THE KOREAN ECONOMIC REVIEW
Volume 26, Number 2, Winter 2010

LIQUIDITY AS PRICE EFFECT ON TIME TO SALE

KEUNKWAN RYU* · HYUN-YEOL SHIN**

This paper proposes a new empirical measure of liquidity, termed "liquidity delta." An asset is considered liquid if it can be traded quickly, in large quantities at low cost with little impact on market price. Trade-off between asking price and sale intensity, is one of the most common characteristics of assets. The new measure, liquidity delta, empirically captures this trade-off. We estimate liquidity delta for sixty major stocks listed on the Korea Stock Exchange. We demonstrate that liquidity delta is a useful measure of liquidity, with liquidity level and its variability showing negative and positive relation, respectively, with the asset's rate of return. The negative relationship shows premium for lack of liquidity whereas the positive one shows premium for liquidity risk.

JEL Classification: C41, G11, G12, G19
Keywords: Liquidity, Excess asking price, Duration analysis, Hazard ratio, Liquidity delta

I. INTRODUCTION

Assets have different degree of liquidity. An asset is considered liquid if it can be traded quickly, in large quantities at low cost with little impact on the market price (Keynes 1936, Glosten and Harris 1988, Pastor and Stambaugh 2003). For all its familiarity as a concept, liquidity is not easy

Received: April 8, 2010. Revised: Aug. 30, 2010. Accepted: Oct. 10, 2010.

* Corresponding Author: Professor, Department of Economics, Seoul National University, E-mail: ryu@snu.ac.kr Tel: 82-2-880-6397, Fax: 82-2-886-4231.

** Senior Economist, Research Department, The Bank of Korea, E-mail: hyshin68@bok.or.kr Tel: 82-2-759-4435, Fax: 82-2-750-6580.

to measure. There is no absolute standard of liquidity (Keynes 1936). The existence of various proxy measures points to the complexity in measuring liquidity.[1]

Bid-ask spread is considered compensation for specialists' liquidity provision in quote driven markets,[2] rendering bid-ask spread a proxy for liquidity. Since Amihud and Mendelson (1986) have adopted bid-ask spread as a measure of liquidity in estimating the relationship between the rate of return and liquidity, a number of papers attempt to develop various liquidity indexes based on bid-ask spread. Among them are Eleswarapu and Reinganium (1993), Brennan and Subrahmanyam (1996), Chalmers and Kadlec (1998), Chordia *et al*. (2000), and Marshal and Young (2003).

Grossman and Miller (1988) mention that bid-ask spread is merely a charge by the market makers for executing their orders, rather than a measure of cost in supplying immediacy to the market. Alternative liquidity measures emphasize "the ability to trade large quantities in a short period of time without significant changes in price." Haugen and Baker (1996), Datar *et al*. (1998), and Chordia *et al*. (2001a) use turnover rate as a liquidity index.

Cooper *et al*. (1985) measure liquidity using the dollar volume of trading necessary to result in a 1% price change. Amihud (2002) and Acharya and Pedersen (2003) use the ratio of daily traded dollar volume to the absolute price change as a proxy for daily liquidity.

Hasbrouck and Schwartz (1988) and George *et al*. (1991) suggest the ratio of short term to long term variances of stock price volatility as a proxy for liquidity. This measure is based on Roll's (1984) observation that spread posed by specialists results in negative serial correlation in short term price movements, suggesting that a high value of short term to long term variance ratio imply lack of liquidity.

Pastor and Stambaugh (2003) propose a liquidity measure based on the degree of temporary price variation stemming from order flows. Gourieroux *et al.* (1999) measure liquidity using the interval between

[1] Markower and Marschak (1938) claimed that liquidity is a bundle of measurable properties.

