the environmental regulations such as the imposition of an emission tax on the polluting industry influence the innovator’s choice on licensing strategies? What are the economic consequences of patent licensing strategies on the society and on the environment? We are trying to answer these questions in this paper.

In the literature of industrial economics on R&D innovation activity without involving environments, many works have studied a patent licensing of cost-reducing innovation and analyzed the welfare consequences. Previous researches with outsider innovator have mainly focused on market structure and regulatory policy, and compare the efficiencies of some different types of licensing. Early work shows that with outside innovation, fixed fee policy is superior to royalty (or auction) policy in perfect competition (Kamien and Tauman, 1984; Katz and Shapiro, 1985), homogenous oligopoly model (Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien et al., 1992), asymmetric Cournot industry (Stamatopoulos and Tauman, 2009), Stackelberg competition (Shin, 2014). But, there are still debates on these results since royalty is preferred to fixed-fee in Bertrand model (Muto, 1993), product differentiation model (Poddar and Sinha, 2004; Bagchi and Mukherjee, 2010), open economy and strategic tax policy (Mukherjee, 2007; Mukherjee and Tsai, 2010), and dynamic frameworks (Saracho, 2011). However, previous literature did not take the government regulation on eco-technology into analysis.

Recently, an important research issue on environmental innovation is to study eco-R&D investment. These studies consider R&D investment as an endogenous variable and analyze how eco-R&D activity is affected by environmental regulation, mostly under the imposition of an emission tax. Hattori (2011) finds that a higher emission tax may increase eco-R&D investment when tax payment is rather small under monopolistic provision of eco-technology. Chien and Hu (2001) compares the non-cooperative eco-R&D activities of firms and show that an R&D cartelization generates the lowest level of eco-R&D activities and the highest level of emissions by regulated firms. Poyago-Theotoky (2007) analyzes the equilibrium outcomes between an independent eco-R&D and an R&D cartel in the presence of spillovers and finds that the relative performance depends on the effectiveness of eco-R&D.

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5 On the other hand, many studies have also focused on patent licensing with inside innovation. With inside innovation, many studies show that royalty is preferred to a fixed fee in Cournot duopoly with a homogenous good (Wang, 1998), differentiated Bertrand duopoly (Wang and Yang, 1999), incumbent innovators in a homogenous Cournot oligopoly (Kamien and Tauman, 2002), leadership structure (Kabiraj, 2005). However, Wang (2002) presents that fixed fee dominates royalty for an insider patentee with a heterogeneous duopoly if the product differentiation is sufficiently large.
activities and the environmental damage. Mcdonald and Payogo-Theotoky (2014) and Tsai and Chang (2014) also consider the organization of R&D on eco-technology and show that the optimal emission tax depends on the type of R&D spillover, input or output spillover, or the type of R&D organizations.

Contrary to the dynamic approach on eco-R&D investment, we treat an eco-technology as exogenously given by an outside innovator. Specifically, this article considers a model of patent licensing of eco-technology between an innovator and oligopolistic polluting firms where polluting firms may purchase a license of pollution abatement technology from an outside innovator, and investigate the innovator’s incentive of eco-technology licensing. We analyze two types of patent licensing contract, royalty licensing and fixed-fee, and examine how patent licensing of eco-innovator on eco-technology affects the incentive of licensees under emission tax. We show that eco-innovator provides non-exclusive license under royalty contract while it might exclude polluting firms under fixed-fee licensing. However, when mixed licensing contract where royalty and fixed-fee contracts are combined together is considered, we show that the eco-innovator provides a non-exclusive license. We also show that, compared to royalty licensing, exclusive fixed-fee contract will increase the welfare but its welfare effect depends on the level of emission tax. Finally, we derive the optimal emission tax and show that, combined with a non-exclusion policy or a profit-cap regulation, an appropriate emission tax can increase the welfare.

The organization of this paper is as follows. Section 2 constructs the basic model of licensing strategies. Section 3 analyzes the market equilibrium of royalty licensing under which licensee should pay usage fee for purchasing an abatement product from innovator. In Section 4, we develop fixed-fee licensing of the innovator where fixed-fee for licensing does not include usage fee for purchasing an abatement product, and examine the behaviors of polluting firms and innovator. Section 5 compares the two patent licensing contracts, royalty licensing and fixed-fee licensing, and examines the case of mixed licensing contract. Section 6 discusses the relation between licensing strategy and emission tax. Furthermore, it derives the optimal emission tax to provide policy implications. The final section provides a conclusion.

II. The Model

Consider the outside innovation case that an innovator licenses eco-technology to polluting firms in the market where the emission tax is imposed, and makes a contract between royalty licensing or fixed fee licensing. When the innovator

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6 Recent survey on eco-technology in Seoul, ‘Low Carbon Green Growth Expo 2012’, reports that
licenses its eco-technology to \( k \) firms out of \( n \) firms, the licensed firms can reduce the pollution and thereby emission tax expenditure.

The inverse demand function for final goods is given by \( P(Q) = A - Q \), where
\[
Q = \sum_{i \in S} \tilde{q}_i^L + \sum_{i \in S} \tilde{q}_i^N
\]
is market output level of final goods, \( \tilde{q}_i^L \) and \( \tilde{q}_i^N \) are the firm’s output level supplied by licensees and non-licensees, respectively, and \( S \) represents the set of licensees. Without loss of generality, as analyzed by Canton, et al. (2012), we specifically consider eco-technology\(^7\) in which emission function of the production is defined as
\[
e_i = e(q_i, a_i) = \frac{(q_i - a_i)^p}{2}, \quad \text{where } a_i \text{ is the amount of abatement goods, purchased by polluters from the innovator to reduce pollution emission level, } 0 \leq a_i \leq q_i^L.
\]
This emission function represents that more production entails more pollution \((\frac{\partial e}{\partial q} > 0)\), more abatement decreases total emissions \((\frac{\partial e}{\partial a} > 0)\), emissions from the last unit produced increase with the production level \((\frac{\partial e}{\partial q} > 0)\), abatement effort is subject to diseconomies of scale \((\frac{\partial^2 e}{\partial a^2} < 0)\), and the higher the abatement, the less the last unit produced generates pollution \((\frac{\partial^2 e}{\partial q^2} < 0)\).\(^8\) Let \( E = n e_i \) be total emission level emitted to the market, and then environmental damages are denoted by \( D(E) = dE = \frac{n}{2}(q_i - a_i)^2 \), which is constant to the total emission level. Finally, for simplicity, we assume that the production cost of polluting firms is zero and the production cost of an innovator is constant, \( c > 0 \).

