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Information Sharing Networks in Oligopoly

Yong-Ju Lee*

We analyze the information sharing incentives of firms in a decentralized environment when firms face a stochastic demand. In order to do that, we develop a two stage model of strategic network formation, which consists of cooperative network formation in the first stage and noncooperative Bayesian Cournot competition in the second stage. Then we derive pure strategy mixed cooperative and noncooperative equilibria that are stable and subgame-perfect, and characterize the equilibrium graph structure. Our main finding is that the incentives of firms for information sharing may vary depending on whether the decision is made in a centralized or a decentralized environment.

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I. Introduction

The purpose of this paper is to investigate the incentives of firms in a decentralized environment to form a network in order to exchange information in an oligopolistic market when firms face an uncertain demand.

Many papers analyze the existence of incentives to share private information in stochastic market environments (Clarke, 1983; Gal-Or, 1985, 1986; Novshek and Sonnenschein, 1982; Raith, 1996; Tives, 1984). And these papers show that it is unclear whether the exchange of information about an uncertain world has a profitable effect on the firms when they compete against each other as Nash competitors in the product market. This is because the results crucially depend on the market's random variables, their distribution, the nature of the competition

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* Assistant Professor, School of Economics and Finance, Yeungnam University, 280 Daehak-ro Gyeungsan 712-749, Korea. Tel: +82-53-810-2847. E-mail: yongjulee@ynu.ac.kr. The author would like to thank John Duffy, Ted Temzelides, Andreas Blume, Utku Unver, and Esther Gal-Or for their helpful comments. This work was supported by the National Research Foundation of Korea Grant funded by the Korean Government (NRF-2013S1A5A8023955).

(Cournot or Bertrand), and the nature of information. On the other hand, it is well known that, with an unknown common demand, information sharing is the unique Nash equilibrium outcome under Bertrand competition and concealing is the unique equilibrium outcome under Cournot competition (Vives, 1984; Gal-Or, 1985). And, with unknown private costs, information sharing is the unique Nash equilibrium outcome under Cournot competition and concealing is the unique equilibrium outcome under Bertrand competition (Gal-Or, 1986). Here we revisit the problem of information sharing in oligopoly and analyze how the results of the literature vary if firms face a decentralized environment.

Also, it is well known that network structures play a significant role in determining the outcome of many important economic relationships.[1] While the previous papers analyze the possibility of a collaboration or cooperation among firms, they impose severe restrictions on the underlying interaction structure among the firms. That is, the literature explicitly or implicitly assumes the existence of an "outside agency" such as a trade association. The role of this agency is to collect private information from each firm and disseminates it throughout the industry. In the terminology of the network literature, they assume that the underlying network structure is a *star network* where the central node is the trade association and the periphery nodes are the firms. And this is equivalent to assuming the *complete network* among the firms without the outside agency.[2] This interpretation is without loss of generality since the literature has not treated the outside agency as a player in the game, but, instead, the outside agency is modeled as a purely mechanical part of the environment with no decisions to make. Therefore, once a firm decides to share its information on its private signal, the firm has to share it with all the other firms at the same time. In this sense, we can say that the previous literature assumes that each firm faces an *exogenously* given *centralized* network structure and makes an *industry-wide* decision.

In this paper, however, we analyze the incentives for information sharing in a *decentralized* setting where the firms make their own decision to choose with which firms to collaborate and share information with the understanding that firms engage in a noncooperative competition in the product market. For this purpose, we endogenize the network formation process explicitly in the model and only the linked firms on the network can share information bilaterally. That is, the present paper studies the information sharing problem in an oligopoly where each firm

[1] For instance, personal networks play important roles in obtaining information about sporadically scattered job opportunities. Such networks of relationship also underlie the trade of goods in non-centralized market, the provision of mutual insurance in developing countries, R&D and collusive alliances among corporations, and international alliances and trade agreements (Refer to Jackson (2010) for an excellent survey.).

[2] By the complete network we mean the graph in which each firm has a direct link with every other firm.

faces a *decentralized* network structure and engages in a *pair-wise* or *group-wise* decision.[3] Collaboration or cooperation among firms is common even in the oligopolistic market (Goyal and Joshi, 2003). As Goyal and Moraga-Gonzalez (2001) indicate, a distinctive feature of the collaboration or cooperation among firms is that they are often bilateral interactions that are embedded in a broader network. Even in situations where firms i and j, and j and k have a cooperative relationship respectively, firms i and k may not have such a relationship. These structural features justify incorporating strategic network formation in the existing information sharing literature.

Another innovative aspect of the paper resides in the choice of the degree of information sharing. In much of the previous literature, firms decide the degree of information sharing by choosing the level of variance of a message, which is a random variable. Given that the network structure is complete as previously mentioned, firms share information uniformly with all the other firms at the same time. For example, they reveal information completely if the variance of message is zero, while they don't share information if the variance is infinity. In our model, the firms choose the degree of information sharing by selecting the set of firms with which they want to form collaborative link. It is assumed that a link involves a commitment on bilateral and truthful information sharing between the two corresponding firms. Therefore, the firms share information completely if the resulting graph is a complete one, while they don't if the resulting graph is an empty one.[4] Therefore, we can say that the previous literature measures the degree of information sharing by "*depth*" while the present paper measures it by "*width*" In this sense, we might view the current paper as complementary to the existing literature on information sharing.

For the analyses, we consider a simple two stage game. In the first stage, the firms strategically form pair-wise links which allow them to obtain the others' private signals on the stochastic market demand. We assume that only the directly linked firms can share information bilaterally, and that they share information truthfully if there is a collaborative link between them. After the link formation, each firm observes its own private signal and then transmits their private information to the linked firms simultaneously. In the second stage, after information transmission, each firm chooses its level of output in the product market. That is, a Bayesian

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[3] We argue that pair-wise information sharing is a more natural form than any others in the oligopoly. First, pair-wise information sharing captures informal and local behaviors among firms in the market. Second, firms in the oligopolistic market may use the agreement on pair-wise information sharing as a pre-step to another stronger forms of coalition such as the cost reduction alliance, joint venture formation, and R&D agreement. Before these decisions are made, firms need some time to know each other well and may use this kind of weak network as a key foundation. The author thanks Esther Gal-Or for pointing this out.

[4] By the empty network we mean the graph in which there is no link between any two firms.

Cournot game is played in the second stage. Following the spirit of d'Aspremont and Jacquemin (1988), we derive "pure strategy mixed cooperative and noncooperative equilibria" that are stable and subgame perfect, and characterize the resulting graphs. That is, given a collaborative network structure firms compete with each other as Nash competitors in the product market in order to maximize their own profits, and firms cooperate bilaterally in the first stage so as to overcome market uncertainty and, hence, to maximize their own profits with the understanding that they engage in noncooperative competition in the second stage. Main questions to be addressed are what is the incentive of firms to collaborate and what is the resulting network structure, what are the effects of strategic network formation on individual and industry-wide performance, and why the incentive of a firm depends on whether network structure is centralized or decentralized.

Our paper can be seen as a contribution to the study of group formation and cooperation in oligopolies. Modeling strategic network formation is inspired by Bala and Goyal (2000), Dutta et al. (1995), and Jackson and Wolinsky (1996) at the pioneer stage. And several papers have addressed similar issues to the ones we address in the present paper. Some examples are Goyal and Moraga-Gonzalez (2001), Goyal and Joshi (2003), and Goyal et al. (2008). These papers highlight the relationship between the firms' incentives for R&D and network formation. However, here we study the relationship between the incentives for strategic information exchange in an uncertain market and network formation. Also, this paper, to the best of our knowledge, is the first to analyze information sharing problem in an oligopoly in the context of a decentralized network formation.

The remainder is organized as follows. Section 2 studies the model, which consists of the cooperative network formation game and the noncooperative oligopoly game with a stochastic common demand. Section 3 derives the equilibria of the game and characterizes the equilibrium network structures. Section 4 introduces heterogeneity to the basic model by assuming that the accuracy of private information is different across firms. Section 5 discusses the possible extensions and suggestions for future research. Section 6 concludes.

