Separation of Two Agencies for Fiscal Policies*

Sangwon Park**

This paper studies the effect of separation of government agencies on fiscal policies. We construct a model with moral hazard which compares the integrated system and the separated system. In the separated system, two independent agencies are in charge of taxes and government expenditure, respectively. Meanwhile, in the integrated system, one agency decides on both policies. In both systems, there is a third party which provides information on the effectiveness of government expenditure only to the budget agency and is willing to overstate in order to acquire more budget. It is shown that the separated system is better at controlling the information provider’s incentive to mislead and can be superior under some parameter values in spite of its coordination problem.

JEL Classification: H61, D78, H11
Keywords: Fiscal Policy, Government Organization, Budget Deficit, Fiscal Discipline

I. Introduction

Taxes, government spending and budget control are the major cornerstone of fiscal policies and their importance can not be overstated. While vast literature exists on these topics, relatively little attention has been given to the governmental institution under which these policies are related each other. In a representative democracy, the responsibility of fiscal policies is usually entrusted to a relatively small group of actors who pursue individual objectives. Observing that the fiscal agencies of the government are the key players in the realm of policy-making, this paper studies the effect of government organization on fiscal outcome, particularly...
by comparing the separation and integration of fiscal functions. In principle, taxes and budget can be independent of each other because tax revenue is collected according to the tax law whereas public spending is allocated through the annual budget process. This procedural independency may justify the separation of agencies whose main jobs are to create public revenue, prepare budget and control public expenditure. However, the more relevant question than just the formal separation, is whether the players have consistent intention or whether they share the same information required for the optimal decision.

It is not only pure academic curiosity, but also reflects, at least to some extent, the realistic matter. As Wanner (2003) explained, the range of functions of fiscal institution can differ across countries and over time. The multiplicity of agencies in the composition usually implies that taxation and budget preparation are conducted by different bodies. For example, while fiscal authorities in countries such as the US, Canada and Australia consist of dual agencies, in other countries, such as Germany and the UK, one minister of a department takes care of all the functions.

Within a country, the different organizational structures have been chosen over time. For example, the Bureau of the Budget (BOB) of the US was established in 1921 as a part of the Department of the Treasury in order to assist the president to prepare the budget. Later on, the BOB was separated and moved to the Executive Office of President. In Korea, the new government introduced in 2008 made an organizational reform which set forth the goal of constructing ‘small but efficient’ entity by reducing the number of government departments. In particular, two agencies whose jobs included the tax policy and budget preparation were combined into the Ministry of Strategy and Finance, aiming to enhance the ability of coordinating the relevant policies and unifying fiscal process.

Of course, many factors should be taken into consideration in evaluating different organizational structures of government, such as the historical background, political system, economy of scope in organization, capacity of bureaucrats and political accountability. In particular, we point out here that this important question relates to how the separation or integration of government fiscal agencies can affect the

---

1 Although the relation between the executive branch and the legislative branch is also an interesting research topic, particularly in a presidential system such as the US, the behavior of agencies within the administration is the concern here. The model here is more suited for a country where the administrative government, rather than the legislative branch, plays the major role in determining the fiscal policy.

2 The BOB is the predecessor of the Office of Management and Budget (OMB).

3 The President Roosevelt’s Executive Order 8248 of September 8, 1939.

4 The draft of the Government Reforming Act of 2008 sent to the assembly.

5 As it is explained in Tomkin (1998), the BOB was introduced due to a surge of public pressure to actively monitor the expenditure. The belief that the monitoring role will be promoted by an independent body can be another simple reason of having the separated system in reality. We thank the anonymous referee to address this point.
relative importance of budget discipline and coordination of policies when there are information asymmetry and moral hazard problems.

To understand the effect of organizational structure of fiscal agencies on fiscal policies, we construct a simple theoretic model and compare the two systems; namely the separated system and the integrated system. In the integrated system, one agency determines both taxes and government spending, taking political responsibility for both. In the separated system, there are two agencies, a tax authority and a budget office, that are in charge of taxes and government spending, respectively. Fiscal policies are the outcome of the interaction between these two players. In both systems, there is uncertainty regarding the benefits from government spending, and this information must be extracted from a third party. This information provider can be interpreted as a spending minister, who implements a public project and wants the greater allocation of the budget in order to promote the interests of the constituents belonging to his/her jurisdiction.

Two effects are worth mentioning. In the integrated system, one fiscal authority determines both policies after receiving messages from the third party, and hence both policies can be adjusted accordingly. In contrast, in the separated system, only the budget allocating agency has access to the information and consequently, a coordination problem may occur. The tax authority’s inability to respond to the message itself makes the separated system perform worse. However, this unresponsiveness may in turn have a positive effect. Since the benefit of the overstatement depends on the degree to which fiscal policies respond, the ignorance of the tax authority will entail less payoff to a false report and allow the separated system to become better at controlling the incentive of the information provider to overstate. Under some parameter values, this effect can dominate the other.

This paper is related to the literature of which the main question is “how political institutions affect fiscal policies.” Some of them have analyzed the effect of electoral systems and others have concentrated on the governmental institutions. Alesina and Perotti (1995, 1999), Alesina et al. (1995) and Eslava (2010) provide comprehensive surveys with both theoretical and empirical results. The analysis on budgetary processes and budget deficit are worth mentioning among these studies. For example, Von Hagen and Harden (1995), Velasco (1999, 2000) and Krogstrup and Wyplosz (2010) presented models in which a budget deficit arises from the common pool problem among spending ministers in a cabinet, interest groups and interacting states. In particular, Von Hagen and Harden (1995) suggested that the bias can be reduced by introducing elements of centralization into the budget process and provided an empirical result. Hallerberg and Von Hagen (1997) and Swank (2002) compared two remedies to the common pool problem, namely the role of the finance minister, who does not have any jurisdictional bias, and binding budget targets imposed by the prime minister. However, the possibility of separating the tax authority from the budget office has been neglected so far.
Studies that show the advantages of multiple agencies in various moral hazard situations also have some relevance here. For example, Lewis and Sappington (1997) explained the merits of separating information collection and production. They revealed, except for forgone economies of scope, that the most preferred outcome is achieved by assigning the planning and production tasks to two different agents and avoiding all contracting distortions. Dewatripont and Tirole (1999) showed how many organizations including the executive and legislative branches of the government, can use competition among enfranchised advocates of special interests in order to improve policy making. However, this paper deals more specifically with fiscal policies and is not based on the principal-agent setup found in the aforementioned studies.

Lastly, studies on earmarked taxes, such as Buchanan (1963), Dhillon and Perroni (2001), Bos (2000), Brett and Keen (2000) and Anesi (2005) are also relevant here. These papers interpret the earmarked tax as a guarantee of budget for a certain area and study its role. Even though earmarked tax literature and this paper share a common approach of considering taxes and public expenditure together, the main purpose of the two studies are clearly different.

II. Model

2.1 Setup

Suppose there is a new public project that the government wants to implement. To avoid the complexity coming from the dynamic consideration, this project is assumed to be a one-time event. The government expenditure amount on the project is denoted by $g$. A lump-sum tax is imposed in order to finance this project, and we let the tax rate be denoted by $t$. We assume that the political cost of imposing $t$ is well represented by a strictly convex function, $C(t)$, which satisfies $\lim_{t \to \infty} C'(t) = \infty$ and $C'(0) = 0$. Likewise, the political support for $g$ is denoted by $\lambda u(g)$. $u(g)$ is assumed to be strictly concave and satisfy $\lim u'(g) = 0$ and $\lim u'(g) = \infty$. Here, $\lambda$ represents the uncertainty regarding the benefit from the public project and is a random variable. Specifically, $\lambda$ takes $\lambda_0$ with probability $p$ and $\lambda_1$ with probability $1-p$ where $0 < \lambda_0 \leq \lambda_1$ and $0 < p < 1$.

$g - t > 0$ (or $g - t < 0$) implies the current budget deficit (or surplus). When there is a deficit (or surplus), the future fiscal policy will be affected. For example, extra taxes or budget cuts in the future should follow a budget deficit. The cost (or benefit) of the burden from the deficit (or surplus) is represented by a strictly convex function $B(g-t)$. $B(\cdot)$ is defined on $\mathbb{R}$ and satisfies $B' > 0$, $B(0) = 0$, $\lim_{x \to \infty} B'(x) = \infty$ and $\lim_{x \to -\infty} B'(x) = 0$. The budget deficit is a political cost and hence $B$ is positive when $g - t > 0$. On the contrary, the budget surplus is political benefit
and hence \( B \) is negative when \( g < t \). The typical example of \( B \) is \( B(g-t) = e^{e^{-t}} - 1 \). \( \frac{\partial}{\partial (g-t)} (\frac{e^{e^{-t}}}{g}) = e^{e^{-t}} > 0 \) implies that the marginal cost of the budget deficit is bigger when its size is larger. In addition, \( \frac{\partial}{\partial (g-t)} (\frac{e^{e^{-t}}}{g}) = -e^{e^{-t}} < 0 \) implies that the marginal benefit of a budget surplus is smaller when its size is larger.