Timing of the game is as follows: In the first stage, given emission tax, an innovator providing eco-technology decides a contract between royalty licensing or fixed fee licensing. Also the innovator announces eco-technology price, royalty or fixed fee. In the second stage, given license contract, polluting firms simultaneously decide whether or not to purchase a license. Finally, when the numbers of \( k \) patents are contracted to purchase licenses, polluting firms choose their outputs production level \((\tilde{q}_i^L, \tilde{q}_i^N)\) and abatement purchasing level \((a_i)\) simultaneously in a Cournot competition in the third stage. Then, the sub-game perfect Nash

\[^7\] In general, there are two-types of eco-technologies, end-of-pipe and clean technology (Requate, 2005; Tsai, et al., 2014). The former refers to equipment installed by a firm that can reduce gross emissions while keeping total output unchanged, while the latter involves a change in a firm’s production process that generates less pollution per unit of output. Thus, the firm’s output and eco-R&D decisions are independent under end-of-pipe technology while those decisions are inter-wined under clean technology. The end-of-pipe technology was specified by David and Sinclair-Desgagne (2005, 2010) and Canton, et al. (2008). Our approach adopts the economic model of clean technology introduced by Canton, et al. (2012) and Tsai, et al. (2014).

\[^8\] This specification satisfies the first-order and second-order conditions for the profit maximization problem in the eco-industry and thus, it ensures the interior solutions of the equilibrium in eco-industry.
equilibrium will be derived by backward induction.

III. Royalty Licensing Contract

Under royalty licensing contract, the eco-innovator sets royalty \( r \) and determines \( k \) numbers of licensees. Then, given emission tax \( t \), polluting firms decide whether or not to buy a license and then, produce final goods and purchase abatement goods in a Cournot competition. In the third stage, let \( \pi^L(k) \) and \( \pi^N(k) \) denote the profit of licensed polluting firms and that of unlicensed polluting firms, respectively, when the number of licensed firms is \( k \). Then, we have the following profit functions:

\[
\pi^L_i(k) = P(Q)q^L_i - ra_i - te^L_i, \quad (1)
\]

\[
\pi^N_i(k) = P(Q)q^N_i - te^N_i, \quad (2)
\]

where \( Q = \sum_{i \in S} q^L_i + \sum_{i \in S} q^N_i \), \( e^L_i = \frac{q^L_i - a_i}{2} \) and \( e^N_i = \frac{q^N_i - a_i}{2} \).

**Lemma 1.** Under royalty licensing contract, the eco-innovator chooses non-exclusive licensing strategy, \( k = n \).

**Proof.** Contradictorily, suppose that the eco-innovator chooses the number of licensees \( k \) where \( 0 < k < n \), which supports non-negative values for outputs, \( q^L_i \) and \( q^N_i \), and the amount of abatement goods, \( a_i \) at equilibrium. Then, it should satisfy the following first-order conditions for profit maximization:

\[
\frac{\partial \pi^L_i}{\partial q^L_i} = A - 2\sum_{i \in S} q^L_i - \sum_{i \in S} q^N_i - q^L_i - t(q^L_i - a_i) = 0 \quad (3)
\]

\[
\frac{\partial \pi^L_i}{\partial a_i} = -r + t(q^L_i - a_i) = 0 \quad (4)
\]

\[
\frac{\partial \pi^N_i}{\partial q^N_i} = A - 2\sum_{i \in S} q^L_i - \sum_{i \in S} q^N_i - q^N_i - tq^N_i = 0 \quad (5)
\]

Solving these first-order conditions simultaneously, we can derive the equilibrium outputs and the amount of abatement goods as follows:

\[
q^L_i = \frac{A(1+t) - r(1+n + t - k)}{(1+n+ t+ tk)}, \quad q^N_i = \frac{A + kr}{(1+n+ t+ tk)}
\]

and
Then, we need to check whether the innovator has an incentive to deviate from this supposed equilibrium where $0 < k < n$. Given the supposed equilibrium, the innovator determines the royalty and the number of licensee to maximize the following profit:

$$\max_{\iota} \pi^M = (r-c)k \alpha_i \text{ s.t. } \alpha_i = \frac{(1+t)\{tA-r(1+n+t)\}}{t(1+n+t+tk)}$$

(6)

Then, from (6) we have the following relation that the innovator’s profit increases with more licensing firms: 9 $\frac{\partial \pi^M}{\partial k} = (r-c)\alpha_i + (r-c)k \frac{\partial \alpha_i}{\partial k} = (r-c)\alpha_i [\frac{1}{1+n+tk}] + (r-c)\alpha_i [\frac{k}{1+n+tk}] > 0$. Thus, the innovator has no incentive to exclude polluting firms at equilibrium under royalty licensing, i.e., $k = n$, which is contradict to the supposed equilibrium.

Note that Lemma 1 holds when the production cost of the innovator is convex function. 10

**Proposition 1.** Under royalty licensing contract, the equilibrium number of licensee is $k = n$ when emission tax is high while $k = 0$ when emission tax is low.

**Proof.** We will first examine the case of $k = n$. Then, excluding the first-order condition in (5) or setting $q_i = 0$, the equilibrium outputs and abatement goods are determined by the first-order conditions in (3) and (4): $q_i^L = \frac{A+c}{A+c+n}$ and $q_i = \frac{tA-r(1+n+t)}{t(1+n+t+tk)}$.

Then, from the first order condition of the innovator in the first stage, $\frac{\partial \pi^M}{\partial k} = k \alpha_i + (r-c)k \frac{\partial \alpha_i}{\partial k} = 0$, we can get $\alpha_i^* = \frac{A+c(1+n+t)}{2(1+n+t)}$, $\alpha_i = \frac{tA-r(1+n+t)}{t(1+n+t+tk)}$, and $q_i^L = \frac{A(2n+2c)(1+n+c)}{2(1+n+c)(1+n+c)}$. Therefore, if $t > \frac{c(1+n)}{A-r}$, we have $\alpha_i > 0$ at equilibrium when $k = n$.

Next, from the viewpoint of licensees’ incentive, we need to compare $\pi^L(k)$ and $\pi^N(k)$ when $0 < k < n$. Then, we can get:

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9 Notice that the elasticity of abatement goods on the number of licensees is inelastic, i.e.,

$-1 < \frac{k \partial q_i}{q_i \partial k} = -\frac{k^2}{(1+n+tk)^2} < 0$.

10 For a proof, see the Appendix II.
\[ \pi^L(k) - \pi^N(k) = \frac{(tA - c(1+n+t))}{8t(1+n+t+tk^2)(1+n+t)} \] 
\[ \left\{ 2(A-c)t^2 + \frac{1}{(7A-5c) + 2n(A-c) + 2k(A+c)t^2}{(1+n)[2t(2A-2c(n+3)+kc) + (tA-c(1+n))] > 0} \right\} \]

for \( 0 < k < n \) when \( t > \frac{c(1+n)}{A-c} \). That is, licensed firms will have higher profit than non-licensed firms and thus, there does not exist the case of \( \pi^L(k) = \pi^N(k) \) for \( 0 < k < n \) when \( t > \frac{c(1+n)}{A-c} \).