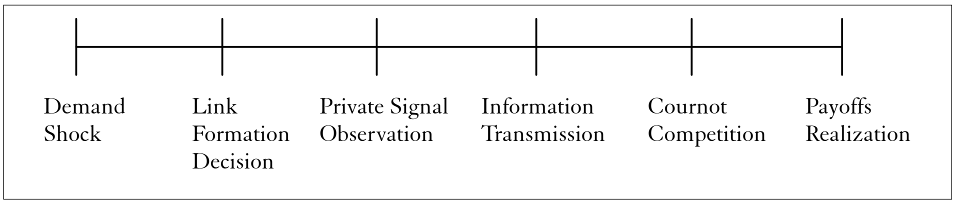
II. The Model

In this section we set up the benchmark model where the game involves a symmetric environment and firms produce homogeneous goods. Section 4 extends the basic model by introducing heterogeneity across firms.

We consider a two stage game. The timing of the game is shown in Figure 1. In the first stage, firms strategically form pair-wise links to obtain information on the stochastic market demand. After link formation each firm observes its own private signal. Then firms simultaneously transmit their information to the linked firms.

We call this stage game the *network formation game*. In the second stage, after information transmission, each firm chooses its level of output in the product market. Also, it is assumed that firms observe the entire network structure before making output decision.[5] We call this second stage game the *oligopoly game*. Our goal is to derive the pure strategy mixed cooperative and noncooperative equilibria of the game. Firms coordinate bilaterally in the first stage so as to overcome market uncertainty and, hence, to maximize their own profits with the understanding that they engage in noncooperative competition in the second stage. We examine the incentives of firms for network formation and the resulting information sharing network structures, and analyze in what degree firms collaborate. Finally, we analyze the effects of the network structure on both individual and market outcomes. We now develop some notations and define our notion of stability and efficiency.

[Figure 1] The Timing of the Game



2.1. The Network Formation Game

Let $N = \{1, 2, \cdots, n\}$, $n \geq 3$ be the set of ex ante identical firms. In the first stage each firm forms pair-wise links which represent commitment both the linked firms must honor regarding bilateral information transmission.[6] For any pair of firms $i, j \in N$, pair-wise relationship between the two firms is represented by a binary variable $g_{ij} \in \{0,1\}$. When $g_{ij} = 1$, this means that the two firms are linked at cost γ respectively, while $g_{ij} = 0$ refers to the case of no link. In our model, $g_{ij} = 1$ means that the two linked firms i and j share their information bilaterally. To avoid reaching a conclusion that critically depends on the link formation cost γ, we assume that the link formation cost γ is negligibly small.[7] The number of

[5] Readers might want to see how the results are affected if each firm only knows its links but does not observe the links that its linked firms have. It may be the case that relaxing this assumption makes the problem much complex. The authors acknowledge that this case might enrich the paper but we leave this for future research.

[6] The present model has a different kind of commitment from the one in the previous literature. In the previous literature there is the commitment between the outside agency and each firm, while in this model commitment is between the pair of corresponding firms. Because of this commitment problem we employ the cooperative approach in modeling network formation.

[7] For simplicity, it can be set to zero.

pairwise links represents the degree of information sharing among firms in the industry. A network g is a collection of links, i.e., $g=\{g_{ij}\}_{i,j\in N}$. Let $g_i=\{g_{ij}\mid g_{ij}=1\}_{j\neq i}$ be the set of links involving firm i, where $g_{-i}=\{g_k\}_{k\neq i}$ is the set of all the firms except firm i. The set of all possible graphs on N is denoted by G. Let $N_i(g)=\{j\in N\setminus\{i\}\mid g_{ij}=1\}$ be the set of firms with which firm i has a link in g, and let $\eta_i(g)$ be the cardinality of the set $N_i(g)$. Let $g-g_{ij}$ denote the network obtained by severing an existing link between firms i and j from the network g, while $g+g_{ij}$ is the network obtained by adding a new link between firms i and j in the network g. A path in g connecting firms i and j is a set of distinct firms $\{i_1,i_2,\cdots,i_k\}$ such that $g_{ii_1}=g_{i_1i_2}=\cdots=g_{i_kj}=1$. We say that a network is connected if there exists a path between any pair $i,j\in N$. A network, $g'\subset g$, is a *component* of g if for all $i,j\in g',i\neq j$, there exists a path in g' connecting i and j, and for all $i\in g'$ and $j\in g,g_{ij}=1$ implies $g_{ij}\in g'$. The profits of firm i in network g are denoted by $\pi_i(g)$, which will be specified in the next subsection.

We shall say that a network g is *pairwise stable* if and only if for all $i,j\in N$:

(i) For $g_{ij}=1,\pi_i(g)\geq\pi_i(g-g_{ij})$ and $\pi_j(g)\geq\pi_j(g-g_{ij})$

(ii) For $g_{ij}=0$, if $\pi_i(g+g_{ij})>\pi_i(g)$, then $\pi_j(g+g_{ij})<\pi_j(g)$.

This definition of stability is taken from Jackson and Wolinsky (1996). These conditions indicate that agents need a bilateral agreement to form a link, while agents can sever the existing link unilaterally. To the extent that larger groups can coordinate their actions in making changes in a network, a stronger solution concept might be needed. Nevertheless, pairwise stability is a natural solution concept in our model, since, as Roth and Sotomayor (1990, p.156) argue that "identifying and organizing large coalitions may be more difficult than making private arrangements between two parties."

Alternatives to pairwise stability that allow for larger coalitions than just pairs of firms to deviate were first considered by Dutta and Mutuswami (1997). The following definition is modified from Jackson and van den Nouweland (2005).

A network $g'\in G$ is *obtainable from* $g\in G$ *via deviation by* S if

(i) $g'_{ij}=1$ in g' and $g_{ij}=0$ in g implies $i,j\in S$, and

(ii) $g_{ij}=1$ in g and $g'_{ij}=0$ in g' implies $\{i,j\}\cap S\neq\varnothing$.

The above definition identifies changes in a network that can be made by a coalition S without the need of consent of any firms outside of S. (i) requires that any new links that are added can only be between firms in S. This reflects the fact that consent of both firms is needed to add a link. (ii) requires that at least one firm of any deleted link be in S. This reflects the fact that either firm incident with a link can unilaterally sever the relationship.

A network g is *strongly stable* if for any $S\subset N, g'$ that is obtainable from g via deviation by S, and $i\in S$ such that $\pi_i(g')>\pi_i(g)$, there exists $j\in S$ such that $\pi_j(g')<\pi_j(g)$.

The definition of strong stability allows for a deviation to be valid if some firms are strictly better off and others are weakly better off, while the definition in Dutta and Mutuswami (1997) considers a deviation valid only if all firms of a coalition are strictly better off. This stronger notion implies pairwise stability. Strong stability provides too strong refinement of pairwise stability, since this concept implies that any group deviation is not possible. Therefore, the concept of strong stability makes sense only in smaller network situations where agents have substantial information about the overall structure and potential payoffs and can coordinate their actions.

We now define some typical networks that play important roles in our analysis. A network is said to be *symmetric* if every firm has the same number of links. Otherwise it is asymmetric. In a symmetric network $\eta_i(g)=\eta_j(g)=\Delta$ for any two firms i and j. We will denote a symmetric network of degree Δ by $g^{\Delta}, \Delta = 0,1,\cdots,n-1$. In particular, if $\Delta=0$, the nework is called the *empty* network, while it is called the *complete* network if $\Delta=n-1$. It can be shown that if the number of firms is even, then there is always a symmetric network of degree $\Delta, \Delta=0,1,\cdots,n-1$ (Goyal and Moraga-Gonzalez, 2001). Among the asymmetric networks, the *dominant group architecture*, $g^{d(k)}$, is characterized by one complete non-singleton component with $k\geq 2$ firms and $n-k$ singleton firms. Thus, there is a set of firms $N^d \subset N$ with the property that $g_{ij}=1$ for every pair $i,j\in N^d$ while for any $m \in N\setminus N^d$, $g_{ml}=0$, $\forall l\in N\setminus\{m\}$ (Goyal and Joshi, 2003). A network is said to be *component-symmetric* if every firm in a component has the same number of links.