A justification for the objective functions above is needed before proceeding. It would be better if we could formalize how fiscal policies affect people’s welfare which, in turn, creates support or resistance through the political system. Instead of formally adding the processes such as voting, reelection or promotion in bureaucratic hierarchy into the model, we take the simplest model by assuming a reduced form of their objectives, hoping that the same intuition works with a full description. For example, the probabilistic voting models such as Coughlin (1982), Hetticah and Winer (1988) and Coughlin et al. (1990) connect the politician’s incentive of maximizing plurality in the election to policies maximizing a social objective function. We believe that \( u, B \) and \( C \) here can be driven by the similar argument.

For the determination of \( g \) and \( t \), we consider two different organizational schemes in the subsequent sections.

### 2.2 Integrated System

In the system we analyze first, there is only one agency that decides both policies simultaneously and takes all the political cost and support. For the sake of the discussion, we call this scheme ‘the integrated system’ and the agency ‘the Department of Finance’(DoF). The objective function of the DoF is \( -C(t) + \lambda u(g) - B(g-t) \). Obviously, the DoF’s choice in \( t \) and \( g \) depends on \( \lambda \). However, it is quite common to believe that the fiscal authority can access limited information on public projects. To model this restriction, it is assumed that the DoF only knows the distribution of \( \lambda \), the value of which is revealed to another government agency who actually implements the public project with the resources given by the DoF. We call this agency ‘Spending Minster’(SM). Following Niskannen (1971), we assume that the SM wants a larger budget spending because he/she can enjoy bigger fringe benefits from it. In this model, SM’s objective function is simply \( g \).

Before making a decision on \( t \) and \( g \), the DoF extracts information about \( \lambda \) from the SM, in the simplest way. That is, after \( \lambda \) is realized, the SM is supposed to report this to the DoF. Of course, the SM may make a false report of \( \lambda \) even though the true state is \( \lambda \). To attenuate this moral hazard, there will be an audit after the completion of the public project. The audit is not complete, meaning that there is a probability of \( \pi \) that an audit is successfully performed on the public project of interest. In addition, given a successful audit, the SM has to incur a cost \( \alpha \) if the original report is found to be false. In sum, \( \pi \alpha \) is the expected cost from
a misreport. Because the social cost of conducting the audit is irrelevant in comparing the two systems, it is assumed to be 0 in order to simplify the exposition.

The following summarizes the procedure in the integrated system.

- \( \lambda \) is realized and known to the SM.
- The SM reports \( \lambda \) to the DoF.
- The DoF chooses \( t \) and \( g \) based on the SM’s report.
- \( C(t) \) and \( \lambda u(g) \) occur.
- The probabilistic audit is performed and the SM incurs the cost, \( \alpha \), in the case of a false report.
- The future cost (benefit) from the current fiscal policy, \( B(t - g) \), is realized.

Let \((R_m, t_m, g_m)\) be a strategy profile in the integrated system. \( R_m \) is the SM’s reporting strategy and it is a function from \( \{ \lambda_l, \lambda_h \} \) to a probability distribution on \( \{ \lambda_l, \lambda_h \} \). For example, \( R_m(\lambda_l) = \lambda_h \) means that the SM’s report is \( \lambda_h \) even though \( \lambda_l \) is actually realized. The DoF’s action may depend on the SM’s report. That is, \( t_m \) and \( g_m \) are functions of the SM’s report and denoted by \( t_m(\lambda) \) and \( g_m(\lambda) \).

Including the SM’s report stage, the overall determination of the fiscal policies in each system is the outcome of the sequential move game, and therefore, we adapt a perfect Bayesian Nash equilibrium. The equilibrium in each system is denoted by *, such as \((R_m^*, t_m^*, g_m^*)\). Technically, a perfect Bayesian Nash equilibrium should include the players’ belief in each information set. However, to avoid notational inconvenience, we will omit the players’ beliefs unless it is necessary to specify them. We say that an equilibrium is separating if the SM reports a true value at each state. An equilibrium is pooling if the SM reports are the same, regardless of the true value of \( \lambda \). Lastly, an equilibrium is mixed if the SM uses a mixed strategy along the equilibrium.

2.3 Separated System

In the system we consider next, there are two agencies that decide on \( t \) and \( g \) independently. For the sake of argument, we call this ‘the separated system’ and the two agencies are called ‘the Department of Taxation’ (DoT) and ‘the Budget Office’ (BO) hereafter. The DoT first selects \( t \) and then \( g \) is determined by the BO sequentially. While the results are not much different under the simultaneous move of the DoT and the BO, the assumption of a sequential move makes the comparison much easier. To focus on the informational difference between the two systems, we assume that DoT and BO have the same objective function as the DoF, which is \(-C(t) + \lambda u(g) - B(g - t)\). The DoT and BO’s choice in \( t \) and \( g \) depend on \( \lambda \) as well. Like the integrated system, the SM is supposed to report this
to the BO, the budget agency, after $\lambda$ is realized. The fact that only the BO, not DoT, can receive the report of the SM plays a key role in this model.

One may question why the DoT and the BO cannot communicate and share information about $\lambda$. However, selfishness of governmental agencies has been reported in many countries. Information sharing among agencies is often limited by technical, organizational, and political barriers as explained by Dawes (1998). For example, Atabakhsh et al. (2004) explained the necessity for government agencies to share data and pointed out obstacles to overcome in order to achieve information sharing, by studying in details two domains in the US: law enforcement and disease informatics. Moreover, it turns out that intentional blockage of information to the DoT ex ante can be even beneficial to the society.

The procedure of the policy making in the separated system can be summarized as follows.

- $\lambda$ is realized and known to the SM.
- The SM reports $\lambda$ to BO.
- The DoT chooses $t$.
- The BO chooses $g$ based on $t$ and the SM’s report.
- $C(t)$ and $\lambda u(g)$ occur.
- The probabilistic audit is performed and the SM incurs the cost, $\alpha$, in the case of a false report.
- The future cost (benefit) from the current fiscal policy, $B(t-g)$, is realized.

Let $(R, t, g)$ denote a strategy profile in the separated system. Note that in the separated system the BO’s strategy is a function of the SM’s report and $t$ as well and is denoted by $g(\lambda, t)$, while the DoT’s strategy only depends on his belief on $\lambda$.

To acquire normative implication, we will use $W(t, g) = -C(t) + \lambda u(g) - B(g - t)$ as the one indicating the social welfare. Though other functions could replace the one above, particularly to accommodate the bias which the government agency might have, we will stick to the most natural one. Let $W^*_s$ and $W^*_m$ be a set of social welfare function values derived from the equilibrium in each system. Formally,

$$W^*_s = \{\text{Expected value of } W(t, g) \mid (t, g) \text{ is an equilibrium outcome in the separated system}\}$$

$$W^*_m = \{\text{Expected value of } W(t, g) \mid (t, g) \text{ is an equilibrium outcome in the integrated system}\}.$$  

With slight abuse of notation, we say $W^*_s < W^*_m$ (or $W^*_m < W^*_s$) if $x < y$ (or $y < x$) for all $x \in W^*_s$ and $y \in W^*_m$. 


III. Analysis of Information Effect

3.1 Integrated System

To formally state an equilibrium in the integrated system, let \( \hat{\lambda} \) and \( \hat{\lambda} \) be the solution to the following simultaneous equations. \( \hat{\lambda} \) and \( \hat{\lambda} \) are the DoF’s choice, given that \( \lambda \) is the expected value according to his/her belief.

\[
-C'(t)+B'(g-t)=0, \\
\lambda u'(g)-B'(g-t)=0.
\]

Let \( K_m^1 \equiv \hat{g}_m(\lambda_1)-\hat{g}_m(\lambda_2) \) and \( K_m^2 \equiv \hat{g}_m(p\lambda_1+(1-p)\lambda_2)-\hat{g}_m(\lambda_2) \). \( K_m^1 \) and \( K_m^2 \) are two key values used in characterizing the equilibrium in terms of \( \pi\alpha \). It is straightforward that \( 0 < K_m^2 < K_m^1 \) holds. Proposition 1 below fully characterizes the equilibrium in the integrated system.