However, when \( t \leq \frac{c(1+n)}{A-c} \), we have \( a_t = 0 \) and \( \pi^L(k) \leq \pi^N(k) \) for \( 0 < k < n \). It implies that \( k = 0 \) at equilibrium. ■

Proposition 1 states that under royalty licensing contract, the innovator has no incentive to exclude polluting firms and all polluting firms decide to either buy licenses or not. Actually, this is well-known result in the traditional patent licensing economics. See, for example, Kamien and Tauman (1986), Katz and Shapiro (1986), Kamien et al. (1992), and Shin (2014). They examined the production cost innovation and showed that under royalty licensing in a homogeneous oligopoly market, the outside innovator sells licenses to all firms because all firms will get the benefit from the cost reduction after licensing and thus the relative marginal costs of the firms are the same, which results in the same profits levels of the firms after licensing. However, in our model, we consider the eco-technology innovation, which can reduce emission tax reduction, and thus the licensing is only beneficial to the polluters when there is a tough environmental regulation. Hence, Proposition indicates that in the presence of emission tax, the emission tax affects the incentive of polluting firms whether to either buy licenses or not. In particular, all polluting firms buy the license when emission tax is high while no firms buy when emission tax is low.

We will now examine and compare two equilibria: \( k = n \) and \( k = 0 \). First, consider a case of \( k = n \) where all firms buy the license. In equilibrium, we have the followings:

\[ P^L(Q) = \frac{A(2n+2t+n+2) + cn(1+n+t)}{2(1+n)(1+n+t)}, \quad Q^L = \frac{n(A(2n+t+2) - c(1+n+t))}{2(1+n)(1+n+t)}, \]
and \( G^L = n a_t = \frac{n(tA-c(1+n+t))}{2t(1+n)} \).

Some comparative static effects with respect to emission tax are as follows:
These comparative static results show that an increase in emission tax induces polluting firms to buy eco-technology more, and to increase both the equilibrium price of eco-technology and the price of final goods by polluting firms, which reduces the outputs of firms.

The profits of the innovator and licensed polluting firms are as follows:

\[ \pi^M = \frac{n(tA - c(1 + n + t))^2}{4t(n + 1)(1 + n + t)} \quad \text{and} \quad \pi^L_i(n) = \pi^L_i(0) = \frac{A^2t(8n^2 + 16n + 2t^2 + n^2 + 10n + 9t + 8) - Atc(8mt + 10t - 2n^2 + 6 + 4t^2 + 10n - 2n^3)}{8t(n + 1)^2(1 + n + t)^2} \]

\[ + \frac{c(4n^3 + 6n^2 + 4n + 5r^2 + 4t + 1 + 8n^2 + n^2 + 6n^2 + 10mt + 2n^3 + n^2 + 2nt)}{8t(n + 1)^2(1 + n + t)^2} \]

Second, consider the other case of \( k = 0 \), in which no polluting firms buy eco-technology. We denotes \( q^N_i \) as non-licensed firm \( i \)'s output. Then, from the first-order-conditions, we can get \( q^N_i = \frac{A}{(1 + n + t)} \), \( a_i = 0 \), \( P^N(Q) = \frac{A(1 + t)}{2(1 + n + t)} \), and \( Q^N = \frac{A}{(1 + n + t)} \). All polluting firms’ profit are same as \( \pi^N_i(0) = \frac{A^2(1 + t)}{2(1 + n + t)^2} \) and the profit of innovator is zero, \( \pi^M = 0 \).

Finally, we will compare the welfares under two equilibria. The welfare is the sum of consumer surplus and the profits of innovator and polluting firms minus environmental damage, which is given by \[ W(Q^L, q^N_i, a_i) = \int_0^Q P(u)du - c(ku - dq^N_i) - dE, \]
where \[ Q = kq^L_i + (n - k)q^N_i, \quad E = ke^L_i + (n - k)e^N_i, \quad e^L_i = \frac{(1 - k)^2}{2}, \quad \text{and} \quad e^N_i = \frac{(1 - k)^2}{2}. \]

Then, we have the following welfares in each case.

\[ W^L = \int_0^Q P(u)du - cnq^L_i - dne^L_i = AQ^L - \frac{(Q^L)^2}{2} - cna_i - \frac{dn(q^L_i - a_i)^2}{2} \]
\[ W^N = \int_0^Q P(u)du - dne^N_i = AQ^N - \frac{(Q^N)^2}{2} - dne^N_i = \frac{A^2n(2 + 2t + n - d)}{2(1 + n + t)^2} \]

When we compare (8) and (9), the welfare difference between two cases is ambiguous. It depends on the size of abatement cost and damage of emissions. Furthermore, when all firms purchase the licensing under any given tax, of which
welfare is (8), tax effect has a trade-off. If emission tax increases, emissions are reduced by the polluting firms since they will buy more eco-technologies, but, it will decrease outputs.

**[Figure 2]** Welfare under royalty licensing \((A = 100, \quad n = 3, \quad c = 5 \text{ and } d = 1)\)

Let us examine an example where \(A = 100, \quad n = 3, \quad c = 5 \text{ and } d = 1\). Notice that \(t_0 = \frac{c(1 + n)}{A - c} = \frac{4}{19} \approx 0.21\) and welfare reaches a maximum value at the emission tax \(t^R = 0.73\). Then, it shows that \(\pi^L(k) > \pi^N(k)\) when \(t > 4/19\) and \(\pi^L(k) < \pi^N(k)\) when \(t < 4/19\). Thus, when emission tax is higher than 4/19, every polluting firm always prefers buying the license to not buying under royalty licensing, but when emission tax is lower than 4/19, no firms buy the license. Figure 2 shows that welfare gradually decreases until \(t_0\) and then it increases as all polluting firms buy eco-technology over \(t_0\).