2.2. The Oligopoly Game

The oligopoly game is based on the existing information sharing literature. The oligopoly consists of n firms producing a product at no cost. We assume the same type of demand uncertainty as in Gal-Or (1985, 1986). The demand function is linear and stochastic:

$$p = a - bQ + u, \quad a,b>0 \tag{1}$$

The prior distribution of u is normal with mean zero and variance σ_u, p is the price and Q is the aggregate quantity produced. Before deciding its output quantity, each firm observes a noisy signal for u, and then transmits it to the linked firms. The signal observed by firm i is x_i. We assume:

$$x_i = u_i + e_i \quad u_i \sim N(0,\sigma), \quad e_i \sim N(0,m) \tag{2}$$

where $Cov(e_i,e_j)=0$, $i\neq j; Cov(u_i,e_j)=0\ \forall i,j;$ $Cov(u_i,u_j)=0,$ $i\neq j;$ and $u=$

$(\Sigma_i u_i)/n$, hence $u \sim N(0, \frac{\sigma}{n})$.[8] e_i is called the signal error. In order to derive explicit forms for their expected values, conditional or unconditional, the assumptions on the normality of these random variables and the linearity of demand function are necessary. The private signals might be positively correlated, but here we simply assume that they are independent.[9] Hence a firm cannot make any inference about the signals observed by the other firms based on its own signal. This fact gives the firms strong incentives for strategic link formation. Since, by assumption, every firm transmits its private signal to the others simultaneously, the transmitted information (signals) from others cannot be used to generate a firm's own message. However, note that there are indirect network effects here. For instance, consider a situation where firms i and j, and j and k have a collaborative link respectively. We can obviously conclude that x_k (or x_i) is unknown information to i (or k), since there is no direct link between i and k. But firm i does know that firm k will use information transmitted from j, x_j, (which is also known to i) to make the optimal decision in the product market in the second stage, and vice versa. That is, under rational expectations there is an indirect network effect even though there are no direct spillovers in the model.

We denote by X the vector of true signals observed by all firms. X_{-i} denotes the vectors of true signals excluding those of firm i.

After information transmission, the transmitted information is subsequently used by each firm to choose its output. The output choice depends on the information available to the firm. For firm i this information consists of the network g, its private information x_i, the received information from other firms $\{x_j \mid g_{ij} = 1\}_{j \neq i}$, and the known values of the parameters m and σ. We denote this information available to i by $y_i = (g, x_i, \{x_j \mid g_{ij} = 1\}_{j \neq 1}, m, \sigma)$. For example, if $n = 5$ and a wheel (loop) network $g = \{g_{12} = g_{23} = g_{34} = g_{45} = g_{51} = 1\}$ is given, then $y_i = (g, x_1, (x_2, x_5), m, \sigma)$. The vector of information that is available to all firms is denoted by y, and information available to all firms except firm i is y_{-i}.

The oligopoly game is a Bayesian Cournot game in which each firm decides its product level based on the information available at the beginning of the second stage. We derive the symmetric equilibrium decision rule for the game beginning at the second stage. Note that, although we allow asymmetric networks throughout the paper, the unique Nash equilibrium decision rule in the game beginning at the second stage is "symmetric" in this limited sense that, given the network g, each firm has the same form of decision rule even though its realizations are different. That is, it is affine in the vector of signals available to the firm. Since all firms are ex

[8] This environment is called a "common values" problem in the auction literature (Gal-Or, 1986).

[9] If private signals are partially correlated, a firm can make partial inferences about the competitors' signals based on its own signal. This lowers firm's incentive for strategic link formation and information sharing. Therefore this assumption only leads to weakening our results since, as analyzed below, there already exist strong negative effects of information sharing under Cournot competition.

ante identical, labeling in the graph is not important, and the only thing that matters is the number of links each firm retains given network structure.

III. Derivation of the Equilibria

The strategy of firm i in the whole game is the pair $\{(g_{ij})_{j\neq i}, q_i(y_i)\}$ where $g_{ij}: N \rightarrow \{0,1\}$, and $q_i: G\times R\times R^{\eta_i(g)-1} \rightarrow R$. We denote by $Q(y)$ the vector of decision rules used by all firms, and by $Q_{-i}(y_{-1})$ the vector of decision rules of all firms except firm i.

The payoff of firm i as a function of the strategies chosen is:

$$\pi_i(g_i, g_{-1}, q_i(y_i), Q_{-i}(y_{-1})) = E_{y,u}\{q_i(y_i)[a - b\sum_k q_k(y_k) + u]\} - \eta_i(g)\gamma \tag{3}$$

where E is the expected value operator.

At the second stage, firm i chooses its decision rule $q_i(\cdot)$ to maximize:

$$W = E_{y_{-i},u}\{q_i(y_i)[a - b\sum_{j=1}^{n} q_j(y_j) + u] \mid y_i\} \tag{4}$$

In (4) profits are conditioned on the realization of y_i. Variables that remain unobserved at the beginning of the second stage are the values of the signals observed by all the unlinked firms $\{x_k \mid g_{ik} = 0\}_{k\neq i}$, the value of random variable u. Equation (4) may be rewritten:

$$W = q_i(y_i)[a - bq_i(y_i) - b\sum_{j\neq i} E_{y_j}(q_j(y_j) \mid y_i) + E_u(u \mid y_i)] \tag{5}$$

Given the decision rules chosen by the other firms, firm i decides its decision rule $q_i(\cdot)$ to maximize (5). Proposition 1 shows the solution for any finite number of firms, when the private signals are independent.

Proposition 1. *With independent signals, for given g, the following decision rule forms the unique Nash equilibrium in the game beginning at the second stage:*

$$q_i(y_i) = A_0^i + \sum_{j\neq i} A_j^i g_{ij} x_j + A_i^i x_i, \quad \forall i \tag{6}$$

where $A_0^i = \frac{a}{b(n+1)}$, $A_j^i = \frac{\sigma}{(\eta_j(g))bn(m+\sigma)}$, $A_i^i = \frac{\sigma}{(\eta_i(g)+2)bn(m+\sigma)}$.

Proof. To maximize (5) while taking $q_j(y_j)$ as given, set:

$$\frac{\partial W}{\partial q_i} = a - b\sum_{j \neq i} E_{y_j}(q_j(y_i) \mid y_i) + E_u(u \mid y_i) - 2bq_i(y_i) = 0$$

and

$$\frac{\partial^2 W}{\partial q_i^2} = -2b < 0$$

Hence

$$q_i(y_i) = \frac{a - b\Sigma_{j\neq i} E_{y_j}(q_j(y_j) \mid y_i) + E_u(u \mid y_i)}{2b}, \quad \forall i \tag{7}$$

Equation (7) is necessary and sufficient condition for the decision rule $q_i(y_i), \forall i$ to be Nash equilibrium decision rule. Using the posterior distribution of u, $E(u \mid y_i) = \frac{1}{n}\frac{\sigma}{m+\sigma}\{\Sigma_{j\neq i} g_{ij} x_j + x_i\}$ Since $E(u \mid y_i) = \frac{1}{n}\Sigma_{k=1}^{n} E(u_k \mid y_i) = \frac{1}{n}[E(u_i \mid x_i) + \Sigma_{j\neq i} g_{ij} E(u_j \mid x_j)]$, and $E(u_i \mid x_i) = \frac{\sigma}{m+\sigma} x_i$. Using the suggested solution of the Proposition,

$$E_{y_j}(q_j(y_j) \mid y_i) = E\{A_0^j + \sum_{k \neq j} A_k^j g_{jk} x_k + A_j^j x_j \mid y_i\} = A_0^j + \sum_{k \neq j} A_k^j g_{jk} x_k + A_j^j g_{ij} x_j \tag{8}$$

Using these in condition (7) and requiring (7) to be satisfied for every possible y_i and y_j, yields a system of equations with the same number of unknowns. Solving this equation system yields the unique solution specified in (6). According to Radner (1962) it is sufficient to restrict attention to decision rules of the generic form expressed by (6), since the decision rules must be affine in the vector of observations available to the firm. ■

Since all firms are ex ante identical (face the same technology and observe signals of the same precision), only the cardinality of the set $N_i(g)$ matters in characterizing the decision rule. From this we observe that the firm imposes the same weights on signals transmitted from other firms as on its own signal as long as they have the same number of links. Without loss of generality, we assume that $q_i(y_i)$ is nonnegative. From equation (6) we can see various effects of information sharing. For example, suppose firm i forms a link to firm l. There are two conflicting direct effects. First, there emerges $A_l^i x_l$ term in the equation (since x_l is now available information to the firm i), which makes $q_i(y_i)$ increasing. Second, firm i imposes less weight in its own information x_i (since the $\eta_i(g)$ component is in the denominator of A_i^i), which makes $q_i(y_i)$ decreasing. Also there are indirect effects embedded in equation (8). Expecting the decision rules of other firms connected to firm l, firm i can deduce that any firm k also use x_l

(now known to firm i) as information in its decision, if $g_{lk} = 1$. We conceive these effects as a kind of network externality. By considering the graph structure explicitly, we can unambiguously capture the effects of information sharing on the decision rules regarding quantity. This decision rule is reduced to the formula of Theorem 1 in Gal-Or (1985) when the complete network structure is exogenously given and each firm truthfully transmit its signal to the linked firms.