**Proposition 1** In the integrated system, the followings hold:

(A) If \( K_m^1 \leq \pi\alpha \), then

\[
R_m^*(\lambda_1)=\hat{\lambda}_1, R_m^*(\lambda_2)=\hat{\lambda}_2, \\
g_m^*(\lambda_1)=\hat{g}_m(\lambda_1), g_m^*(\lambda_2)=\hat{g}_m(\lambda_2), \\
\hat{\tau}_m^*(\lambda_1)=\hat{\tau}_m(\lambda_2), \hat{\tau}_m^*(\lambda_2)=\hat{\tau}_m(\lambda_2).
\]

(B) If \( K_m^2 < \pi\alpha < K_m^1 \), then

\[
R_m^*(\lambda_1)=\lambda_1 \text{ with probability } q_m \text{ and } \lambda_2 \text{ with probability } 1-q_m, \\
R_m^*(\lambda_2)=\hat{\lambda}_2, \\
g_m^*(\lambda_1)=\hat{g}_m(\lambda_1), \\
g_m^*(\lambda_2)=\hat{g}_m\left(\frac{p(1-q_m)}{p(1-q_m)+(1-p)}\lambda_1+\frac{1-p}{p(1-q_m)+(1-p)}\lambda_2\right), \\
\hat{\tau}_m^*(\lambda_1)=\hat{\tau}_m(\lambda_2), \\
\hat{\tau}_m^*(\lambda_2)=\hat{\tau}_m\left(\frac{p(1-q_m)}{p(1-q_m)+(1-p)}\lambda_1+\frac{1-p}{p(1-q_m)+(1-p)}\lambda_2\right)
\]

where \( q_m \) satisfies \( \hat{g}_m\left(\frac{p(1-q_m)}{p(1-q_m)+(1-p)}\lambda_1+\frac{1-p}{p(1-q_m)+(1-p)}\lambda_2\right)-\hat{g}_m(\lambda_2)=\pi\alpha \).

(C) If \( \pi\alpha \leq K_m^2 \), then
\[ R_N^*(\lambda_i) = R_N^*(\lambda_h) = \lambda_h, \]
\[ g_m^*(\lambda) = \hat{g}_m(p\lambda_i + (1-p)\lambda_h), \]
\[ t_m^*(\lambda) = \hat{t}_m(p\lambda_i + (1-p)\lambda_h). \]

**Proof**: Since it is straightforward that \( R_N^*(\lambda_i) = \lambda_i \), we are going to check up only the best response of \( \lambda_h \) type.

(A) \( K_m^1 = \hat{g}_m(\lambda_i) - \hat{g}_m(\lambda_h) \leq \alpha \) case:

If the DoF follows \( g_m(\lambda_i) = \hat{g}_m(\lambda_i), \ g_m(\lambda_h) = \hat{g}_m(\lambda_h), \ t_m(\lambda) = \hat{t}_m(\lambda) \), and \( t_m(\lambda_i) = \hat{t}_m(\lambda_i) \), then the payoff to the \( \lambda_i \) type from the strategy \( R_m(\lambda_i) = \lambda_i \) is \( \hat{g}_m(\lambda) \). The SM’s payoff from deviation to \( R_m(\lambda_h) = \lambda_h \) is \( \hat{g}_m(\lambda_h) - \alpha \). Because \( \hat{g}_m(\lambda_i) = \alpha \leq \hat{g}_m(\lambda_h) \), the deviation is not profitable. If the SM reports truthfully, \( (\hat{g}_m(\lambda_i), \hat{t}_m(\lambda_i)) \) is obviously the DoF’s optimal choice with the report of \( \lambda_i \). Likewise, \( (\hat{g}_m(\lambda_h), \hat{t}_m(\lambda_h)) \) is the DoF’s optimal choice given the true report of \( \lambda_h \). To show that there is no other equilibrium, assume that \( R_m(\lambda) = \lambda_h \) with positive probability \( q > 0 \) at the equilibrium. Then, the DoF’s best response should be \( \hat{t}_m \left( \frac{p(1-q)}{p(1-q) + (1-p)} \lambda_i + \frac{1-p}{p(1-q) + (1-p)} \lambda_h \right) \) and \( \hat{g}_m \left( \frac{p(1-q)}{p(1-q) + (1-p)} \lambda_i + \frac{1-p}{p(1-q) + (1-p)} \lambda_h \right) \). However, given \( \hat{t}_m \left( \frac{p(1-q)}{p(1-q) + (1-p)} \lambda_i + \frac{1-p}{p(1-q) + (1-p)} \lambda_h \right) \) and \( \hat{g}_m \left( \frac{p(1-q)}{p(1-q) + (1-p)} \lambda_i + \frac{1-p}{p(1-q) + (1-p)} \lambda_h \right) - \alpha \), because

\[ \hat{g}_m \left( \frac{p(1-q)}{p(1-q) + (1-p)} \lambda_i + \frac{1-p}{p(1-q) + (1-p)} \lambda_h \right) < \hat{g}_m(\lambda_h) - \alpha \]
\[ \leq \hat{g}_m(\lambda_i). \]

Therefore, \( R_m(\lambda_i) = \lambda_i \) with any positive probability \( q \) cannot be an equilibrium.

(B) \( K_m^2 < \alpha \leq K_m^1 \) case:

\( \alpha < \hat{g}_m(\lambda_i) - \hat{g}_m(\lambda_h) \) implies that \( R_m(\lambda_i) = \lambda_i \) and \( R_m(\lambda_h) = \lambda_h \) cannot be an equilibrium, because the SM with \( \lambda_h \) will have a profitable deviation of reporting \( \lambda_i \). In addition, consider \( R_m(\lambda_i) = R_m(\lambda_h) = \lambda_h \). By following \( R_m(\lambda_i) = \lambda_i \) the SM with \( \lambda_i \) only gets \( \hat{g}_m(p\lambda_i + (1-p)\lambda_h) - \alpha \), whereas deviating to \( R_m(\lambda_i) = \lambda_i \) guarantees at least \( \hat{g}_m(\lambda_i) \). Hence, \( \hat{g}_m(p\lambda_i + (1-p)\lambda_h) - \hat{g}_m(\lambda_i) < \alpha \) implies that \( R_m(\lambda_i) = R_m(\lambda_h) = \lambda_h \) cannot be an equilibrium either. The argument up to now means that the only possible equilibrium is a mixed strategy by the SM with type \( \lambda_i \). To be a mixed strategy, \( R_m(\lambda_i) = \lambda_i \) and \( R_m(\lambda_h) = \lambda_h \) should provide the same payoff to type \( \lambda_i \). In addition, the DoF forms his/her belief based on the mixed strategy of type \( \lambda_i \). From these two observations, we can derive
\( g_m^*(\lambda_i), g_m^*(\lambda_h), t_m^*(\lambda_i), t_m^*(\lambda_h) \) and \( q_m \) as above.

(C) \( \pi \alpha \leq K_m^2 \) case:

\[ \pi \alpha \leq \hat{g}_m(p\lambda_i + (1-p)\lambda_h) - \hat{g}_m(\lambda_i) < \hat{g}_m(\lambda_h) - \hat{g}_m(\lambda_i) \] implies that \( R_m(\lambda_i) = \lambda_i \) and \( R_m(\lambda_h) = \lambda_h \) cannot be an equilibrium, because \( \lambda_i \) type will deviate by reporting \( \lambda_h \). Consider the possibility of a mixed strategy of \( \lambda_i \) type. \( \hat{g}_m(q\lambda_i + (1-q)\lambda_h) - \pi \alpha \), the payoff from \( R_m(\lambda_i) = \lambda_i \), is strictly larger than \( \hat{g}_m(p\lambda_i + (1-p)\lambda_h) - \pi \alpha \), which is also bigger than \( g_m(\lambda_i) \). Hence, reporting \( \lambda_h \) is strictly better than \( \lambda_i \) and moreover, a mixed strategy cannot be any part of the equilibrium. The only possibility is that \( R_m(\lambda_i) = R_m(\lambda_h) = \lambda_i \). Given \( R_m(\lambda_i) = R_m(\lambda_h) = \lambda_h \), the DoF’s best response is \( \hat{g}_m(p\lambda_i + (1-p)\lambda_h) - \pi \alpha \). To complete equilibrium description, we need to construct the DoF’s belief and his/her action after observing the report of \( \lambda_i \). To assign 0 probability to type \( \lambda_h \) after observing \( \lambda_i \) is one possible belief. Then the best response of the DoF to the report of \( \lambda_i \) is \( \hat{g}_m(\lambda_i) \). Therefore, as long as \( \pi \alpha \leq \hat{g}_m(p\lambda_i + (1-p)\lambda_h) - \hat{g}_m(\lambda_i) \) holds, \( \lambda_i \) type has no incentive to deviate from \( \lambda_i \) to \( \lambda_h \). \\

The lesson in Proposition 1 is clear. The SM’s incentive of the false report depends on its benefit and cost. The expected cost from the audit when the SM, with \( \lambda_i \), makes a false report of \( \lambda_h \) is \( \pi \alpha \). Its benefit derives from the DoF’s decision on \( g \), which depends on the truthfulness of the report on the equilibrium. If DoF expects truth-telling from the both types, then \( \hat{g}_m(\lambda_i) \) and \( \hat{g}_m(\lambda_h) \) should be the government expenditure, given the reports of \( \lambda_i \) and \( \lambda_h \), respectively. It means that \( K_m^1 = \hat{g}_m(\lambda_i) - \hat{g}_m(\lambda_h) \) is the benefit that \( \lambda_i \) type received from the overstatement. If \( K_m^1 \leq \pi \alpha \), then the equilibrium should prescribe the SM to truly report \( \lambda_i \).