**IV. Fixed-Fee Licensing Contract**

In this section, we consider fixed-fee licensing contract, in which eco-innovator can control the profits of licensees by restricting the number of licensees, \(k\), and by announcing fixed-fee, \(f\). Then, we examine how it is possible for the innovator to exclude the number of licensees to increase its profit. The profit functions of a licensed firm, \(\pi^L(k)\), and a non-licensed firm, \(\pi^N(k)\), when the number of licensee is \(k\), are as follows:

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\(^{11}\) Some detailed analysis on optimal emission tax will be discussed in section 6. See, equation (20). Notice also that we can have the optimal emission tax when \(t < t_0\) at \(d = 1\), using the welfare in (9). That is, zero emission tax is optimal when no polluting firms buy eco-technology.
\[ \pi_i^L(k) = P(Q)q_i^L - te_i^L - f \quad \text{where} \quad e_i^L = \frac{(q_i^L - a_i)^2}{2}. \]  
(10)

\[ \pi_i^N(k) = P(Q)q_i^N - te_i^N \quad \text{where} \quad e_i^N = \frac{(q_i^N)^2}{2}. \]  
(11)

Notice that under fixed-fee licensing, licensed firms will set \( a_i = q_i^L \) and thus, \( e_i^L = 0 \) at the optimal choice since there is an emission tax on the emission level while the usage price of abatement goods is zero. That is, \( \pi_i^L(k) = P(Q)q_i^L - f \).

The first-order conditions for profit maximization are as follows:

\[ \frac{\partial \pi_i^L}{\partial q_i^L} = A - Q - q_i^L = 0 \]  
(12)

\[ \frac{\partial \pi_i^N}{\partial q_i^N} = A - Q - q_i^N - t q_i^N = 0 \]  
(13)

Then, we have \( q_i^L(k) = a_i(k) = \frac{A(1+t)}{(1+n+tk)} \) and \( q_i^N(k) = \frac{A}{(1+n+tk)} \) at equilibrium, which gives \( \pi_i^L(k) = \frac{A(2+t)}{2(1+n+tk)} \) and \( \pi_i^N(k) = \frac{A(2+t)}{2(1+n+tk)} \). Notice that \( \frac{\partial q_i^L}{\partial k} > 0 > \frac{\partial q_i^N}{\partial k} \).

Thus, emission tax will encourage the production of licensed firms while it will discourage the production of non-licensed firms. Notice also that \( \frac{\partial q_i^L}{\partial k} < \frac{\partial q_i^N}{\partial k} < 0 \).

Thus, as the number of licensee increases, the production level of both licensed and non-licensed firms will be reduced. But, output-decreasing effect on licensed firms is stronger than that on non-licensed firms.

For the incentive compatibility of licensed firms and non-licensed firms, the optimal fixed-fee should be equal to the profit difference of each licensee between accepting and rejecting the licensing offer, that is, \( \pi_i^L(k) - \pi_i^N(k-1) = 0 \) when \( k \geq 1 \). Specifically, we have

\[ f = \frac{A^2(3r^2 + 2r^3 + 4nrt^2 + 2r^2 + 6tn + 2t)k + 3n^2 + 2rn^2 + 2n - 2t + 2m - t^2 - 1}{2(1 + n + k)t^2(1 + n + t + k)^2} \]  
(14)

Notice also that \( \frac{\partial f}{\partial k} < 0 \) and \( \frac{\partial f}{\partial t} > 0 \). That is, fixed-fee will be reduced as the number of licensee increases and as the level of emission tax decreases, because the profit difference from licensing will be reduced.

Then, the profit maximization problem of eco-innovator is as follows:

\[ \max_{k} \pi_k^M = k(f - ca_i) \quad \text{st.} \quad f = \pi_i^L(k) - \pi_i^N(k-1) \quad \text{and} \quad a_i = \frac{A(1+t)}{1 + n + t + tk} \]  
(15)
Considering integer problem\textsuperscript{12} in choosing $k$, the optimal decision on the number of licensees $k^*$ should satisfy that $\pi_k^M \geq \pi_k^M$ for $k \neq k^*$.

Proposition 2. Under fixed-fee licensing contract, the eco-innovator chooses exclusive licensing strategy, $0 \leq k \leq n$.

The proposition 2 shows that under fixed-fee licensing contract, the monopolistic innovator can induce $k$ licensed firm and $n-k$ non-licensed firm to coexist at the equilibrium over specific ranges of emission tax.\textsuperscript{13} Thus, we have: 

$$W^E = \int_0^Q P(u)du - c\pi_k - dE$$

$$= \frac{A[A(n+tk)(n+2+2t+tk)+A(k-n)d-2(k(n+1+t)(1+n+t+tk))]}{2(1+n+t+tk)^2}$$

Finally, the welfare function is as follows:

$$W^E = \int_0^Q P(u)du - c\pi_k - dE$$

where $Q = k^Lq_n^L + (n-k^L)q_n^N$ and $E = \frac{(n-k^L)q_n^L}{2}$ since $a_k^L = q_n^L$ and $e_k^L = 0$.

For the further analysis, let $\pi_k^M(t)$ denote the profit of the innovator when the number of licensed firms is $k$ and the emission tax level is $t$. Notice that $k$ is fixed in a certain range even though $t$ is changed. We also define $t_k$ as the emission tax at which the innovator indifferently chooses fixed-fee licensing to give the license to $k$ firms, i.e., $t_k$ satisfies $\pi_k^M(t_k) \geq 0$ and $\pi_k^M(t_k) = \pi_k^M(t_{k-1})$.

Let us examine the simulation results for the optimal choices of the innovator and welfare under fixed-fee licensing contract when $A=100$, $n=3$, $c=5$ and $d=1$. The optimal choice of the innovator can be shown in Figure 3. For a non-negative profit of the innovator, when licensing to only one firm, it should be

\textsuperscript{12} If there is no integer problem in choosing $k$, the first-order condition gives the implicit optimal number of license: $k^* = \frac{\frac{\partial \pi_k}{\partial k} - \frac{\partial \pi_k^M}{\partial k}}{\frac{\partial \pi_k^M}{\partial n} - \frac{\partial \pi_k}{\partial k}}$ where $\frac{\partial \pi_k}{\partial k} < 0$ and $\frac{\partial \pi_k^M}{\partial k} < 0$.

\textsuperscript{13} Similar results can be found in the traditional patent licensing economics. See, for example, Kamien and Tauman (1986), Katz and Shapiro (1986) and Kamien et al. (1992).
satisfied that \( t \geq t_1 = 0.23 \) since \( \pi_M^0(t_1) = 0 \). Notice that \( t_0 > t_1 = 0.21 \). This implies that fixed-fee licensing will be proposed by the innovator only when royalty licensing can be also effective to induce polluting firms to purchase. Thus, if emission tax is larger than \( t_1 \), it is possible for the innovator to choose fixed-fee licensing. The optimal number of licensed firms depends on the emission tax level. Specifically, optimal number of licensees are \( k^1 = 1 \) when \( t_1 < t < t_2 \), \( k^2 = 2 \) when \( t_2 < t < t_3 \), and \( k^3 = 3 \) when \( t > t_3 \). Therefore, under fixed-fee licensing contract, the innovator can induce \( k \) licensed firm and \( n-k \) non-licensed firm to coexist at equilibrium when emission tax is high, \( t > t_1 \). Figure 3 also presents that welfare increases with the number of licensed firms.