Now we use this equilibrium decision rule to derive the payoffs of the subgame that starts at the second stage, denoted $w_i(y_i; g)$ for firm i. The payoff function in this subgame starting at the second stage is then used to derive the payoff in the game that starts at the first stage. Denote this last function by

$$\pi_i(g) = \pi_i(q_i(g), q_{-i}(g)) = E_{y_i}[w_i(y_i; g)] - \eta_i(g)\gamma = bE_{y_i}[q_i(y_i)]^2 - \eta_i(g)\gamma$$

The last equality follows directly from the payoff function expressed in (5) and the form of the Nash equilibrium decision rule expressed in (7). We can rewrite $\pi_i(g)$ explicitly as follows:

$$\begin{aligned} \pi_i(g) &= bE_{y_i}[A_0^i + \sum_{j \neq i} A_j^i g_{ij} x_j + A_i^i x_i]^2 - \eta_i(g)\gamma \\ &= \frac{a^2}{b(n+1)^2} + \sum_{j \neq i} g_{ij} \frac{\sigma^2}{(\eta_j(g)+2)^2 bn^2(m+\sigma)} + \frac{\sigma^2}{(\eta_i(g)+2)^2 bn^2(m+\sigma)} - \eta_i(g)\gamma \quad (9) \end{aligned}$$

Equation (9) follows since $Var(x_k) = m + \sigma, \forall k$, and $Cov(x_i, x_j) = 0, \forall i, j, i \neq j$. Note that this derivation of the payoff functions is possible only under the assumptions of the linearity of market demand function and the normality of the signals. We can check the conflicting effects of network formation and information sharing on the payoff function in equation (9). With additional link formation (deletion), firm i receives a negative (positive) effect on the third term in equation (9), and experiences a positive (negative) effect from the second term at the same time. Marginal benefit and marginal cost from an additional link with j are $\frac{\sigma^2}{((\eta_j(g)+1)+2)^2 bn^2(m+\sigma)}$ and $\frac{\sigma^2(2\eta_i(g)+5)}{(\eta_i(g)+2)^2(\eta_i(g)+3)^2 bn^2(m+\sigma)}$ respectively, while marginal benefit and marginal cost of severing an existing link to j are $\frac{\sigma^2(2\eta_i(g)+3)}{(\eta_i(g)+1)^2(\eta_i(g)+2)^2 bn^2(m+\sigma)}$ and $\frac{\sigma^2}{(\eta_j(g)+2)^2 bn^2(m+\sigma)}$ respectively, where $\eta_i(g)$ and $\eta_j(g)$ are the cardinalities of $N_i(g)$ and $N_j(g)$ before link formation(deletion).

3.1. Pairwise Stable Networks

Now we characterize the stable collaboration networks under Bayesian quantity competition. It turns out that there exist multiple (symmetric and asymmetric) pairwise stable networks. Recall that we assume that the link formation cost, γ, is negligibily small since our main goal is to study the incentives for information sharing (equivalently, the benefit and cost of information sharing) when firms face uncertainty. The interesting findings, as we will see, are that even in the case where $\gamma = 0$, firms are not willing to form as many links as they possibly can. This reflects that information sharing itself has a nonnegligibly negative effect on the payoff of the firm.

We think it is natural and important to start by checking the commonly known results of the existing information sharing literature. Recall that, under unknown common demand, no information sharing is the unique Nash equilibrium outcome under Cournot competition (See, Vives, 1984; Gal-Or, 1985).

Proposition 2. *Under Assumptions (1), (2) and* $\sigma > 0$*, both the empty and the complete network are pairwise stable when* $n \geq 3$.

Proof. First we show that the empty network is pairwise stable. Notice that the stability condition (i) is trivially satisfied. Thus we only need to check whether condition (ii) is satisfied. Suppose that firms i and j form a link. The resulting network is $g^0 + g_{ij}$. We next check whether firms i and j find such a deviation profitable. Using equation (9), we have $\pi_i(g^0) - \pi_i(g^0 + g_{ij}) = \frac{\sigma^2}{36bn^2(m+\sigma)} + \gamma > 0$. So condition (ii) is satisfied. Now we show that the complete network is pairwise stable. By a similar reasoning, we only need to check whether pairwise stability condition (i) is satisfied. Consider again that one of the firms i and j sever a link. The resulting network is $g^{n-1} - g_{ij}$. Using equation (9), we have $\pi_i(g^{n-1}) - \pi_i(g^{n-1} - g_{ij}) = \pi_j(g^{n-1}) - \pi_j(g^{n-1} - g_{ij}) = \frac{\sigma^2}{bn^2(m+\sigma)}\{\frac{n^2-2n-1}{(n+1)^2 n^2}\} - \gamma > 0$ if $n \geq 3$. So condition (i) is satisfied. Hence the complete network is also pairwise stable. ■

This result indicates that complete information sharing is also a pairwise stable equilibrium. Note that centralized (and industry-wide) decision making, by definition, is exactly the same as decentralized (and pairwise) decision making if $n = 2$. So we reasonably expect that both produce the same result. The following result shows that the empty network (no information sharing) is the unique equilibrium outcome in a duopoly as in the centralized setting.

Remark. In a duopoly (i.e, $n = 2$), the empty network (no information sharing) is a unique equilibrium outcome. From Proposition 2, $\pi_i(g^0) - \pi_i(g^0 + g_{ij}) = \frac{\sigma^2}{36bn^2(m+\sigma)} + \gamma > 0$ and $\pi_i(g^{n-1}) - \pi_i(g^{n-1} - g_{ij}) = \frac{\sigma^2}{bn^2(m+\sigma)}\{\frac{n^2-2n-1}{(n+1)^2 n^2}\} - \gamma < 0$ when $n = 2$.

With this, we can derive the following result directly from Proposition 2. Recall that the dominant group architecture, $g^{d(k)}$, is characterized by one complete non-singleton component with $k \geq 2$ firms and $n-k$ singleton firms. Thus, there is a set of firms $N^d \subset N$ with the property that $g_{ij}=1$ for every pair $i, j \in N^d$ while for any $m \in N \setminus N^d, g_{ml}=0, \forall l \in N \setminus \{m\}$ (Goyal and Joshi, 2003).

Proposition 3. *Under Assumptions (1), (2) and* $\sigma > 0$ *the dominant group architecture,* $g^{d(k)}$*, is pairwise stable when* $k \geq 3$.

Proof. Consider any firm $i \in N^d$. From Proposition 2, this firm has no incentives to sever the existing link. That is, $\forall i, \pi_i(g^{k-1}) - \pi_i(g^{k-1} - g_{ij}) > 0$ when $k \geq 3$. So the stability condition (i) is easily satisfied. Now we need to check whether this firm has any incentive to form a link to an isolated firm m.
$\pi_i(g) - \pi_i(g+g_m) = \{\frac{a^2}{b(n+1)^2} + k\frac{\sigma^2}{b(k+1)^2 n^2 (m+\sigma)}\} - \{\frac{a^2}{b(n+1)^2} + (k-1)\frac{\sigma^2}{b(k+1)^2 n^2 (m+\sigma)} + \frac{\sigma^2}{b(k+2)^2 n^2 (m+\sigma)}$
$+\frac{\sigma^2}{9bn^2(m+\sigma)}\} + \gamma < 0$ when $k \geq 2$.
This means that firm i wants to form an additional link to the isolated firm m. The remaining thing is to check the incentives of the firm m.
$\pi_m(g) - \pi_m(g+g_{im}) = \{\frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{4bn^2(m+\sigma)}\} - \{\frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{9bn^2(m+\sigma)} + \frac{\sigma^2}{b(k+2)^2 n^2 (m+\sigma)}\} - \gamma > 0$
when $k \geq 2$.
So, the isolated firm i wants to remain isolated. Also, following from the previous Corollary, firm m does not have any incentives to form a link to $l \in N \setminus \{m\}$. Therefore, the stability condition (ii) is satisfied. ■

This shows that partial and asymmetric information sharing appears as an equilibrium if firms make a decentralized decision, even though the firms are ex ante identical. In addition, there exist other (symmetric and asymmetric) equilibrium structures specified below. The following result incorporates all findings studied above as special cases.