[2] Bid-ask spread is believed to be determined by the profit maximizing behavior of dealers, and to be comprised of four economic factors: order processing cost, inventory holding cost, specialists' monopolistic rent, and, most importantly, adverse selection cost caused by asymmetric information between dealers and privately informed traders. See Glosten and Harris (1988), Brennan and Subrahmanyam (1996).

independent consecutive trades and also the interval between order placement and execution. Lippman and McCall (1986) define liquidity in terms of time until an asset is exchanged for money. They present time-to-sell as an operational measure of an asset's liquidity. Beside these, there are several other liquidity measures.[3]

In general, time-to-sell decreases (liquidity increases) as bids are more frequent, as there are less impediments to transfer of legal title, as the cost of holding an asset decreases, and as the asking price falls. In the stock markets, however, asking and bidding prices are the most important determinant of a stock's liquidity. This is because ownership transfer is just a matter of "one click" and the transaction charge is minimal due to well established institutional trading mechanism.

In this paper, we measure liquidity as the "time-to-sell reduction effect of lowering the asking price." Our measure builds on Lippman and McCall (1986). Our liquidity measure captures the trade-off between asking price and (order) execution hazard rate.

Rest of the paper is organized as follows. In chapter II, we present an empirical framework for measuring an asset's liquidity. In chapter III, we describe the data to be used for liquidity estimation. In chapter IV, we present the estimation results including the results of robustness check from alternative model specification, and document correlations among different liquidity measures, correlations between rates of return and alternative measures of liquidity. Chapter V concludes the paper.

II. MEASUREMENT OF LIQUIDITY

1. Intuition

Liquidity of an asset is sometimes defined as the value which can be secured as soon as it is put up for sale. This definition acknowledges the trade-off between asking price and speed of sale execution. One must lower the price of an asset to get it sold immediately upon sale announcement. This definition is neither complete in the sense that it cannot determine liquidity of those assets which cannot be sold

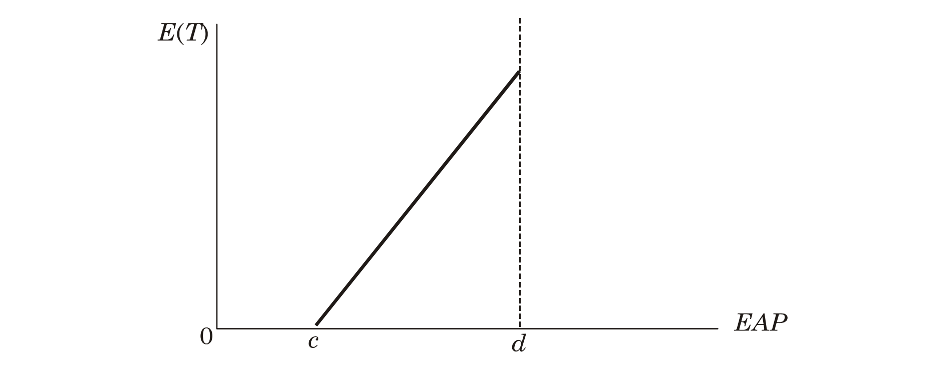
[3] See Bayer, Borell, and Moslener (2005).

immediately upon announcement, nor satisfactory for assets which can be sold at a reasonable value within a reasonable time horizon but only at a huge discount if sold immediately.

To construct a more complete and satisfactory liquidity measure, we would like to explicitly consider the trade-off between asking price and time till sale execution. Cash is of course perfectly liquid: without lowering its asking price below its nominal level at all, cash can be exchanged immediately for cash at that nominal price (tautological). When an asset is less than perfectly liquid, it does not make sense to talk about its price without reference to its time till sale execution. The lower the asking price of an asset, the faster it will be sold, resulting in an equilibrium trade-off between the asking price and the expected time till sale.

Let EAP be the ratio of excess asking price to the market price, i.e., $EAP = (P_0^A - P_0)/P_0$, where P_0^A is the asking price and P_0 is the market price at the time of sale announcement.[4] Let $T\,(\geq 0)$ denote the time until an asset is exchanged for cash (hereafter we also call it 'duration').