[Figure 3] Optimal choices and welfare under fixed-fee licensing (\( A = 100 \), \( n = 3 \), \( c = 5 \), and \( d = 1 \))

V. Comparison and Combination

5.1. Royalty Licensing vs. Fixed-Fee Licensing

Under fixed-fee licensing contract, the increase of production of abatement goods will reduce the emission level, but will increase abatement production cost. From the viewpoint of welfare, therefore, there is a trade-off between marginal public cost
(environmental damage) and marginal private cost (abatement production cost). Comparing the welfares in (8) under royalty licensing and (16) under fixed-fee licensing, the resulting welfare depends on the marginal damage of reduced emission level and the marginal cost of increased abatement production level.

Let us first consider an example where $A=100$, $n=3$, $c=5$ and $d=1$. Figure 4 shows that the innovator gradually extends the number of licensees $k$ in response to emission tax. In particular, the innovator will chooses fixed-fee licensing with $k=1$ if $t > t_F$, where $t_F$ satisfies that the profit under royalty licensing equals to that under fixed fee licensing with $k=1$. If $t$ is between $t_F$ and $t_2$, the innovator will only license to one firm and if $t_2 < t < t_3$, the innovator will license to two firms. The lower part of Figure 4 also shows that if an innovator can choose fixed-fee licensing contract, it would improve the welfare. Specifically, welfares under fixed licensing, $W_1^F$, $W_2^F$ and $W_3^F$, are always higher than $W_R$. Therefore, the licensing contract which is selected by the innovator depends on the levels of emission tax. As a result, fixed-fee licensing is socially desirable over the feasible ranges of emission tax.

![Figure 4](image)

**Proposition 3.** Fixed-fee licensing contract is dominating when emission tax is high while royalty is dominating when emission tax is low.
For an extensive discussion, we will examine various simulation results and compare the welfares between royalty and fixed-fee licensing strategies. Figure 5 shows the equilibrium and socially desirable licensing contract. The left side in Figure (5a) shows the choice of profit-maximizing innovator while the right side in Figure (5b) shows the choice of welfare-maximizing government.

In Figure (5a), the innovator does not produce abatement goods in the left side of solid line $t_0$ while it gives licenses to all polluting firms under royalty licensing contract in the right side of line $t_0$. However, in the right side of line $t_F$, fixed-fee licensing is available to the innovator where it can control the number of licensees. For example, given emission tax level, when production costs between $t_F$ and $t_1$, the innovator gives a license to only one polluting firm because fixed-fee licensing gives more profit than royalty licensing.

Figure (5b) provides the socially optimal choice for the maximized welfare in the simulation. Notice that $C$ are the loci of the highest cost levels which give more social welfare than the social welfare under no-providing eco-technology into market. Notice that royalty licensing or fixed-fee licensing with $k=1$ is not the socially optimal choice. Notice also that the socially optimal number of licensees under fixed-fee licensing is $k=2$ in the upper line of $W_{FF}^P = W_1^P$, while $k=3$ in the lower line of $W_{FF}^P = W_3^P$.

Comparing Figure (5a) and (5b), we can easily find that the optimal choices of innovator are not always socially desirable, i.e., there exists welfare loss. For example, the shaded regions in Figure (5b) provides the welfare losses from the choice of innovator. Notice also that depending on the emission tax level, it is possible that exclusive fixed-fee licensing can be socially desirable when production cost is higher.

[Figure 5] The choice of innovator and welfare losses ($A=100$, $n=3$ and $d=1$)
5.2. Mixed Licensing Strategies

In the previous section, we have considered the exclusive choice of the innovator between royalty licensing and fixed-fee licensing. In this sub-section, we will examine the optimal combination of mixed licensing contracts between royalty and fixed-fee under emission tax.

The inverse demand function for final goods is similarly given by \( P(Q) = A - Q \), where \( Q = \sum_{i \in S} q'_i + \sum_{i \in S} q''_i + \sum_{i \in S} q''_i \) is market output level of final goods where \( q'_i \), \( q''_i \), and \( q''_i \) are the firm’s output level supplied by fixed-fee licensees, royalty licensees and non-licensees, respectively, and \( S \) represents the set of licensees.

Timing of the game is as follows: In the first stage, given emission tax, when the innovator provides eco-technology, it first decides fixed-fee schedule, the numbers of \( f_k \) licensees and fixed-fee \( f \). After announcing fixed fee schedule, polluting firms simultaneously decide whether or not to purchase a fixed-fee license in the second stage. In the third stage, the innovator then announces royalty schedule to non-licensed firms which did not buy fixed-fee licensing, \( n - f_k \). After announcing royalty \( r \) and \( k_r \) numbers of royalty licensees, non-licensed polluting firms simultaneously decide whether or not to purchase a royalty license in the fourth stage. Finally, when the numbers of \( f_k \) and \( k_r \) licensees are contracted to purchase a license, polluting firms choose their outputs level \((q'_i, q''_i, q''_i)\) and abatement level \((a'_i, a''_i)\) in a Cournot fashion. Then, the sub-game perfect Nash equilibrium will be derived by backward induction.

**Proposition 4.** Under mixed licensing contract, the eco-innovator chooses non-exclusive licensing strategy, \( k_f + k_r = n \).

**Proof.** see the Appendix I.

This proposition states that under mixed licensing contract, the innovator sells all the licenses to the polluting firms, composed of fixed fee license to \( f_k \) firms and royalty licenses to \( n - k_f \) firms. For the further analysis, let us consider an example where \( A = 100 \), \( n = 3 \), \( c = 5 \), and \( d = 1 \). Let \( \pi^M(k_f, k_r) \) denote the profit of the innovator under mixed licensing. Then, Figure 6 shows that the eco-innovator gradually extends the number of licenses \( k_f \) and lessens the number of licenses \( k_r \) in response to the increased emission tax. For example, the innovator sells all licenses under royalty licensing if \( t < t_o \). But, the innovator will chooses mixed licensing if \( t > t_p \) where \( t_p \) satisfies that the profit under royalty licensing equals to that under mixed licensing with \( k_f = 1 \) and \( k_r = 2 \). If \( t \) is between \( t_p \) and \( t_p \), the innovator will sell licenses to one firm by fixed-fee licensing and to two firms by royalty
licensing and if \( t^i_2 < t < t^i_3 \), the innovator will license to two firms by fixed-fee licensing and to one firm by royalty licensing. If \( t > t^i_3 \), the innovator will license to all firms by fixed-fee licensing.