Theorem 4. *Let* $\mathcal{F}_1(N) = \{N_1, N_2, \cdots, N_p\}$ *be a partition of* N *such that* $\forall i \in \{1, \cdots, p\}, |N_i| \neq 2$*, and* $\forall i \in \{1, \cdots, p-1\}, |N_{i+1}| > \frac{(|N_i|+1)(|N_i|+2)}{\sqrt{2|N_i|+3}} - 2$. *And* $g^{|N_i|-1}$ *denotes the complete network over* N_i *for all* $i \in \{1, \cdots, p\}$*, Then*

(1) $g(\mathcal{F}(N)) = \bigcup_{i=1}^{p} g^{|N_i|-1}$ *is a pairwise stable network.*

(2) If g *is component-symmetric and pairwise stable, then* $g \in g(\mathcal{F}(N)) = \bigcup_{i=1}^{p} g^{|N_i|-1}$.

Proof. *For the proof of (1):* Take any network $g(\mathcal{F}(N) = \bigcup_{i=1}^{p} g^{|N_i|-1}$ satisfying the conditions specified. Without loss of generality, take any three firms i, j, and k such that $i \in N_i$, $j \in N_j$, $k \in N_{j+1}$ where $|N_i| = 1$, $|N_j| \geq 3$. First, we show that the firms j and k have no incentive to sever their existing link. In Proposition 2 we have shown that $\pi_j(g^{|N_j|-1}) - \pi_j(g^{|N_i|-1} - g_{jl}) > 0$ if $|N_i| \geq 3$, $l \in N_j$. This is true for the case of firm k. So condition (i) of pairwise stability is satisfied. Let's check whether condition (ii) of pairwise stability is satisfied.

Obviously, the isolated firm i has no incentives for link formation since $\pi_i(g)-\pi_i(g+g_{ij})>0$ and $\pi_i(g)-\pi_i(g+g_{ik})>0$. Then we only need to investigate the incentives of firms j and k by checking

$$\pi_j(g)-\pi_j(g+g_{jk})=\left\{\frac{a^2}{b(n+1)^2}+|N_j|\frac{\sigma^2}{b(|N_j|+1)^2 n^2(m+\sigma)}\right\}-\left\{\frac{a^2}{b(n+1)^2}\right\}$$

$$+(|N_j|-1)\frac{\sigma^2}{b(|N_j|+1)^2 n^2(m+\sigma)}+\frac{\sigma^2}{b(|N_j|+2)^2 n^2(m+\sigma)}$$

$$+\frac{\sigma^2}{b(|N_{j+1}|+2)^2 n^2(m+\sigma)}-\gamma$$

$$=\frac{\sigma^2}{bn^2(m+\sigma)}\left\{\frac{1}{(|N_j|+1)^2}-\frac{1}{(|N_j|+2)^2}-\frac{1}{(|N_{j+1}|+2)^2}\right\}-\gamma>0$$

$$\text{if } |N_{j+1}|>\frac{(|N_j|+1)(|N_j|+2)}{\sqrt{2|N_j|+3}}-2.$$

This means that firm j does not agree to form an additional link with k even if firm k tries to. This implies that j does not form a link to any other firm with more than $|N_{j+1}|-1$ links. Therefore condition (ii) of pairwise stability is also satisfied.

For the proof of (2): It suffices to show that each component $g'\in g$ is complete. We denote a symmetric component of degree Δ by g'^{Δ}, $\Delta=2,3,\cdots,|N_i|-1$. Suppose that $|N_i|\geq 3$. Then $\forall i$, $j\in N_i$, $\pi_i(g'^{\Delta})-\pi_i(g'^{\Delta}+g_{ij})=\frac{\sigma^2\{(\Delta+3)^2-2(\Delta+2)^2\}}{bn^2(m+\sigma)(\Delta+3)^2(\Delta+2)^2}<0$ for $\Delta=2,3,\cdots,|N_i|-2$, and $\forall i$, $j\in N_i$, $\pi_i(g'^{|N_i|-1})-\pi_i(g'^{|N_i|-1}-g_{ij})>0$. Therefore, if g is component-symmetric and pairwise stable, each component $g'\in g$ must be complete. ■

Theorem 4 illustrates the basic feature of pairwise stable network structures. The more links a firm has, the stronger incentive it gets for additional information. This property becomes clear when we introduce heterogeneity to the model in section 4. From this we can summarize our main finding: Unlike the result of previous literature on "centralized" information sharing, there emerges a broader level of information sharing as pairwise stable equilibria.

Example. Suppose $n=4$. Then $\{g^0,g^{d(3)},g^3\}$ is the set of all pairwise stable networks.

Here we feel it necessary to interpret the results of the present model qualitatively.

When firms behave as Nash competitors in the market, the effects of pooling private information on the profits of the firms are unclear. When more accurate information is available to the firms, the strategies can be chosen more accurately. And increased accuracy has an unambiguously positive effect on the payoff of the firm (which is called "Information effect"). On the other hand, the increase of common information by pooling of private information raises the correlation among the firms' decision rules. This increased correlation has ambiguous effects on the firm (which is called "Correlation effect"). Suppose that a firm observes a signal of low demand and shares it with others. Then it reduces the likelihood that its competitors overproduce. But when it observes a signal of high demand and reveals it to others, it reduces the likelihood that its competitors underproduce. While the first case raises the profits of the firm, the second reduces them. Hence, it is unclear whether firms will transmit their private information to the rival firms. In the previous literature, as Gal-or (1985) indicates, the main result is that benefits from pooling information and obtaining a more accurate measure of demand are dominated by the losses from increasing the correlation of firms' output decisions. Due to the underlying complete network structure, the models in the previous literature have much higher correlation among the decision rules than the present model. Thus, the negative effect dominates and no information sharing is the equilibrium outcome. In the present model, due to the (decentralized) pairwise interaction structure, correlation of the decision rules is lower than that of the previous literature even though there still exist strong negative effects, so there emerges a broader level of information sharing as pairwise stable equilibria. In this sense, this paper throws a new implication on the information sharing literature: that is, underlying network structure plays a significant role because the correlation effect varies according to the network structure.

Until now we have considered pairwise stability as a solution concept which allows unilateral and pairwise deviation of the firms. As is well known in the network formation literature, individual or pairwise based solution concepts may lead to multiple stable networks, so that they provide broad predictions. Nevertheless, we'd like to emphasize this result, since we think that pairwise stability is relevant and natural equilibrium concept in the analysis of the decentralized oligopolistic market. In our context, however, we cannot exclude the possibility of communication among firms that may allow a number of them to coordinate their choices of links. It is also probable that information sharing depends on the coalition structure. With this coalitional consideration, we now study strongly stable networks as a natural way for making tighter predictions.

3.2. Strongly Stable Networks

Strong stability of networks is a very demanding property, since it means that no set of firms could benefit through any rearranging of the links that they are involved in (including those linking to firms outside the coalition). If there exist such networks, they are essentially impossible to destabilize, since there is no possible reorganization that would be improving for all firms whose consent is needed. We now characterize the strongly stable collaboration networks under quantity competition.

Example. Under Assumptions (1), (2) and $\sigma > 0$, neither the complete network, g^{n-1}, nor the *dominant group architecture*, $g^{d(k)}$, are strongly stable.

We can show this by way of counterexample. First, consider a certain firm, i, in the complete network g^{n-1}, and suppose that firm i by itself deviates by severing all its links at once. Strong stability concept captures this kind of deviation while pairwise stability doesn't. Then, firm i is the only singleton firm and the remaining $(n-1)$ firms form one complete non-singleton component. Formally, the dominant group architecture $g^{d(n-1)} \in G$ is obtained from $g^{n-1} \in G$ via a deviation by $S = \{i\}$, and we can check that $\pi_i(g^{d(n-1)}) - \pi_i(g^{n-1}) = \{\frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{4bn^2(m+\sigma)}\} - \{\frac{a^2}{b(n+1)^2} + \frac{n\sigma^2}{(n+1)^2 bn^2(m+\sigma)} - (n-1)\gamma\} = \frac{\sigma^2}{bn^2(m+\sigma)}$ $\{\frac{(n-1)^2}{4(n+1)^2}\} + (n-1)\gamma > 0$. That is, this kind of deviation is profitable. Therefore g^{n-1} is not strongly stable. Similar arguments hold in the case of the dominant group architecture $g^{d(k)} \in G$. Consider again a certain firm j in the complete component, (i.e., $j \in N^d$), and suppose firm j severs all its links at once. Then we can find that another dominant group architecture $g^{d(k-1)} \in G$ is obtainable from $g^{d(k)} \in G$ via a deviation by $S = \{j\}$, and we can check that

$$\pi_j(g^{d(k-1)}) - \pi_j(g^{d(k)}) = \left\{\frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{4bn^2(m+\sigma)}\right\}$$
$$-\left\{\frac{a^2}{b(n+1)^2} + \frac{k\sigma^2}{(k+1)^2 bn^2(m+\sigma)} - (k-1)\gamma\right\} = \frac{\sigma^2}{bn^2(m+\sigma)}\left\{\frac{(k-1)^2}{4(k+1)^2}\right\}$$
$$+(k-1)\gamma > 0 \quad \text{when} \quad k \geq 2$$

Therefore $g^{d(k)} \in G$ is not strongly stable.