On the other hand, if \( \pi \alpha \) is too low then only \( \lambda_h \) will be reported. Specifically consider the equilibrium in which both types report \( \lambda_h \). \( \hat{g}_m(p\lambda_i + (1-p)\lambda_h) \) is the DoF’s choice of \( g \), in the case of \( \lambda_h \) message, because it does not give any additional information and \( p \) is the ex ante probability. His decision in the case of \( \lambda_h \) message depends on his out-of-equilibrium belief. The minimum benefit that the true report can attain arises when DoF believes that it is the low type and therefore, chooses \( \hat{g}_m(\lambda_h) \) after receiving the \( \lambda_h \) report. Hence, if \( \pi \alpha \) is lower than \( K_m^1 = \hat{g}_m(p\lambda_i + (1-p)\lambda_h) - \hat{g}_m(\lambda_i) \), then both types will report \( \lambda_h \) and DoF will choose \( \hat{g}_m(p\lambda_i + (1-p)\lambda_h) \) along the equilibrium.

A mixed strategy of the low type can be part of the equilibrium if \( \pi \alpha \) falls in the middle between \( K_m^1 \) and \( K_m^1 \). The SM with \( \lambda_h \) reports randomly and the DoF chooses \( (t, g) \) based on the posterior belief which is obtained from the SM’s mixed strategy.

Assuming there is no audit cost (or more generally the audit cost is independent
of $\pi\alpha$), it is straightforward that the social welfare is increasing in $\pi\alpha$. The figure below shows an example of social welfare function in the integrated system.\(^6\)

![Figure 1] Social Welfare as a Function of $\pi\alpha$ in the Integrated System

### 3.2 Separated System

The characterization of the equilibrium in the separated system is similar to the one in the previous subsection but more complicated expression is needed because the two agencies with different beliefs may interact after the SM’s report. For a formal description, we define $\hat{\lambda}_i(x,y,z)$, $\hat{g}_i(x,y,z)$ and $\hat{g}_i(x,y,z)$ as $(t, g^l, g^h)$, which satisfies the following simultaneous equations,

$$-C'(t) +xB'(g^l - t) + (1-x)B'(g^h - t) = 0, \quad (1)$$
$$y\lambda_i + (1-y)\lambda_i u'(g^l) - B'(g^l - t) = 0, \quad (2)$$
$$z\lambda_i + (1-z)\lambda_i u'(g^h) - B'(g^h - t) = 0. \quad (3)$$

Simply put, $\hat{\lambda}_i(x,y,z)$ is the optimal choice of the DoT, given his belief that with a probability of $x$, the BO’s action will be $g^l$ and a probability of $1-x$, it will be $g^h$. $\hat{g}_i(x,y,z)$ is the BO’s best response if he believes, after the SM’s report of $\lambda_i$, that $y$ is the probability of $\lambda_i$. Likewise, $\hat{g}_i(x,y,z)$ is the BO’s best response if he believes, after the SM’s report of $\lambda_i$, that $z$ is the probability of $\lambda_i$.

\(^6\) It is obtained under $u(g) = \ln g$, $B(g-t) = e^{e^t}-1$, $C(t) = e^{e^t} - 1$, $\lambda_i = 1$, $\lambda_i = 10$, and $\rho = \frac{1}{2}$.
We use the subscripts $l$ and $h$ to distinguish between the BO’s actions depending on the SM’s report. For example, suppose that $R_l(\lambda_t) = \lambda_l$ and $R_h(\lambda_t) = \lambda_h$ are given. Then the DoT is supposed to believe that the BO will receive the report of $\lambda_l$ with probability $p$. Hence, the $x$ in $(x,y,z)$ should be $p$. In addition, after the SM’s report of $\lambda_l$, the BO’s belief is to assigns 1 to $\lambda_l$ and 0 to $\lambda_h$, which makes the $y$ in $(x,y,z)$ 1. Likewise after the report of $\lambda_h$, the BO assigns 1 to $\lambda_h$ and 0 to $\lambda_l$, which makes the $z$ in $(x,y,z)$ 0. In sum, $\hat{\gamma}_l(p,1,0), \hat{\gamma}_h(p,1,0)$, and $\hat{\gamma}_l(p,1,0)$ are the DoT’s and BO’s best response given $R_l(\lambda_t) = \lambda_l$ and $R_h(\lambda_t) = \lambda_h$.

Note that in the separated system, the DoT and the BO move sequentially. Therefore, in his/her maximization, the DoT may consider the effect of his choice $t$ on the BO’s optimal choice of $g$. However, since the BO has the same objective function as the DoT, this secondary effect is always 0, as long as they share the same expectation on the SM’s reporting strategy.\footnote{Technically, we need the requirement that the BO’s ex post belief only depends on the SM’s strategy. In other words, even after observing the out-of-equilibrium $t$, the BO maintains the same belief on the SM’s type as the one along the equilibrium $t$. This requirement is embedded in the definition of perfect Bayesian Nash equilibrium.}

**Lemma 1** For any \( q \in (0,1) \)

(A) \( \hat{\gamma}_l^0(pq,1, p(p-q)) - \hat{\gamma}_l^0(pq,1, p(p-q)) < \hat{\gamma}_l^0(p,1,0) - \hat{\gamma}_l^0(p,1,0) \)

(B) \( \hat{\gamma}_h^0(0,1,p) - \hat{\gamma}_h^0(0,1,p) < \hat{\gamma}_h^0(pq,1, p(p-q)) - \hat{\gamma}_h^0(pq,1, p(p-q)) \)

**Proof**: In the appendix.

Three values in Lemma 1, $\hat{\gamma}_l^0(p,1,0), \hat{\gamma}_h^0(0,1,p), \hat{\gamma}_h^0(pq,1, p(p-q)) - \hat{\gamma}_h^0(pq,1, p(p-q))$ and $\hat{\gamma}_l^0(0,1,p) - \hat{\gamma}_l^0(0,1,p)$ are benefits of overstatement to the SM with low $\lambda_l$, in each separating, mixed and pooling equilibrium, respectively. In a separating equilibrium, the BO anticipates a true report and the report of $\lambda_h$ will convince him that the benefit of public expenditure is really high. Hence, it induces the largest $g$. By the same token, the report of $\lambda_l$ engenders the smallest $g$ and the difference in $g$ between the two reports in the separating equilibrium should be large. In a pooling equilibrium, the SM’s message does not bring any information. The SM’s $\lambda_h$ reports causes a limited impact on $g$ and the difference in $g$ between the two reports is minimal. The one from the mixed equilibrium obviously falls between the two values and we obtain Lemma 1. On the contrary, the cost of overstatement is $\pi \alpha$ regardless of the type of equilibrium. Combining this observation and the result of Lemma 1 provides us with proposition 2, which characterizes the equilibrium of the separated system in terms of $\pi \alpha$.\footnote{Technically, we need the requirement that the BO’s ex post belief only depends on the SM’s strategy. In other words, even after observing the out-of-equilibrium $t$, the BO maintains the same belief on the SM’s type as the one along the equilibrium $t$. This requirement is embedded in the definition of perfect Bayesian Nash equilibrium.}
Proposition 2 Let $K^i_1 \equiv \hat{g}_i^i(p,1,0) - \hat{g}_i^i(p,1,0)$ and $K^i_2 \equiv \hat{g}_i^i(0,1,p) - \hat{g}_i^i(0,1,p)$. The followings hold.

(A) If $K^i_1 \leq \pi \alpha$, the equilibrium should satisfy

\[
R^i_1(\lambda_i) = \lambda_s, \\
R^i_2(\lambda_s) = \lambda_s, \\
\ell^i = \hat{\ell}(p,1,0), \\
g^i_1(\lambda_i, \hat{\ell}(p,1,0)) = \hat{g}_i^i(p,1,0), \\
g^i_2(\lambda_s, \hat{\ell}(p,1,0)) = \hat{g}_i^i(p,1,0).
\]

(B) If $K^i_2 < \pi \alpha < K^i_1$, the equilibrium should satisfy

\[
R^i_1(\lambda_i) = \begin{cases} \lambda_i, \text{with } q \text{ prob.} \\ \lambda_s, \text{with } 1-q \text{ prob.} \end{cases}, \\
R^i_2(\lambda_s) = \lambda_s, \\
\ell^i = \hat{\ell}\left(pq, 1, \frac{p(1-q)}{p(1-q) + (1-p)}\right), \\
g^i_1(\lambda_i, \hat{\ell}(p,1,0)) = \hat{g}_i^i\left(pq, 1, \frac{p(1-q)}{p(1-q) + (1-p)}\right), \\
g^i_2(\lambda_s, \hat{\ell}(p,1,0)) = \hat{g}_i^i\left(pq, 1, \frac{p(1-q)}{p(1-q) + (1-p)}\right).
\]

where $q$ satisfies $\hat{g}_i^i\left(pq, 1, \frac{p(1-q)}{p(1-q) + (1-p)}\right) - \hat{g}_i^i\left(pq, 1, \frac{p(1-q)}{p(1-q) + (1-p)}\right) = \pi \alpha$.