[Figure 6] Optimal choices under mixed licensing (\( A = 100 \), \( n = 3 \), \( c = 5 \) and \( d = 1 \))

VI. Discussions on Environmental Regulation

In the previous analysis, we have found that the optimal decision on licensing contract depends on the level of emission tax. Therefore, under the strategic relationship between licensing contract and emission tax, we will first examine an optimal emission tax under royalty contract and provide some optimal policy implications on the licensing strategies of eco-technology.

First, when the innovator doesn’t license the patent when \( t \leq \frac{c(1+n)}{c+\epsilon} \), we can obtain an optimal emission tax from the welfare in (9). Using the market equilibrium without licensing, \( k = 0 \), we have the optimal emission tax, \( t^N = d - 1 \). Notice that the first positive term is marginal environmental damage from emission and the second negative term is output distortion in market. In other words, when we recalculate marginal output tax rate, we have \( t^N q = dq - q \) from total output tax \( t^N q = t^N q^2 / 2 \), and the second term is the difference between market price and marginal revenue, \( P'q = -q \). Therefore, the optimal emission tax could be either positive or negative, depending on the relative size of the distortions from environmental damages and polluting firm’s market power, where a negative value for the environmental tax would correspond to a subsidy.14

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14 On this result of two-fold type of tax/subsidy under imperfect competition, see Buchanan (1969)
maximized welfare will be \( W^N(t^N) = \frac{a_l}{\pi^N(t^N)}. \)

Next, when \( t > \frac{a_l}{\pi^N(t^N)} \) and thus \( k = n \), we can obtain an optimal emission tax from the welfare in (8). Then, the optimal emission tax should satisfy the following conditions:

\[
\frac{\partial W^L}{\partial t} = \sum \frac{\partial W^L}{\partial q_i} \frac{\partial q_i}{\partial t} + \sum \frac{\partial W^L}{\partial a_i} \frac{\partial a_i}{\partial t} = 0
\] (17)

Or, equivalently,

\[
\frac{\partial q_i}{\partial t}(A - Q - d(q_i - a_i)) = \frac{\partial a_i}{\partial t}(c - d(q_i - a_i))
\] (17')

Using the first-order conditions for profit-maximization in (3) and (7), we can derive the following equation:

\[
\frac{\partial q_i}{\partial t}(q_i + (t - d)(q_i - a_i)) = \frac{\partial a_i}{\partial t}(-ta_i + (t - d)(q_i - a_i))
\] (18)

By calculations, the expression of the optimal tax can be shown as follows:

\[
t^* = d + \frac{q_i \frac{\partial q_i}{\partial t} + ta_i \frac{\partial a_i}{\partial t}}{(q_i - a_i) \left( \frac{\partial q_i}{\partial t} - \frac{\partial q_i}{\partial t} \right)} = \frac{d(q_i - a_i) \left( \frac{\partial a_i}{\partial t} - \frac{\partial q_i}{\partial t} \right) + q_i \frac{\partial q_i}{\partial t}}{(q_i - a_i) \left( \frac{\partial q_i}{\partial t} - \frac{\partial q_i}{\partial t} \right) - a_i \frac{\partial a_i}{\partial t}}
\] (19)

Notice that \( \frac{\partial a_i}{\partial t} < 0 \) and \( \frac{\partial q_i}{\partial t} > 0 \). Thus, the denominator necessarily positive, but the first term of the numerator is negative while the second term of the numerator is positive as far as emission tax is positive. This implies that the optimal emission tax can be higher or lower than marginal damage, depending upon the relative size of emission abatement goods, \( a_i \), and outputs of polluting market, \( q_i \). Therefore, this equation underlines the trade-off between externality from emissions and market power from imperfect competition that faces a benevolent regulator. In particular, in order to better understand the important variables influencing the regulator’s decision, we can compare the optimal emission tax and marginal

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15 Under vertical structure of eco-industry, Canton, et al. (2008) examined an optimal emission tax with exogenous number of polluting firms while Lee and Park (2011) analyzed the emission tax and abatement subsidy with endogenous number of firms.

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damage as follows:

$$R_{ii} \geq dq_i \hat{e} t_q \hat{e} q_i \hat{e} t$$

(20)

The two opposite incentives are now isolated on each side of the inequality, where LHS implies the change of abatements from the tax and RHS implies the change of outputs from the tax.

Putting the results of equilibrium and comparative statics into (20) gives:

$$t^R > \frac{c[tA - c(1 + n + t)]}{t^2} > \frac{A[t(2n + t + 2) - c(1 + n + t)]}{(1 + n + t)^3}$$

(20')

This relation enables us to emphasize the key elements affecting the trade-off and to do comparison between two antagonistic effects. First, the higher the number of polluting firms, the more likely the tax will be set above marginal damage. That is, when the number of firms increases in the polluting market, the degree of competition increases as well, reducing the mark-up between price and marginal cost. Consequently, the regulator has less incentive to distort the tax from marginal damage. Second, the lower the production cost of abatement, the more likely the tax will be set above marginal damage. That is, when the abatement cost is less, the regulator has much incentive to set higher tax to reduce emissions. Therefore, the overall effect depends on the number of firms and production cost.

Finally, we will examine the efficiency of optimal tax in (19) under the fixed-licensing contract of the innovator. Using the same simulation results, the optimal emission tax in (19) is depicted as $t^R$ in Figure 7. Then, we can see that the optimal emission tax will not always provide the social optimum. For example, in the right side of line $t_1$, the innovator’s choice of $k = 3$ under optimal emission tax will provide the social optimum. But, in the left side of line $t_1$, the exclusive fixed-fee contract with $k = 2$ might provide lower welfare than social optimum. Therefore, exclusive fixed-fee contract should be prohibited in some regions. However, it is not always efficient since exclusive fixed-fee contract might provide social optimum when the production cost is high (unshaded areas). Thus, when licensing is welfare-reducing, the new policy should be proposed. One possible way to protect welfare loss under exclusive fixed-fee contract when optimal emission tax

---

16 LHS is certainly decreasing as the number of firms increases while the effect on RHS is ambiguous, that is, \( \frac{\partial LHS}{\partial n} = - \frac{c}{n} < 0 \) and \( \frac{\partial RHS}{\partial n} = 0 \) if \( 2(n + 1)(c - 2A) = -2(2c - A) \).

17 RHS is certainly decreasing as the production cost of abatement goods increases while the effect on LHS is ambiguous, that is, \( \frac{\partial RHS}{\partial c} = - \frac{c}{(t + c)^2} < 0 \) and \( \frac{\partial LHS}{\partial c} = 0 \) if \( c \leq \frac{4(2c - A)}{2(n + 1)} \). In particular, when \( c \) is very small, LHS is increasing in \( c \).
can be imposed is to set an upper limit on the fixed-fee or the profit of innovator.\(^{18}\) That is, new policy with profit-cap constraints might be useful. For example, when the production cost \(c = 9\) in the figure, the optimal emission tax will be \(t^K = 0.85\), in which the choice of innovator is \(k = 2\) while the social optimal choice \(k = 3\). Thus, the profit cap of \(\pi^M_2(t^K) \leq \pi^M_3(t^K)\) could be imposed. Then, the innovator will choose \(k = 3\) under optimal emission tax.