[Figure 1] Relationship between EAP and $E(T)$



Note: If the seller's EAP is between c and d, then the time until sale execution is stochastically increasing in the excess asking price, resulting in a positively sloped equilibrium relationship between EAP and $E(T)$.

[4] In this paper, we measure the market price as a simple average of the highest bid price and the lowest ask price just before the time of sale announcement. In other situations where the market price is not observable, one may use a "hedonic" regression function to estimate it. In used car markets, one may also substitute "blue book" price for the market price.

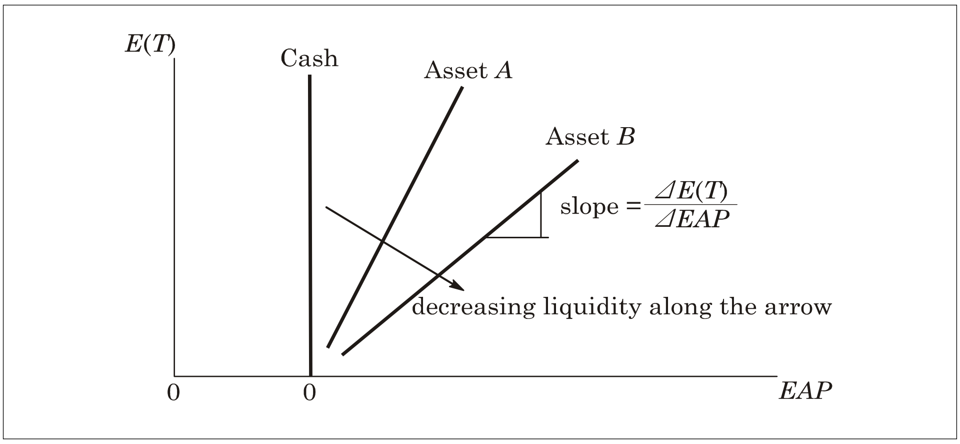
Figure 1 shows the equilibrium relationship between EAP, the excess asking price, and $E(T)$, the expected time till sale execution:

If the seller's EAP is lower than c, buyers will take the seller's offer immediately, resulting in $E(T) = 0$. If the seller's EAP is higher than d, no buyer will take the offer, yielding $E(T) = \text{infinity}$. Finally, if the EAP is between c and d, then the time until sale execution is stochastically increasing in the excess asking price, resulting in a positively sloped equilibrium relationship between EAP and $E(T)$ for $c < EAP < d$.

Different assets have different equilibrium relationships. Consider cash again. At any asking price higher than its nominal value, cash would never be accepted by anybody. In contrast, at any asking price lower than the nominal value, cash would be taken immediately by anybody. Thus, the equilibrium relationship between EAP and $E(T)$ in the case of cash can be represented by a vertical line at $EAP = 0$. In general, the lower the liquidity of an asset, the flatter the slope of the equilibrium relationship.

In Figure 2, the liquidity of asset A is higher than that of asset B. The expected duration of asset A's sale execution responds more sensitively than that of asset B to unit change in EAP.

[Figure 2] Asset specific relationship between EAP and $E(T)$



Note: The liquidity of asset A is higher than that of asset B. The expected duration of asset A's sale execution responds more sensitively than that of asset B to unit change in EAP. Cash enjoys the maximum level of liquidity.

In this paper, we propose to measure the degree of liquidity (or simply liquidity) of an asset by utilizing the slope of the corresponding equilibrium relationship. With an electronic order processing system, although other factors such as ownership transfer procedure and share holding costs could affect expected duration, the effect of *EAP* on duration is most important.[5]