(C) If $\pi \alpha \leq K^i_2$, then $R^i_1(\lambda_i) = \lambda_s$. In addition, there is an equilibrium satisfying

\[
R^i_1(\lambda_i) = R^i_2(\lambda_s) = \lambda_s, \\
\ell^i = \hat{\ell}(0,1,p), \\
g^i_1(\lambda_i, \hat{\ell}(0,1,p)) = \hat{g}_i^i(0,1,p), \\
g^i_2(\lambda_s, \hat{\ell}(0,1,p)) = \hat{g}_i^i(0,1,p).
\]

Proof: In the appendix.

Proposition 2 has a similar structure to Proposition 1 and provides two critical values in terms of $\pi \alpha$. $K^i_1 = \hat{g}_i^i(p,1,0) - \hat{g}_i^i(p,1,0)$, corresponding to $K^i_1$ in Proposition 1, is one that the low type SM gets from the false report when BO and
DoT anticipate a true report. If \( \pi \alpha \) is larger than this value, the low type will not overstate \( \lambda \) along the equilibrium and only truth-telling equilibrium will prevail. Likewise, \( K^2_s = \hat{g}^s(0,1,p) - \hat{g}^s(0,1,p) \) is a payoff that the SM with low \( \lambda \) received from the false report when BO and DoT anticipate the report of \( \lambda_s \) regardless of the true state. It corresponds to \( K^2_m \) in Proposition 1. If \( \pi \alpha \) is lower than this value, the benefit of the false report is larger than the cost and further, an equilibrium in which both types report \( \lambda_s \) exists. If \( \pi \alpha \) is in the middle, then the equilibrium should prescribe a mixed action, that is, the low type reports \( \lambda_l \) with \( q \) probability and \( \lambda_s \) with \( 1-q \) probability.

The social welfare in the separated system is also increasing in \( \pi \alpha \). The figure below is obtained under the same assumption of Figure 1.

3.3 Comparison of the Two Systems

Given Propositions 1 and 2, the comparison of the two systems boils down to a relative standing of the critical values. The observation in Lemma 2, which states that the critical values in the separated system are smaller than the corresponding ones in the integrated system, is interesting as itself. To see the intuition, first consider a true report equilibrium in each system. In the integrated system, after obtaining the \( \lambda_s \) report, DoF is going to set \( t \) and \( g \) relatively big because the government expenditure is supposed to be beneficial. In the separated system, without the access to this informative report, DoT assigns only \( 1-p \) probability to \( \lambda_s \) and chooses \( t \) accordingly. Because \( g \) is strategically complementary to \( t \),
in the integrated system responds more sensitively than that in the separated system. This sensitivity makes the integrated system more vulnerable to the false report and therefore, the critical value of $\pi\alpha$ which prevents it, should be larger.

The same argument holds for the critical values of the existence of the pooling equilibrium, meaning that the one for the integrated system is larger than that of the separated system.

**Lemma 2**

$$
\hat{g}^s(p,1,0) - \hat{g}^s_p(p,1,0) < \hat{g}_m(\lambda_s) - \hat{g}_m(\lambda_s),
\hat{g}^l(0,1,p) - \hat{g}^l_p(0,1,p) < \hat{g}_m(p\lambda + (1-p)\lambda_s) - \hat{g}_m(\lambda_s).
$$

**Proof:** Given $t$ and $\lambda$, we can define the BO's best response as

$$\hat{g}(t,\lambda) = \arg\max_g \lambda u(g) - C(t) - B(g - t).$$

$\hat{g}(t,\lambda)$ is well-defined and satisfies the first order condition, $B'(g - t) = \lambda u'(g)$. In addition, we can easily show that $0 < \frac{\partial C(t)}{\partial t} < 1$ and $0 < \frac{\partial B(t)}{\partial t}$. To prove $\hat{g}^s(p,1,0) - \hat{g}^s_p(p,1,0) < \hat{g}_m(\lambda_s) - \hat{g}_m(\lambda_s)$, note that $\hat{t}_s(p,1,0)$ satisfies the following equation,

$$-C'(t) + p B'(\hat{g}(t,\lambda_s) - t) + (1-p) B'(\hat{g}(t,\lambda_s) - t) = 0 \quad (4).$$

In addition, $\hat{t}_m(\lambda_s)$ and $\hat{t}_m(\lambda_s)$ satisfy (5) and (6), respectively.

$$\hat{t}_m(\lambda_s) : -C'(t) + B'(\hat{g}(t,\lambda_s) - t) = 0 \quad (5),$$

$$\hat{t}_m(\lambda_s) : -C'(t) + B'(\hat{g}(t,\lambda_s) - t) = 0 \quad (6).$$

From (4), (6) and the convexity of $B$ and $C$, we can conclude that $\hat{t}_s(p,1,0) > \hat{t}_m(\lambda_s)$. By the same token, from (4) and (5), we get $\hat{t}_s(p,1,0) < \hat{t}_m(\lambda_s)$. Therefore the followings hold:

$$\hat{g}_m(\lambda_s) = \hat{g}(\hat{t}_m(\lambda_s),\lambda_s) > \hat{g}(\hat{t}_s(p,1,0),\lambda_s) = \hat{g}^s(p,1,0),$$

$$\hat{g}_m(\lambda_s) = \hat{g}(\hat{t}_m(\lambda_s),\lambda_s) < \hat{g}(\hat{t}_s(p,1,0),\lambda_s) = \hat{g}^l(0,1,p).$$

The above two inequalities lead us to $\hat{g}^s(p,1,0) - \hat{g}^s_p(p,1,0) < \hat{g}_m(\lambda_s) - \hat{g}_m(\lambda_s)$.

Now let us show $\hat{g}^l(0,1,p) - \hat{g}^l_p(0,1,p) < \hat{g}_m(p\lambda + (1-p)\lambda_s) - \hat{g}_m(\lambda_s)$. Both $\hat{t}_s(0,1,p)$ and $\hat{t}_m(p\lambda + (1-p)\lambda_s)$ satisfy the following equation,
\[
C'(t) + B'(g(t, p \lambda + (1-p) \lambda_h) - t) = 0 .
\]

It means that \( \hat{\iota}_n(0,1,p) = \hat{\iota}_m(p \lambda + (1-p) \lambda_h) \). Hence,

\[
\hat{g}_m(p \lambda + (1-p) \lambda_h) = \hat{g}(\hat{\iota}_m(p \lambda + (1-p) \lambda_h), \lambda_h) = \hat{g}(\hat{\iota}_m(0,1,p), \lambda_h) = \hat{g}(0,1,p) .
\]

(7)

In addition, \( \hat{\iota}_m(\lambda_i) \) satisfies the following equation,

\[
\hat{\iota}_m(\lambda_i) - C'(t) + B'(g(t, \lambda_i) - t) = 0 .
\]

Because \( \tilde{g}(t, p \lambda + (1-p) \lambda_h) > \tilde{g}(t, \lambda_i) \) for any \( t \), we can conclude that \( \hat{\iota}_m(\lambda_i) < \hat{\iota}_m(0,1,p) \). Therefore, the following holds.

\[
\hat{g}_m(\lambda_i) = \hat{g}(\hat{\iota}_m(\lambda_i), \lambda_i) < \hat{g}(\hat{\iota}_m(0,1,p), \lambda_i) = \hat{g}(0,1,p)
\]

(8)

From (7) and (8), \( \hat{g}_m(0,1,p) - \hat{g}_m(0,1,p) < g_m(p \lambda + (1-p) \lambda_h) - g_m(\lambda_i) \) is obtained. //

Given Propositions 1, 2 and Lemma 2, we are ready to present the main result of this paper. There are two factors working in the opposite direction. The first one is the direct effect from the tax agency’s inability to make an adjustment. In the integrated system, the DoF has access to the SM’s report and his decision on \( t \) can depend on it. However, in the separated system, the DoT’s choice is only based on his belief for the SM’s report and this inability becomes a weakness of the system. An interesting observation is that the more informative the SM’s message is the larger the first effect is. In other words, if the SM’s report does not bring any additional information along the equilibrium, such as pooling, then this advantage of the separated system disappears. The other effect comes from the relation to the SM, the information provider. Because \( g \) and \( t \) are positively related, the DoT’s insensitivity to the SM’s message also induces that \( g \) is relatively small even with the SM’s report of high \( \lambda \). Specifically, the benefit that the SM gets from overstatement must be smaller and therefore, the SM’s moral hazard will be less severe in the separated system. Hence, the fiscal discipline works in the favor of the separated system.