VII. Conclusion

This article has focused on the provision of pollution abatement technology in a patent licensing contract framework. In the presence of emission tax, we have examined an optimal patent licensing strategy of an innovator between royalty licensing and fixed-fee licensing contracts, and compared its welfare consequences. We have shown that eco-innovator provides non-exclusive license under royalty contract while it might exclude polluting firms under fixed-fee licensing contract. However, when mixed licensing contract where royalty and fixed-fee contracts are used together is available, we have shown that eco-innovator provides non-exclusive license. We have also shown that compared to royalty licensing, exclusive fixed-fee contract would increase the welfare, but its welfare effect of fixed-fee contract

\(^{18}\) In the previous literature, governmental intervention on licensing strategies is also discussed. For example, Shapiro (1985), Fauli-Oller and Sandonis (2002), Mukherjee and Tsai (2013), and Niu (2013) proposed the constraints on royalty rate or optimal tax/subsidy, and Erkal (2005), Gonzalez-Maestre (2008), and Shinha (2010) suggested the disallowance of the use of certain licensing contracts.
depends on the level of emission tax. Finally, we have examined the optimal emission tax and shown that an appropriate emission tax combined with non-exclusion policy or profit-cap regulation can increase the welfare.

As future research directions, the generalization of functional forms on demand, production cost, environmental damage, and eco-technology is necessary. In particular, it is worthwhile to notice that licensing contracts between fixed-fee or royalty could be determined by the characteristic of eco-technology such as equipment size, service duration, abatement efficiency and so on. Also, from the viewpoint of dynamic issue of eco-technology licensing, the innovative analysis on eco-R&D investment is required. Furthermore, other licensing mechanisms, such as auctioning or two-part tariff licensing contracts, and other strategic behaviors, such as the competition patterns of polluting oligopolistic firms and monitoring cost to abatement activities, should be also examined. These issues are challenges for future research on environmental policy with eco-industry.
Appendix I: Proof of proposition 4

The process of the proof is similar with the proof of Proposition 1 and thus we will prove the equilibrium for choosing the number of royalty licenses only.

In the final stage, let \( \pi'_f(k'_f, k', k'_n) \), \( \pi'_r(k'_r, k'_f, k'_n) \), and \( \pi'_N(k'_r, k'_f, k'_n) \) denote the profit of licensed downstream firms and that of unlicensed downstream firms, respectively, when the number of licensed firms is \( k'_f \) and \( k'_r \). Then, we have the following profit functions:

\[
\begin{align*}
\pi'_f(k'_f, k'_r, k'_n) &= P(Q)q'_f - r q'_f - t e'_f & \text{(A1)} \\
\pi'_r(k'_r, k'_f, k'_n) &= P(Q)q'_r - t e'_r - f & \text{(A2)} \\
\pi'_N(k'_r, k'_f, k'_n) &= P(Q)q'_N - t e'_N & \text{(A3)}
\end{align*}
\]

where \( Q = \sum_{i \in S} q'_f + \sum_{i \in S} q'_r + \sum_{i \in S} q'_N, e'_f = \frac{(q'_f - q'_r)}{2}, e'_r = \frac{(q'_r - q'_N)}{2}, \) and \( e'_N = \frac{(q'_N - q)}{2} \).

Suppose that the eco-innovator chooses the number of \( k'_r \) where \( 0 < k'_r < n - k'_f \), which supports non-negative values for outputs, \( q'_f, q'_r, \) and \( q'_N \), and the amount of abatement goods, \( a'_f \) and \( a'_r \) at equilibrium. Then, at the supposed equilibrium, it should satisfy the following first-order conditions for profit maximization:

\[
\begin{align*}
\frac{\partial \pi'_f}{\partial q'_f} &= A - \sum_{i \in S} q'_f - \sum_{i \in S} q'_r - \sum_{i \in S} q'_N - q'_f - t(q'_f - a'_f) = 0 & \text{(A4)} \\
\frac{\partial \pi'_r}{\partial a'_r} &= -r + t(q'_r - a'_r) = 0 & \text{(A5)} \\
\frac{\partial \pi'_f}{\partial q'_r} &= A - \sum_{i \in S} q'_f - \sum_{i \in S} q'_r - \sum_{i \in S} q'_N - q'_r = 0 & \text{(A6)} \\
\frac{\partial \pi'_N}{\partial q'_N} &= A - \sum_{i \in S} q'_f - \sum_{i \in S} q'_r - \sum_{i \in S} q'_N - t q'_N = 0 & \text{(A7)}
\end{align*}
\]

Solving these first-order conditions simultaneously, we can derive the equilibrium outputs and the amount of abatement goods as follows:

\[
\begin{align*}
q'_f &= \frac{(1 + k'_f)[A(1 + t) + ta'_f(1 + n + t + rk'_f - k'_r)]}{(1 + t)(1 + (n + k'_f) + t(1 + k'_r^2))}, \quad q'_r = \frac{(1 + t)(A + rt k'_r)}{(1 + n + t + rk'_f - k'_r)}, \\
q'_N &= \frac{(1 + k'_r)[A(1 + t) - ta'_r k'_f]}{(1 + t)(1 + (n + k'_r) + t(1 + k'_f^2))}, \quad a'_r = \frac{(1 + t)\{tA - r(1 + n + t + rk'_r)\}}{t(1 + n + t + rk'_f + rk'_r)}.
\end{align*}
\]
and \[ a_i' = \frac{(1 + t)(A + rtk_f)}{(1 + n + t + tk_f + tk_i)}. \]

Then, we need to check whether the firms (an innovator and polluting firms) have an incentive to deviate from this equilibrium when \( 0 < k_i < n - k_f \). The innovator determines the royalty of eco-technology and the number of licenses to maximize its profit:

\[
\max_{r,k_i} \pi^N(k_f,k_i) = (r-c)k_i a_i' + (f - ca_i')k_f
\]

s.t. \( a_i' = \frac{(1 + t)(tA - r(1 + n + t + tk_f))}{t(1 + n + t + tk_f + tk_i)} \) and \( a_f' = \frac{(1 + t)(A + rtk_f)}{(1 + n + t + tk_f + tk_i)}. \)

Then, from (A8) we have the following relation that the innovator’s profit increases with more licensing firms: \( \frac{\partial \pi^N}{\partial k_i} = (r-c)a_i' + (r-c)k_i \frac{\partial a_i'}{\partial k_i} - ck_f \frac{\partial a_i'}{\partial k_f} > 0 \). This is because \( 1 + \frac{k_i}{a_i'} \frac{\partial a_i'}{\partial k_i} > \frac{1 + n + tk_f}{(1 + n + t + tk_f + tk_i)} > 0 \) and \( \frac{\partial a_i'}{\partial k_f} = -\frac{1}{(1 + n + t + tk_f + tk_i)} < 0 \). Thus, the innovator has no incentive to exclude polluting firms at equilibrium under royalty licensing, i.e., \( k_i = n - k_f \), which is contradict to the supposed equilibrium.