This example says that complete information sharing is not an equilibrium outcome if we allow any range of coordination and cooperation among the group of firms in the industry. Also it captures the idea that a firm would not benefit from severing any single link but would benefit from severing several links simultaneously which is not accounted for under the concept of pairwise stability. A little discussion on this issue will follow in section 5.

Theorem 5. *Under Assumptions (1), (2) and* $\sigma > 0$*, no information sharing (and the resulting empty network) is the unique strongly stable equilibrium outcome.*

Proof. Suppose that the empty network g^0 is not strongly stable. Consider the deviating coalition $S \ni i, j$, and the obtainable graph $g' \in G$ from $g^0 \in G$ via a deviation by S. Needless to say, g' must be nonempty. Suppose $\eta_i(g') = 1$, $g_{ij} = 1$. Then

$$\pi_i(g^0) - \pi_i(g') = \left\{ \frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{4bn^2(m+\sigma)} \right\} - \left\{ \frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{9bn^2(m+\sigma)} + \frac{\sigma^2}{(\eta_j(g')+2)^2 bn^2(m+\sigma)} - \gamma \right\}$$

$$= \frac{\sigma^2}{bn^2(m+\sigma)} \left\{ \frac{5(\eta_j(g')+2)^2 - 36}{36(\eta_j(g')+2)^2} \right\} + \gamma > 0$$

That is, forming a single link is not profitable. Hence, every deviating firm in must form at least two links simultaneously. Now suppose that every firm in forms at least two links. We can show this kind of deviation is not profitable either. That is,

$$\pi_i(g^0) - \pi_i(g') = \left\{ \frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{4bn^2(m+\sigma)} \right\} - \left\{ \frac{a^2}{b(n+1)^2} + \frac{\sigma^2}{(\eta_j(g')+2)^2 bn^2(m+\sigma)} + \sum g_{ij} \frac{\sigma^2}{(\eta_j(g')+2)^2 bn^2(m+\sigma)} - \eta_i(g')\gamma \right\}$$

$$= \frac{\sigma^2}{bn^2(m+\sigma)} \left\{ \frac{1}{4} - \frac{1}{(\eta_i(g')+2)^2} - \sum g_{ij} \frac{1}{(\eta_j(g')+2)^2} \right\} + \eta_i(g')\gamma > 0$$

where $\eta_i(g') \geq 2,\ \eta_j(g') \geq 2$ if $g_{ij} = 1, \forall j$.

Therefore, a profitable deviation is not possible. This contradicts that g^0 is not strongly stable. Also, this implicitly proves the uniqueness of strongly stable networks. If there is a nonempty graph g where a typical firm i has a single link, then this firm will benefit from a unilateral deviation by severing the link. Suppose that there is another graph g' where firms have at least two links or remain isolated. Then the firm with more than two links will benefit from severing all the existing links simultaneously. This completes the proof. ■

We reach the same no-information-sharing result as in the previous literature by allowing firms to form any range of coalition for deviation and coordinate their choices simultaneously. It is never a strange coincidence that both approaches produce the same result. In fact, the concept of strong stability in the present model moves the analysis closer to that of the existing literature: By allowing a wider level of coalition and cooperation among firms, we allow the firms to make a industry-wide decision. The wider level of coalition is allowed, the more similar effects are produced. In this sense, our result strengthens earlier findings about the incentives of oligopolistic firms, and can be regarded as complementary to the existing literature. Note that strong stability is a very demanding property, in the sense that once formed such networks are essentially impossible to destabilize, as there is no possible reorganization that would be improving for all of the firms whose consent is needed. Therefore, this no-information-sharing result can not understate pairwise stable equilibrium outcomes of the model analyzed above. Furthermore, we can carefully state that the results of the existing literature were obtained in very restricted situations, since they reach such results under the exogenously given grand coalition assumption. In sum, this paper, by characterizing both pairwise stable and strongly stable networks, demonstrates that the incentives for information sharing critically depend on the coalition structure as well.

IV. Heterogeneous Firms

In this section we introduce heterogeneity across firms by assuming that firms observe a noisy signal for u with different degree of accuracy. We sustain the other assumptions as above. This environment enables us to analyze which firm has the lager incentive to share information when the accuracy of private information is different across firms. So, we can examine the divergence of incentives for information sharing and the difference of behavioral characteristics among firms. While our goal is still characterizing the set of stable network structures, we can check the robustness of the basic model by trembling the model a little.

The signal observed by firm i is π_i. We assume:

$$\pi_i = u_i + e_i,\ u_i \sim N(0,\sigma_i),\ e_i \sim N(0,m_i) \tag{10}$$

where $Cov(e_i,e_j)=0, i \neq j; Cov(u_i,e_j)=0 \forall i,j; Cov(u_i,u_j)=0, i \neq j; \sigma_i < \infty, m_i < \infty$, $\forall i$; and $u=(\Sigma_i u_i)/n$, hence $u \sim N(0,\frac{1}{n^2}\Sigma\sigma_i)$. With a little calculation, the following decision rule forms the unique Nash equilibrium in the game beginning at the second stage:

$$q_i(y_i) = A_0^i + \Sigma_{j\neq i} A_j^i g_{ij} x_j + A_i^i x_i,\ \forall i \tag{11}$$

where $A_0^i = \frac{a}{b(n+1)}$, $A_j^i = \frac{\sigma_j}{(\eta_j(g)+2)bn(m_j+\sigma_j)}$, $A_i^i = \frac{\sigma_i}{(\eta_i(g)+2)bn(m_i+\sigma_i)}$. The payoff function starting at the first stage is given by:

$$\pi_i(g) = \frac{a^2}{b(n+1)^2} + \sum_{j\neq i} g_{ij} \frac{\sigma_j^2}{(\eta_j(g)+2)^2 bn^2(m_j+\sigma_j)} + \frac{\sigma_i^2}{(\eta_i(g)+2)^2 bn^2(m_i+\sigma_i)} - \eta_i(g)\gamma \qquad (12)$$

Now we characterize the stable networks among heterogeneous firms.

Proposition 6. *Under Assumptions (1), (10) and* $\sigma_i > 0 \forall i$, *the empty network is always pairwise stable, and the complete network is pairwise stable if* $n \geq 3$ *and* $\frac{2n+1}{n^2} < \frac{\sigma_j^2(m_i+\sigma_i)}{\sigma_i^2(m_j+\sigma_j)} < \frac{n^2}{2n+1}$, $\forall i, j$.

Proof. First we show that the empty network is pairwise stable. Using equation (12), we have $\pi_i(g^0) - \pi_i(g^0 + g_{ij}) = \frac{5\sigma_i^2(m_j+\sigma_j) - 4\sigma_j^2(m_i+\sigma_i)}{36bn^2(m_i+\sigma_i)(m_j+\sigma_j)} + \gamma$. If $5\sigma_i^2(m_j+\sigma_j) - 4\sigma_j^2(m_i+\sigma_i) > 0$, it's done. Suppose $5\sigma_i^2(m_j+\sigma_j) - 4\sigma_j^2(m_i+\sigma_i) < 0$. Then we have $\pi_j(g^0) - \pi_j(g^0 + g_{ij}) = \frac{5\sigma_j^2(m_i+\sigma_i) - 4\sigma_i^2(m_j+\sigma_j)}{36bn^2(m_i+\sigma_i)(m_j+\sigma_j)} + \gamma > 0$. So condition (ii) is satisfied. By similar way, we can show that $\pi_i(g^{n-1}) - \pi_i(g^{n-1} - g_{ij}) > 0$ and $\pi_j(g^{n-1}) - \pi_j(g^{n-1} - g_{ij}) > 0$ if $n \geq 3$ and $\frac{2n+1}{n^2} < \frac{\sigma_j^2(m_i+\sigma_i)}{\sigma_i^2(m_j+\sigma_j)} < \frac{n^2}{2n+1}$, $\forall i, j$. ■

We can draw many implications from Proposition 6, but there are two points we want to make about this result here. First, the empty network is stable because a firm with the less accurate information of the two declines the link formation. It is still true that for any given g, if link formation fails between any two firms, it is the firm with inaccurate information that refuses to collaborate. Second, the complete network is stable only when firms with the similar level of accuracy comprise the network. When firms are symmetric, the complete network is always pairwise stable if $n \geq 3$ (Proposition 2).