**Proposition 3** The followings hold:

\[^{8}\]

\[^{8}\] Three intervals in (A), (B) and (C) are not exhaustive. If \( \hat{\omega}_m(p \lambda + (1-p) \lambda_h) - \hat{\omega}_m(\lambda_i) < \pi \alpha < \)
(A) If \( \hat{g}_m(\lambda) - \hat{g}_s(\lambda) \leq \pi\alpha \), then the SM truthfully reports in any equilibrium of both systems and \( W_m^* < W_s^* \).

(B) If \( \pi\alpha \leq \hat{g}_s^*(0,1,p) \), then SM always reports \( \lambda_b \) in any equilibrium of both systems. Therefore \( W_m^* = W_s^* \).

(C) If \( \hat{g}_s^*(0,1,p) < \pi\alpha \), then the probability of SM’s false report in any equilibrium of the integrated system is strictly larger than that of any equilibrium of the separated system and \( W_m^* < W_s^* \).

Proof: In the appendix.

If \( \pi\alpha \) is big enough, as in (A) of Proposition 3, only the separating equilibrium prevails in both cases. This means that both systems are equally successful in containing the SM’s moral hazard problem. However, the coordination problem still occurs in the separating system because the \( t \) can not adjust accordingly. Hence, the social welfare is larger in the integrated system. If \( \pi\alpha \) is small, as in (B) of Proposition 3, then only a pooling equilibrium exists in both systems. This means that the SM’s incentive to overstate could not be controlled at all, even in a separated system. In addition, the advantage of the separated system does not come to the play either because the SM’s message is uninformative. The two systems generate the same polices and further, social welfare coincides as well.

(C) of Proposition 3 states that, there is some interval of \( \pi\alpha \) such that the separated system provides a better outcome than the integrated system. Lemma 2 has shown that under some parameter values, the SM’s will overstate in the integrated system, but will tell the true \( \lambda \) (at least partially) in the separated system. It means that the separated system is successful more in the information extraction from SM. In addition, the merit of the integrated system does not occur because the SM reports only \( \lambda_b \) in any case. Hence, the separated system is superior to the integrated one.

The figure below shows the social welfare function in each system under the same assumption of Figure 1 and 2.

It would not be an easy task to measure \( \pi\alpha \) and determine the better system in the real world. However, lessons and policy implications are quite clear. As the stronger discipline is required because of lack of monitoring power, the larger advantage of the separated system appears in which the independent entity takes in charge of preparing the budget. If coordination is more important with the variation in policy conditions, the integrated system should be the better choice.

\( \hat{q}_m(\lambda_b) - \hat{q}_s(\lambda_b) \), then the advantage of controlling SM’s overstatement and the incoordination problem in the separated system occur together and the comparison between \( W_m^* \) and \( W_s^* \) creates ambiguity.
IV. Conclusion

The main purpose of this paper is to gain insight into the pros and cons of separation and integration of fiscal authorities, while considering the informational imperfection and disciplinal need for public spending.

This paper begins with the belief that modern government consists of different entities and that they want to achieve each of their own individual goals. In addition, the fiscal authority in the government lacks full information that is required for the optimal decision and therefore, needs to consult a third party. Then, different government organizations for fiscal agencies may have different outcomes. Specifically, we compared two systems. In an integrated system, one agency determines both taxes and government spending, taking political responsibility for both. In contrast, the separated system commands division of labor such that the two fiscal agencies, the tax authority and the budget office, interact and fiscal policies are its outcome. In both systems, there is uncertainty regarding the possible benefits from government spending, and this information must be extracted by the budget agency from a spending minister. The spending minister (SM) typically wants a greater allocation of the budget and therefore, has an incentive to overstate.

The tax authority cannot fully adjust accordingly in the separated system because the message is not delivered to him/her. This disadvantage is more relevant as more valuable information on the public spending is reported. In contrast, and somewhat
surprisingly, this insensitivity to the report helps to control the spending subject’s incentive to overstate his private information, which can be beneficial to the society at the end. The more fiscal policies respond to the report, the bigger payoff the SM gets from the false report. In other words, the SM will have less incentive to misreport if it brings a relatively small payoff due to lack of change in the tax authority’s action. Hence, the separated system performs better in discouraging the SM from sending the wrong message, which is good for the society. Which effect is more important depends on the parameter values, which may, at least partially, explain such variety in the organizational structure for different countries that are also depending on the political and economic circumstances.
References


Niskanen, W. (1971), Bureaucracy and Representative Government, Chicago: Aldine-
Atherton.
Appendix

Proof of Lemma 1:
To shorten the notations, let
\[
\beta = \frac{p(1-q)}{p(1-q) + (1-p)},
\]
\[
\hat{q}^A_s(s) = \hat{q}^A_s(p,1,0), \quad \hat{q}^A_i(s) = \hat{q}_i^A(p,1,0), \quad \hat{t}_i(s) = \hat{t}_i(p,1,0),
\]
\[
\hat{q}^A_s(m) = \hat{q}^A_s\left(pq,1,\frac{p(1-q)}{p(1-q) + (1-p)}\right), \quad \hat{q}^A_i(m) = \hat{q}_i^A\left(pq,1,\frac{p(1-q)}{p(1-q) + (1-p)}\right),
\]
\[
\hat{t}_i(m) = \hat{t}_i\left(pq,1,\frac{p(1-q)}{p(1-q) + (1-p)}\right),
\]
\[
\hat{q}^A(o) = \hat{q}^A(0,1,p), \quad \hat{q}^A_i(o) = \hat{q}_i^A(0,1,p) \quad \text{and} \quad \hat{t}_i(o) = \hat{t}_i(0,1,p).
\]

By definition, \(\hat{q}^A_s(s), \hat{q}^A_i(s), \hat{t}_i(s), \hat{q}^A_s(m), \hat{q}^A_i(m), \hat{q}^A(o), \hat{q}^A_i(o)\) and \(\hat{t}_i(o)\) satisfy the followings.\(^9\)

\[
-C'(\hat{t}_i(s)) + pB'(\hat{q}^A_s(s) - \hat{t}_i(s)) + (1-p)B'(\hat{q}^A_i(s) - \hat{t}_i(s)) = 0 
\]
\[
\lambda_s u'(\hat{q}^A_s(s)) - B'(\hat{q}^A_s(s) - \hat{t}_i(s)) = 0
\]
\[
\lambda_s u'(\hat{q}^A_i(m)) - B'(\hat{q}^A_i(m) - \hat{t}_i(m)) = 0
\]
\[
-C'(\hat{t}_i(m)) + pqB'(\hat{q}^A_s(m) - \hat{t}_i(m)) + (1-pq)B'(\hat{q}^A_i(m) - \hat{t}_i(m)) = 0
\]
\[
\lambda_s u'(\hat{q}^A_i(m)) - B'(\hat{q}^A_i(m) - \hat{t}_i(m)) = 0
\]
\[
(\beta \lambda_s + (1-\beta)\lambda_i) u'(\hat{q}^A_s(m) - \hat{t}_i(m)) = 0
\]
\[
-C'(\hat{t}_i(o)) + B'(\hat{q}^A_s(o) - \hat{t}_i(o)) = 0
\]
\[
\lambda_s u'(\hat{q}^A_i(o)) - B'(\hat{q}^A_s(o) - \hat{t}_i(o)) = 0
\]
\[
(p\lambda_s + (1-p)\lambda_i) u'(\hat{q}^A(o) - \hat{t}_i(o)) = 0
\]

Proof of (A):
Step 1: Let’s show \(\hat{q}^A_s(m) < \hat{q}^A_i(s)\). Assume to the contrary that \(\hat{q}^A_i(s) \leq \hat{q}^A_s(m)\).

\[
\hat{q}^A_s(s) \leq \hat{q}^A_s(m) \Rightarrow (\beta \lambda_s + (1-\beta)\lambda_i) u'(\hat{q}^A_s(m)) < \lambda_s u'(\hat{q}^A_i(s)) \quad (\because \ u \text{ is concave})
\]
\[
\Rightarrow B'(\hat{q}^A_s(m) - \hat{t}_i(m)) < B'(\hat{q}^A_i(s) - \hat{t}_i(s)) \quad (\because \ (11) \text{ and } (14))
\]

---

\(^9\) As it is explained before, the DoT does have to consider the marginal effect of his choice \(t\) on his objective function through the BO’s choice of \(g\) because it is always 0.
\[\Rightarrow \hat{q}_r^b(m) - \tilde{i}_r(m) < \hat{q}_r^b(s) - \tilde{i}_r(s) \quad (\because B \text{ is convex})\]
\[\Rightarrow \tilde{i}_r(s) < \tilde{i}_r(m)\]