Next, we will examine the case of \( k_i = n - k_f \). Then, excluding the first-order condition in (A7) or setting \( q_i^N = 0 \), the equilibrium outputs and abatement goods are determined by the first-order conditions in (A4) and (A6):

\[
q_i' = \frac{A - r(1 + k_f)}{(1 + n)}, \quad q_f' = \frac{A + r(n - k_f)}{(1 + n)}, \quad a_i' = \frac{A - r(1 + n + t + tk_f)}{t(1 + n)}
\]

\[ a_f' = \frac{A + r(n - k_f)}{(1 + n)} \]

Then, from the first order condition of the innovator in the third stage, \( \frac{\partial \pi^N}{\partial r} = k_i a_i' + (r-c)k_i \frac{\partial a_i'}{\partial r} - ck_f \frac{\partial a_i'}{\partial k_f} = 0 \), we can get \( r^* = \frac{\partial a_i'}{\partial r} = \frac{(1 + n + tk_f)}{(1 + n + t + tk_f + tk_i)} \),

\[ q_i^N = \frac{2(1 + n + t + tk_f) - (1 + n + tk_f) + (1 + n + t + tk_f) + (1 + n + t + tk_f)}{2(1 + n + t + tk_f) + (1 + n + t + tk_f) + (1 + n + t + tk_f) + (1 + n + t + tk_f)} \]

\[ q_f^N = \frac{2(1 + n + t + tk_f) + (1 + n + tk_f) + (1 + n + t + tk_f) + (1 + n + t + tk_f)}{2(1 + n + t + tk_f) + (1 + n + t + tk_f) + (1 + n + t + tk_f) + (1 + n + t + tk_f)} \]

If \( t > \frac{c(1+n)}{4-r} \), we have \( a_i' > 0 \) at equilibrium when \( k_i = n - k_f \).

Next, from the viewpoint of licensees’ incentive, we have \( \pi^L(k_i) > \pi^N(k_i) \) for \( 0 < k_i < n - k_f \) when \( t > \frac{c(1+n)}{4-r} \). That is, royalty-licensed firms will have higher profit than non-licensed firms and thus, there does not exist the case of \( \pi^L(k_i) = \pi^N(k_i) \) for \( 0 < k_i < n - k_f \) when \( t > \frac{c(1+n)}{4-r} \). However, when \( t \leq \frac{c(1+n)}{4-r} \), we have \( a_i' = 0 \) and \( \pi^L(k_i) < \pi^N(k_i) \) for \( 0 < k_i < n - k_f \). It implies that \( k_i = 0 \) at
equilibrium.

For the equilibrium of remaining game on deciding the number of fixed-fee licensing firms, \( k_f \), is the same with the case of fixed-licensing and thus omitted. (It is noteworthy that when the innovator decides \( k_f \) in (A8), the fixed-fee will be determined at the following relation: \( \pi_f'(k_f, n-k_f) - \pi_f'(k_f-1, n-k_f+1) = 0 \).)

**Appendix II: Convex production cost under royalty contract**

We provide two approaches on the proof of Lemma 1 when the production cost is a convex function under royalty contract, i.e., \( c(k \cdot a_i) \), where \( \epsilon' > 0 \) and \( \epsilon'' > 0 \). The eco-innovator determines the royalty and the number of licensee to maximize the following profit function:

\[
\pi^M = r \cdot k - c(k \cdot a_i) \\
\text{s.t.} \\
(1 + t) \left( tA - r(1 + n + t) \right) = \frac{t(1 + n + t + tk)}{}.
\]

Given \( k \), we have the following first-order condition for profit maximization:

\[
\frac{\partial \pi^M}{\partial r} = ka_i + rk \frac{\partial c}{\partial r} - c'(k \cdot a) \frac{\partial a}{\partial r} = k \left[ a_i + (r - c') \frac{\partial a}{\partial r} \right] = 0. 
\]

From (B1), we have \( (r - c') = -a_i \frac{\partial c}{\partial r} > 0 \) since \( \frac{\partial c}{\partial r} < 0 \) and \( -1 < e_{ak} < 0 \) where \( e_{ak} = \frac{k \cdot \partial c}{\partial k} \) from footnote 10. Then, we have the following relation:

\[
\frac{\partial \pi^M}{\partial k} = ra_i + rk \frac{\partial c}{\partial k} - c'(a_i + k \frac{\partial a}{\partial k}) = (r - c') (a_i + k \frac{\partial a}{\partial k}) = (r - c') a_i (1 + k \frac{\partial c}{\partial k}) > 0 \ (B2)
\]

Thus, we have \( k = n \).

Alternatively, suppose that there exists \( k^* < n \) for maximizing the profit of eco-innovator under royalty contract. With \( k^* \), the eco-innovator determines \( r^* \) and then we derive \( a^*(r^*, k^*) \), which is the amount of abatements goods that each licensed firm purchases. Then, the total amount of abatement goods of licensed firms can be defined as \( z^* = a^* k^* \). On the other hand, if the eco-innovator sells the same total amount of abatement goods to all \( n \) firms, we can define the amount of abatement goods of each firm as \( a^{**} = \frac{z^*}{n} \). Then, it must be \( a^{**} < a^* \) since \( k^* < n \). That is, given the same royalty \( r^* \), the eco-innovator can raise the same amount of total revenue and thus the same amount of profit by either selling \( a^* \) to \( k^* \) firms or selling \( a^{**} \) to \( n \) firms. However, given the same royalty \( r^* \), the market
on the abatement goods will be higher than $a^{*\ast}$ because the demand function of abatement goods is inelastic, i.e., $\epsilon_{ka} = \frac{k}{a} \frac{\partial a}{\partial k} < 0$. Thus, due to the negative slope of demand function on abatement goods, the eco-innovator can raise the royalty to sell $a^{*\ast}$ to all firms, which will increase total revenue without changing total cost. Therefore, the eco-innovator can get more profits by increasing the number of licensed firms from $k$ to $n$. This is a contradiction.
References


Seung-Leul Kim · Sang-Ho Lee: Eco-Technology Licensing under Emission Tax