Remark. In any equilibrium information is shared among the firms with similar accuracy of information. And if information sharing fails between any two firms for any network structure, it is the firm with inaccurate information that refuses to collaborate.

Example. Under Assumptions (1), (10) and $\sigma_i > 0 \forall i$, the dominant group architecture, $g^{d(k)}$, is pairwise stable if (1) $k \geq 3$, (2) $\frac{2k+1}{k^2} < \frac{\sigma_j^2(m_i+\sigma_i)}{\sigma_i^2(m_j+\sigma_j)} < \frac{k^2}{2k+1}$, $\forall i, j \in N^d$, and (3) for any $i \in N^d$ and any $h \in N \setminus N^d$, $\frac{\sigma_h^2(m_i+\sigma_i)}{\sigma_i^2(m_h+\sigma_h)} > \frac{36}{5(k+1)^2}$ or

$\frac{\sigma_h^2(m_i+\sigma_i)}{\sigma_i^2(m_h+\sigma_h)} > \frac{9(2k+3)}{(k+1)^2(k+2)^2}$

$g^{d(k)}$ is pairwise stable when the complete non-singleton component is formed by the k firms with similar accuracy (represented by conditions (1) and (2)), and the dominant group of k firms and $n-k$ singleton firms are significantly differentiated in their private information (represented by condition (3)). Also this shows that partial or asymmetric information sharing emerges as equilibria when heterogeneous firms make a decision in the decentralized environment. The following result characterizes all pairwise stable networks, and incorporates all findings studied above as special cases.

Theorem 7. *Let* $N=\{1,2,\cdots,n\}$*,* $n\geq 3$ *be the set of rearranged firms such that* $\frac{\sigma_i^2}{(m_i+\sigma_i)} > \frac{\sigma_{i+1}^2}{(m_{i+1}+\sigma_{i+1})}$*,* $\forall i\in\{1,2,\cdots,n-1\}$*. So, firm 1 is the one with the least accurate private information, while firm n is the one with the most accurate private information. Let* $\mathcal{F}_2(N)=\{N_1,N_2,\cdots,N_{\mathrm{p}}\}$ *be a partition of* N *such that* $\forall i\in\{1,2,\cdots,\mathrm{p}-1\}$*,* $\forall i\in N_i$*,* $j\in N_{i+1}$*,* $\frac{\sigma_i^2}{(m_i+\sigma_i)} > \frac{\sigma_j^2}{(m_j+\sigma_j)}$*. Suppose that* $g(N_i)$ *is the network over* N_i *for all* $i\in\{1,\cdots,\mathrm{p}\}$ *Then* $g(\mathcal{F}_2(N))=\bigcup_{i=i}^{\mathrm{p}} g(N_i)$ *is a pairwise stable network iff*

(1) If N_i *is non-singleton,* $|N_i|\geq 3$ *and* $\eta_i(g(N_i))\geq 2, \forall i\in N_i$

(2) For $g_{ii+h}=1, \frac{\sigma_i^2(m_{i+h}+\sigma_{i+h})}{\sigma_{i+h}^2(m_i+\sigma_i)} < \frac{(\eta_i(g)+1)^2(\eta_i(g)+2)^2}{(\eta_{i+h}(g)+2)^2(2\eta_i(g)+3)}$*, and*

for $g_{ii+h}=0, \frac{\sigma_i^2(m_{i+h}+\sigma_{i+h})}{\sigma_{i+h}^2(m_i+\sigma_i)} > \frac{(\eta_i(g)+2)^2(\eta_i(g)+3)^2}{(\eta_{i+h}(g)+3)^2(2\eta_i(g)+5)}$*,* $\forall i, i+h\in N_i$*,*

(3) $\forall i\in\{1,2,\cdots,\mathrm{p}-1\}, \forall i\in N_i, j\in N_{i+1}, \frac{\sigma_i^2(m_j+\sigma_j)}{\sigma_j^2(m_i+\sigma_i)} > \frac{(\eta_i(g)+2)^2(\eta_i(g)+3)^2}{(\eta_j(g)+3)^2(2\eta_i(g)+5)}$.

Proof. Just use the definition of pairwise stability. Condition (1) allows the existence of singleton components, condition (2) indicates stability within a component, and condition (3) guarantees stability between any two firms from distinct components. ■

This is somewhat abstract, so here we take a special example which captures all the basic features of the model.

Example: Assortative Networks. Let $\mathcal{F}_2(N)=\{N_1,N_2,\cdots,N_{\mathrm{p}}\}$ be a partition of N such that $\forall i\in\{i,2,\cdots,\mathrm{p}-1\}, \forall i\in N_i, j\in N_{i+1}, \frac{\sigma_i^2}{(m_i+\sigma_i)} > \frac{\sigma_j^2}{(m_j+\sigma_j)}$. So, N_1 is the component which consists of firms with the least accurate information, while N_{p} is the component which consists of firms with the most accurate information. Suppose that $g^{|N_i|-1}$ is the complete network over N_i for all $i\in\{1,\cdots,\mathrm{p}\}$. Then $g(\mathcal{F}_2(N))=\bigcup_{i=1}^{\mathrm{p}} g^{|N_i|-1}$ is a pairwise stable network if

(1) If N_i is non-singleton, $|N_i|\geq 3$ and $\frac{2|N_i|+1}{|N_i|^2} < \frac{\sigma_h^2(m_i+\sigma_i)}{\sigma_i^2(m_h+\sigma_h)} < \frac{|N_i|^2}{2|N_i|+1}, \forall i,h\in N_i$,

(2) $\forall i \in \{1,2,\cdots,\mathrm{p}-1\}, \forall i \in N_i, j \in N_{i+1}, \frac{\sigma_i^2 (m_j+\sigma_j)}{\sigma_j^2 (m_i+\sigma_i)} > \frac{(|N_i|+1)^2(|N_i|+2)^2}{(|N_j|+2)^2(2|N_i|+3)}$

This result is similar to Theorem 4. Condition (1) guarantees the completeness of a component. Condition (2) requires that signals should be differentiated enough that firms from the different components do not form a link to each other. Partition $\mathcal{F}_2(N)$ exactly corresponds to partition $\mathcal{F}_1(N)$. The size of the component in partition $\mathcal{F}_1(N)$ has the same role as the accuracy of firms' information in the component in partition $\mathcal{F}_2(N)$ Here we can check the basic features of decentralized information sharing among heterogeneous firms: In any equilibrium information is shared among the firms with similar accuracy of information. And if information sharing fails between any two firms for any network structure, it is the firm with inaccurate information that refuses to collaborate. Also this implies partial and asymmetric information sharing results.

Lemma 8. *Under Assumptions (1), (11) and* $\sigma > 0$*, no information sharing (and the resulting empty network) is the unique strongly stable equilibrium outcome.*

The results of this section reinforce the findings of the previous section in that partial and asymmetric information sharing emerges when firms make a decision in the decentralized way.

In sum, we introduce heterogeneity across firms by assuming that firms observe a noisy signal for μ with different degree of accuracy, and show that, unlike the result of the previous literature on the centralized information sharing, there emerges a broader level of information sharing as a pairwise stable equilibrium.

V. Discussion on Further Researches

5.1. A Noncooperative Game of Network Formation

Myerson (1991) suggests a noncooperative game of network formation. For every firm i, the strategy set is an $n-1$ tuple of 0 and 1, $G_i = \{0,1\}^{n-1}$. Let g_{ij} denote the j th coordinate of g_i. If $g_{ij} = 1$, firm i indicates its willingness to form a link with firm j. Given the strategy profile g, an undirected network g is formed by letting firms i and j linked if and only if $g_{ij} g_{ji} = 1$. In words, the formation of a link requires the consent of both firms.