By the similar way of using concavity of \(u\), convexity of \(B\), (10) and (13); we can show that \(\tilde{i}_r(s) < \tilde{i}_r(m)\) implies \(\hat{q}_r^b(s) < \hat{q}_r^b(m)\). Lastly,

\[\hat{i}_r(s) < \hat{i}_r(m)\]
\[\Rightarrow \begin{bmatrix} pB'(q_r^s(s) - \tilde{i}_r(s)) \\ + (1 - p)B'(q_r^b(s) - \tilde{i}_r(s)) \end{bmatrix} < \begin{bmatrix} pqB'(q_r^s(m) - \tilde{i}_r(m)) \\ + (1 - pq)B'(q_r^b(m) - \tilde{i}_r(m)) \end{bmatrix} \quad (\because (9) \text{ and } (12))\]

\[\Rightarrow \begin{bmatrix} p\lambda u'(q_r^s(s)) \\ + (1 - p)\lambda u'(q_r^b(s)) \end{bmatrix} < \begin{bmatrix} pq\lambda u'(q_r^s(m)) \\ + (1 - pq)\beta\lambda u'(q_r^b(m)) \end{bmatrix} \quad (\because (10) \text{ and } (11))\]

\[\Rightarrow \begin{bmatrix} p\lambda u'(q_r^s(s)) \\ + (1 - p)\lambda u'(q_r^b(s)) \end{bmatrix} < \begin{bmatrix} pq\lambda u'(q_r^s(m)) \\ + (1 - pq)\lambda u'(q_r^b(m)) \end{bmatrix} \quad (\because u \text{ is concave})\]

\[\Rightarrow \begin{bmatrix} p\lambda u'(q_r^s(s)) \\ + (1 - p)\lambda u'(q_r^b(s)) \end{bmatrix} < \begin{bmatrix} pq\lambda u'(q_r^s(m)) \\ + (1 - pq)(1 - \beta)\lambda u'(q_r^b(m)) \end{bmatrix} \quad (\because \lambda \lambda \beta \lambda \beta \lambda \beta = 0)\]

\[\Rightarrow \hat{q}_r^b(m) < \hat{q}_r^b(s) \quad \text{or} \quad \hat{q}_r^b(m) < \hat{q}_r^b(s) \quad (\because u \text{ is concave})\]

The last inequality is a contradiction because we already showed that \(\hat{q}_r^b(s) < \hat{q}_r^b(m)\) and \(\hat{q}_r^b(s) \leq \hat{q}_r^b(m)\).

Step 2: Let's show \(\hat{q}_r^b(m) - \hat{q}_r^b(m) < \hat{q}_r^b(s) - \hat{q}_r^b(s)\). If \(\hat{q}_r^b(s) \leq \hat{q}_r^b(m)\), (A) is easily driven because of step 1. Hence, assume the other case. That is, \(\hat{q}_r^b(m) < \hat{q}_r^b(s)\). Using concavity of \(u\), convexity of \(B\), (10) and (13); we can show that \(\hat{q}_r^b(m) < \hat{q}_r^b(s)\) implies \(\tilde{i}_r(m) < \tilde{i}_r(s)\) and \(\tilde{i}_r(s) - \tilde{i}_r(s) < \hat{q}_r^b(m) - \hat{q}_r^b(s)\). In addition,

\[\hat{i}_r(s) < \hat{i}_r(m)\]
\[\Rightarrow \begin{bmatrix} pqB'(q_r^s(s) - \tilde{i}_r(s)) \\ + (1 - pq)B'(q_r^b(s) - \tilde{i}_r(s)) \end{bmatrix} < \begin{bmatrix} B'(q_r^s(s) - \tilde{i}_r(s)) \\ + (1 - p)B'(q_r^b(s) - \tilde{i}_r(s)) \end{bmatrix} \quad (\because (9) \text{ and } (12))\]
\[ pB'(\hat{q}^b_i(m)-\hat{i}(m)) \] 
\[ + (1-p)B'(\hat{q}^b_i(m)-\hat{i}(m)) \] 
\[ \Rightarrow \hat{q}^b_i(m)-\hat{i}(m) < \hat{q}^b_i(s)-\hat{i}(s) \quad (\because \hat{q}^b_i(s)-\hat{i}(s) < \hat{q}^b_i(m)-\hat{i}(m) \quad \text{and} \quad B \text{ is convex}) \]

\[ \hat{q}^b_i(s)-\hat{i}(s) < \hat{q}^b_i(m)-\hat{i}(m) \quad \text{and} \quad \hat{q}^b_i(m)-\hat{i}(m) < \hat{q}^b_i(s)-\hat{i}(s) \] 
imply that \( \hat{q}^b_i(m) - \hat{q}^b_i(s) < \hat{q}^b_i(s) - \hat{q}^b_i(m) \).

Proof of (B).

Step 1: Let's show \( \hat{q}^b_i(o) < \hat{q}^b_i(m) \). Assume to the contrary that \( \hat{q}^b_i(m) \leq \hat{q}^b_i(o) \).

\[ \hat{q}^b_i(m) \leq \hat{q}^b_i(o) \]
\[ \Rightarrow (p\lambda_i + (1-p)\lambda_o)u'(\hat{q}^b_i(o)) < (\beta\lambda_i + (1-\beta)\lambda_o)u'(\hat{q}^b_i(m)) \]  
\[ (\because \beta < p \text{ and } u \text{ is concave}) \]
\[ \Rightarrow B'(\hat{q}^b_i(o)-\hat{i}(o)) < B'(\hat{q}^b_i(m)-\hat{i}(m)) \quad (\because (14) \text{ and } (17)) \]
\[ \Rightarrow \hat{q}^b_i(o)-\hat{i}(o) < \hat{q}^b_i(m)-\hat{i}(m) \quad (\because B \text{ is convex}) \]
\[ \Rightarrow \hat{i}(m) < \hat{i}(o) \]

By the similar way of using concavity of \( u \), convexity of \( B \), (13) and (16); we can show that \( \hat{i}(m) < \hat{i}(o) \) implies \( \hat{q}^b_i(m) < \hat{q}^b_i(o) \). Lastly,

\[ \hat{i}(m) < \hat{i}(o) \]
\[ \Rightarrow \]
\[ (p\lambda_i + (1-p)\lambda_o)u'(\hat{q}^b_i(m)) \]
\[ + (1-pq)B'(\hat{q}^b_i(m)-\hat{i}(m)) \] 
\[ \Rightarrow \]
\[ pq\lambda_i u'(\hat{q}^b_i(m)) \]
\[ + (1-pq)\beta\lambda_o u'(\hat{q}^b_i(m)) \]
\[ < (p\lambda_i + (1-p)\lambda_o)u'(\hat{q}^b_i(o)) \] 
\[ \Rightarrow \]
\[ pq\lambda_i u'(\hat{q}^b_i(m)) \]
\[ + (1-pq)(1-\beta)\lambda_o u'(\hat{q}^b_i(m)) \] 
\[ < (p\lambda_i + (1-p)\lambda_o)u'(\hat{q}^b_i(o)) \] 
\[ \Rightarrow \]
\[ pq\lambda_i u'(\hat{q}^b_i(m)) \]
\[ + (1-pq)(1-\beta)\lambda_o u'(\hat{q}^b_i(m)) \] 
\[ < (p\lambda_i + (1-p)\lambda_o)u'(\hat{q}^b_i(o)) \] 
\[ (\because u \text{ is concave}) \]
Sangwon Park: Separation of Two Agencies for Fiscal Policies

\[ \Rightarrow \left[ \begin{array}{c} p\lambda_u(q^*_h(m)) \\ +(1-pq)(1-\beta)\lambda_u(q^*_s(m)) \end{array} \right] < (p\lambda_s + (1-p)\lambda_s)u(q^*_s(o)) \]

(∵ \( pq + (1-pq)\beta = p \))

\[ \Rightarrow \left[ \begin{array}{c} p\lambda_u(q^*_h(m)) \\ +(1-p)\lambda_u(q^*_s(m)) \end{array} \right] < (p\lambda_s + (1-p)\lambda_s)u(q^*_s(o)) \]

(∵ \( (1-pq)(1-\beta)=1-p \))

\[ \Rightarrow q^*_s(o) < q^*_s(m) \quad (\because \ u \ is \ concave) \]

The last inequality is a contradiction to the assumption of \( q^*_s(m) \leq q^*_s(o) \).

Step 2: Let’s show \( q^*_s(o)-q^*_s(o) < q^*_s(m)-q^*_s(o) \). If \( q^*_s(m) \leq q^*_s(o) \), (B) is easily driven because of step 1. Hence, assume the other case that \( q^*_s(o) < q^*_s(m) \). Using concavity of \( u \), convexity of \( B \), (13) and (16), we can show that \( q^*_s(o)-q^*_s(o) < q^*_s(m)-q^*_s(o) \). In addition, using concavity of \( u \), convexity of \( B \), (14) and (17), we can also show that \( q^*_s(o)-q^*_s(o) < q^*_s(m)-q^*_s(o) \) and \( q^*_s(o)-q^*_s(o) < q^*_s(m)-q^*_s(o) \) leads us to \( q^*_s(o)-q^*_s(o) < q^*_s(m)-q^*_s(m) \).

Proof of Proposition 2:

As it is explained previously, \( g^*_s(p,1,0), g^*_s(p,1,0) \) and \( \hat{i}_s(p,1,0) \) are the BO’s and the DoT’s best response to \( R(\lambda_t) = \lambda_t \) and \( R(\lambda_s) = \lambda_s \). In addition, \( \hat{g}^*_s(pq,1,\frac{p(1-q)}{1-p+q\beta-\beta^p}) \), \( \hat{g}^*_s(pq,1,\frac{p(1-q)}{1-p+q\beta-\beta^p}) \) and \( \hat{i}_s(pq,1,\frac{p(1-q)}{1-p+q\beta-\beta^p}) \) are the BO’s and DoT’s best response to the SM’s mixed strategy of playing \( R(\lambda_t) = \lambda_t \) with \( q \) probability, \( R(\lambda_s) = \lambda_s \) with \( 1-q \) probability and \( R(\lambda_t) = \lambda_s \). Lastly, \( \hat{g}^*_s(0,1,p), \hat{g}^*_s(0,1,p) \) and \( \hat{i}_s(0,1,p) \) are the BO’s and DoT’s best response to \( R(\lambda_t) = R(\lambda_s) = \lambda_t \) as long as they believe it is \( \lambda_t \) type, observing an out-of-equilibrium report of \( \lambda_t \). Since it is straightforward that \( R^*(\lambda_s) = \lambda_s \), we need to check up only the optimality of the \( \lambda_t \) type’s action.

(A) Assume \( K^*_t \leq p\alpha \) and consider the separating equilibrium. Given the BO’s and DoT’s strategy of \( g^*_s(p,1,0), g^*_s(p,1,0) \) and \( \hat{i}_s(p,1,0) \), if the SM with \( \lambda_t \) reports \( \lambda_t \), his/her payoff becomes \( g^*_s(p,1,0) \). If \( \lambda_s \) type deviates by choosing \( \lambda_s \) instead, then his/her payoff becomes \( g^*_s(p,1,0)-\pi\alpha \). Therefore, \( K^*_t \leq p\alpha \) guarantees that \( R(\lambda_t) = \lambda_t \) and \( R(\lambda_s) = \lambda_s \) constitute the equilibrium.

Consider now any possibility of pooling equilibrium. The difference between \( g \) obtained by reporting \( \lambda_s \) and one from reporting \( \lambda_t \) becomes largest when the out-of-equilibrium report of \( \lambda_t \) makes the BO and the DoT believe that it is \( \lambda_t \) type. Hence, if \( K^*_t \leq p\alpha \), then the pooling equilibrium can not exist because, by Lemma 1, \( K^*_t = K^*_s \) holds. Lastly, the possibility of any mixed strategy should be considered. \( g^*_s(pq,1,\frac{p(1-q)}{1-p+q\beta-\beta^p}) \) and \( \hat{g}^*_s(pq,1,\frac{p(1-q)}{1-p+q\beta-\beta^p}) \) are \( g \) the SM expects from reporting \( \lambda_s \) and \( \lambda_t \), respectively. Hence, \( g^*_s(pq,1,\frac{p(1-q)}{1-p+q\beta-\beta^p})-\hat{g}^*_s(pq,1,\frac{p(1-q)}{1-p+q\beta-\beta^p}) < K^*_t \leq p\alpha \) implies that no mixed equilibrium exists.
(B) Assume \( K^2 < \pi \alpha < K' \). \( \hat{g}^s(p,1,0) - \pi \alpha > \hat{g}^l(p,1,0) \) implies that separating equilibrium can not exist because the low type is willing to deviate by reporting \( \hat{\lambda}_l \). Moreover, \( \hat{g}^s(0,1,p) - \pi \alpha < \hat{g}^l(0,1,p) \) implies that the pooling equilibrium can not exit either because the low type is better off by playing truth-telling. Hence, the only possibility is being a mixed equilibrium of \((\hat{\lambda}_l, \hat{\lambda}_s) = (\hat{\lambda}_l, \hat{\lambda}_s) \) with probability \( q \). \( \hat{g}^s(pq,1, \frac{p(1-q)}{p(1-q)+q}) \) and \( \hat{g}^l(pq,1, \frac{p(1-q)}{p(1-q)+q}) \) are the BO’s and DoT’s best response to this SM’s mixed strategy. In addition, by the definition of \( q \), the low type is indifference between the two reports, that is, \( \hat{g}^s(pq,1, \frac{p(1-q)}{p(1-q)+q}) - \pi \alpha = \hat{g}^l(pq,1, \frac{p(1-q)}{p(1-q)+q}) \).

(C) Now assume \( \pi \alpha \leq K^2 \). It implies that \( \hat{g}^l(0,1,p) < \hat{g}^s(0,1,p) - \pi \alpha \) and \( \hat{g}^l(pq,1, \frac{p(1-q)}{p(1-q)+q}) < \hat{g}^s(pq,1, \frac{p(1-q)}{p(1-q)+q}) - \pi \alpha \). Hence, neither the separating nor the mixed equilibrium exists because the low type has the incentive to choose deviation of reporting \( \hat{\lambda}_l \).

Proof of Proposition 3-(A): By Lemma 2, we know that \( \hat{g}^s(\lambda_l) - \hat{g}^m(\lambda_l) \leq \pi \alpha \) implies \( \hat{g}^s(p,1,0) - \hat{g}^l(p,1,0) < \pi \alpha \). Hence, only the separating equilibrium exists in both systems (see Proposition 1 and 2). Given \( R^s_m(\lambda_l) = \hat{\lambda}_l \) and \( R^m_m(\lambda_l) = \hat{\lambda}_s \), the DoF can obtain at least \( W^* \) by mimicking the equilibrium strategy of the separated system. That is, the DoF can always choose \( t^*_m(\lambda_l) = t^*_m(\lambda_s) \) and \( g^* = \hat{g}^s(p,1,0) \). Thus, we can say \( W^* \leq W^m \). In addition, \( t^*_m(\lambda_l) \neq t^*_m(\lambda_s) \) and the uniqueness of the DoF’s best response to \( R^s_m(\lambda_l) = \hat{\lambda}_l \) and \( R^m_m(\lambda_s) = \hat{\lambda}_s \) imply that \( W^*_m \) is strictly bigger than \( W^* \).

Proof of Proposition 3-(B): By Lemma 2, we know that \( \pi \alpha \leq \hat{g}^s(0,1,p) - \hat{g}^l(0,1,p) \) implies \( \pi \alpha < \hat{g}^s(p,\lambda_l + (1-p)\lambda_l) - \hat{g}^m(\lambda_l) \). Then, the pooling equilibrium only exists in both systems (see Proposition 1 and 2). Therefore, the SM’s report always is \( \hat{\lambda}_s \) along the equilibrium of both systems. In addition, DoF’s potential ignorance in the separated system does not make any difference. Hence, \( W^*_m = W^m \) holds.

Proof of Proposition 3-(C): Let \( \hat{g}^s(0,1,p) - \hat{g}^l(0,1,p) < \pi \alpha \leq \hat{g}^m(p,\lambda_l + (1-p)\lambda_l) - \hat{g}^m(\lambda_l) \). According to Lemma 2, \( \hat{g}^s(0,1,p) - \hat{g}^l(0,1,p) < \hat{g}^m(p,\lambda_l + (1-p)\lambda_l) - \hat{g}^m(\lambda_l) \).
\( \hat{g}_m(\lambda) \). Proposition 1 states that the pooling equilibrium only exists in the integrated system. On the other hand, either separating or mixed equilibrium will prevail in the separated system. If the DoT had chosen \( \hat{i}_m(p\lambda + (1-p)\lambda) \) in the separated system, the BO's payoff would be strictly bigger than the DoF's payoff in the integrated system. It is because the BO also were able to mimic the DoF by choosing \( \hat{g}_m(p\lambda + (1-p)\lambda) \) and he/she did not. Because the SM's report is either separating or mixed along the equilibrium of the separated system, \( \hat{g}_m^*(\lambda, i_m(p\lambda + (1-p)\lambda)) \neq g_m^*(\lambda) = \hat{g}_m^*(p\lambda + (1-p)\lambda) \). Since the DoT's objective function is the same as the BO's, the DoT's payoff along the equilibrium of the separated system is also strictly larger than that of DoF in the integrated system. Therefore, we can conclude that \( W_m^* < W_f^* \).