A strategy profile g is a *Nash equilibrium* if and only if, for all i, all strategies g_i', in G_i, $\pi_i(g) \geq \pi_i(g_i', g_{-i})$ and g is called Nash stable. It is easy to see that the concept of Nash stability is too weak as a concept for modeling network formation when links are bilateral. For instance, the empty network is always a

Nash network, regardless of the payoff structure. Moreover, any network where no player could gain by severing some links is a Nash network, regardless of how attractive it might be to add additional links (Bloch and Jackson, 2005). Thus, further refinement is necessary, and different directions must be proposed.

5.2. When Uncertainty is about Unknown Private Demands

Until now we have examined how firms' incentives for information sharing vary depending upon the network structure when uncertainty is about an unknown common demand. As is well known, if uncertainty reflects unknown private demands, our results will be significantly affected. Let's consider this case. The game is played in the same way as in the previous sections. Now a market consists of the firms, each producing a heterogeneous product and each firm faces an individual demand shock. The market demand is still linear, namely

$$p_i = a - b_i q_i - \sum_{j \neq i} b_j q_j + u_i , \quad a, b_i > 0, b_i > | b_j |, \forall j \neq i \tag{13}$$

where p_i is the price and q_i the amount of product i produced. Since b_j can be positive or negative the two products can be substitutes or complements, and since $b_i > | b_j |$, $\forall j \neq i$, "cross effects" are dominated by "own effects" (Gal-Or, 1986). As the coefficients b_i and b_j are closer to each other, the two products are less differentiated. Putting a constant a is without loss of generality, since u_i captures heterogeneity in the intercept of the function.

The market demand is stochastic, i.e., u_i is normally distributed with mean zero and variance σ_i.[10] The signal observed by firm i is x_i. We continue to assume that $\pi_i = u_i + e_i$, $e_i \sim N(0, m_i)$ where $Cov(e_i, e_j) = 0, i \neq j$; $Cov(u_i, e_j) = 0, \forall i, j$; $Cov(u_i, u_j) = 0, i \neq j$. We can also allow asymmetry in network structure. We can reasonably expect that the analysis requires the use of numerical rather than analytical methods since asymmetry in network structure and heterogeneity in oligopoly causes great complexity and the calculations are likely to become cumbersome.

5.3. Some other Issues

First, we should take up the issue of spillovers. Our model does not accommodate direct spillovers across the collaborative links of firms, since, by assumption, information transmissions happen simultaneously. We can study the incentives for information sharing under the correlated signals assumption. By doing so, we may

[10] This environment is called a "private values" problem in the auction literature (Gal-Or, 1986).

vary the amount of initial correlation among the signals to investigate how various degrees of initial correlation affect the incentives for network formation and information sharing.

Another extension would be to investigate whether the incentives for network formation and information exchange are affected by other sources of uncertainty in the market. In particular, much stronger incentives may arise if technology rather than demand is stochastic, or if prices rather than quantities are chosen. However, generalizing the analysis to a different class of demand function may require the use of numerical rather than analytical methods.

In the present paper, we have restricted our analysis to an ex ante symmetric environment. We have shown that the resulting outcome may be ex post asymmetric. It would be another interesting direction to analyze an ex ante asymmetric environment where one firm has access to more precise information or enjoys a superior technology.

VI. Conclusion

We have developed a simple two stage model of strategic network formation and Cournot competition in order to analyze the incentive of firms to share information in an oligopolistic market where firms face an uncertain demand. Before firms observe a private signal they decide whether to form links to the other firms in order to exchange information on market uncertainty. After link formation each firm observes its own private signal, and then they transmit their private information to the linked firms simultaneously. After network formation and information transmission each firm chooses its level of output. Our interest has been in the interaction between the incentives of firms to collaborate for information sharing (and the resulting network structure) responding to the market uncertainty and the competition in the product market. We have derived pure strategy mixed cooperative and noncooperative equilibria that are stable and subgame perfect, and characterized the resulting graphs.

Our analysis has attempted to clarify the nature of collaboration structures under market uncertainty and different scopes of cooperation or coordination among the oligopolistic firms in the market. An important finding is that even in the setting where firms face an unknown common demand, complete information sharing, no information sharing, and partial-asymmetric information sharing emerge as pairwise stable and subgame perfect equilibrium outcomes under Cournot competition, if firms make a pairwise decision in the decentralized environment. This result shows a strong contrast to the existing literature on centralized information sharing. The result is both interesting and important, since pairwise stable outcome captures relevant and natural equilibrium state in the analysis of the decentralized oligopolistic market. It becomes obvious when we consider the real world in which forming a large and credible coalition is quite difficult in oligopoly which consists of Nash competitors. Finally, the unique strongly stable equilibrium outcome involves no information sharing and the resulting empty network. This illustrates that information sharing among firms facing an uncertain environment critically depends on coalition structure.

References

Bala, V. and S, Goyal (2000), "A Noncooperative Model of Network Formation," *Econometrica,* Vol. 68, No. 5, 1181-1230.

Clarke, R. (1983), "Collusion and the Incentives for Information Sharing," *Bell Journal of Economics*, Vol. 14, No. 2, 383-394.

d'Aspremont, C. and A. Jacquemin (1988), "Cooperative and Noncooperative R&D in Duopoly with Spillovers," *American Economic Review*, Vol. 78, No. 5, 1133-1137.

Dutta, B. and S. Mutuswami (1997), "Stable Networks," *Journal of Economic Theory*, Vol. 76, No. 2, 322-344.

Gal-Or, E. (1985), "Information Sharing in Oligopoly," *Econometrica*, Vol. 53, No. 2, 329-343.

Gal-Or, E. (1986), "Information Transmission - Cournot and Bertrand Equilibria," *Review of Econmic Studies*, Vol. 53, No. 1, 85-92.

Goyal, S. and S. Joshi (2003), "Networks of Collaboration in Oligopoly," *Games Economic Behavior*, Vol. 43, No. 1, 57-85.

Goyal, S., J. L. Moraga-Gonzalez, and A. Konovalov (2008), "Hybrid R&D," *Journal of the European Economic Association*, Vol. 6, No. 6, 1309-1338.

Goyal, S. and J. L. Moraga-Gonzalez (2001), "R&D Networks," *RAND Journal of Economics*, Vol. 32, No. 4, 686-707.

Jackson, M. and A. van den Nouweland (2005), "Strongly Stable Networks," *Games and Economic Behavior*, Vol. 51, No. 2, 420-444.

Jackson, M. and A. Watts (2002), "The Evolution of Social and Economic Networks," *Journal of Economic Theory*, Vol. 106, No. 2, 265-295.

Jackson, M. (2010), Social and Economic Networks, Princeton University Press.

Jackson, M. and A. Wolinsky (1996), "A Strategic Model of Social and Economic Networks," *Journal of Economic Theory*, Vol. 71, No. 1, 44-74.

Kirby, A. (1988), "Trade Associations as Information Exchange Mechanisms," *RAND Journal of Economics*, Vol. 19, No. 1, 138-146.

Kranton, R. and D. Minehart (2001), "A Theory of Buyer-seller Networks," *American Economic Review*, Vol. 91, No. 3, 485-508.

Myerson, R. (1991), Game Theory: Analysis of Conflict, Harvard University Press.

Myerson, M. (1977), "Graphs and Cooperation in Games," *Mathematics of Operations Research*, Vol. 2, No. 3, 225-229.

Novshek, W. and H. Sonnenschein (1982), "Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release," *Bell Journal of Economics*, Vol. 13, No. 1, 214-218.

Radner, R. (1962), "Team Decision Problems," *Annals of Mathematical Statistics*, Vol. 33, No. 3, 857-881.

Roth, A. and M. Sotomayor (1989), Two Sided Matching, Econometric Society Monographs No. 18, Cambridge University Press.

Suzumura, K. (1992), "Cooperative and Noncooperative R&D in an Oligopoly with

Spillovers," *American Economic Review*, Vol. 82, No. 5, 1307-1320.

Vives, X. (1984), "Duopoly Information Equilibrium: Cournot and Bertrand," *Journal Economic Theory*, Vol. 34, 71-94.

Watts, A. (2001), "A Dynamic Model of Network Formation," *Games and Economic Behavior*, Vol. 34, No. 2, 331-341.