We would like to compare our liquidity measure with that of Lippman and McCall (1986; hereinafter denoted LM). First, our liquidity measure looks at 'slope' whereas LM's measure just looks at expected time to sale execution under "reasonable price" or "optimized price." Second, concerning the relationship between interest rate and liquidity, LM's liquidity measure increases as the market interest rate increases. It is simply because asset holders would like to get rid of the asset as soon as possible when market interest rates are high. Our liquidity measure might decrease as market interest rate increases, since potential buyers become choosy as market interest rates go up. Third, liquidity offers flexibility in asset choice and thus it provides a kind of option value. This flexibility allows holders of a liquid asset to act quickly as new investment opportunities arise. Our measure is more suited to this notion of flexibility than LM's.[6] Fourth, LM's liquidity measure depends on asset holder's subjective optimization. It depends on asset holder's preference, private value, search cost, opportunity cost of money, etc. On the contrary, our measure is free from asset holder's subjective optimization. Fifth, "quick sale" and "reasonable price or fair market price" are not compatible concepts in LM's liquidity measure. Our measure, however, explicitly captures the relationship between asking price and time to sale execution with greater applicability.

[5] Our liquidity measure is closely related to that of Hirshleifer (1972) and that of Markower and Marschak (1938). Hirshleifer says that "Illiquid assets... are those characterized by a relatively large discount for 'premature' realization" (p. 137). Markower and Marschak describe saleability as "the relationship between the selling price and the time which the seller must wait in order to get it." (p. 280). Thus, for them, 'saleability' is a measure of market imperfection and a measure of liquidity.

[6] Hicks (1974) argues that "by holding the imperfectly liquid asset the holder has narrowed the band of opportunities which may be open to him."

2. Model

Liquidity i.e., the degree of responsiveness of $E(T)$ to changes in *EAP*, can be explicitly measured through the effect of *EAP* on the sale execution hazard rate. The hazard rate at any moment in time denotes the conditional intensity of sale execution within a short time interval, conditional on that sale has not occurred yet. If the execution hazard rate is high, "duration" or "time to sale" becomes short in a stochastic sense, and vice versa.[7]

Let $h(t|x)$ denote the conditional hazard rate at time t given a set of covariates, say x. The hazard rate can be represented as

$$h(t|x) = \frac{f(t|x)}{S(t|x)} \tag{1}$$

where $f(t|x)$ is the density function and $S(t|x)$ is the survival function. From $h(t|x) = -d\ln S(t|x)/dt$, we derive

$$S(t|x) = \exp\left[-\int_0^t h(\tau|x)d\tau\right]. \tag{2}$$

Using equations (1) and (2), the likelihood value of observing a completed duration at t, given x, is

$$f(t|x) = h(t|x)\exp\left[-\int_0^t h(\tau|x)d\tau\right]. \tag{3}$$

Meanwhile, the likelihood value of observing a right censored duration at t, given x, is $S(t|x)$. An example of right-censored duration data is sell limit orders which are either cancelled or left unsold by time of market close.

[7] Cho and Nelling (2000) also used duration analysis to identify the trade-off between a price gap (which is a gap between the limit order price and the market price) and the sale/purchase transaction hazard rate. They analyzed both sell and buy limit orders, while we only focus on sell limit orders. We are interested in identifying the trade-off between the excess asking price and the sale transaction hazard rate and thus in suggesting a new liquidity measure by estimating that trade-off.

Let $\{t_1, \cdots, t_n; t_1^*, \cdots, t_m^*\}$ denote a collection of observed durations, where durations without * denote complete spells and those with * denote right censored spells. Let $f(t_i \mid x_i)$ and $S(t_j^* \mid x_j^*)$ be the conditional density function and the conditional survival function given covariate values. Assuming independence across observations, the log likelihood function is

$$\ln L = \sum_{i=1}^{n} \ln f(t_i \mid x_i) + \sum_{j=1}^{m} \ln S(t_j^* \mid x_j^*) . \tag{4}$$

To complete the model, let us specify the hazard rate function as follows:

$$h(t \mid x) = h_0(t) \exp(x'\delta) . \tag{5}$$

This is a proportional hazard model well known in the literature (Cox and Oakes, 1984). A set of covariates, x, shifts the so called baseline hazard function $h_0(t)$ up or down proportionally. Of course, δ is a vector of unknown parameters.[8] The baseline hazard function $h_0(t)$ captures the overall duration dependency of the hazard rate function.[9]

EAP is one of the covariates. It is expected that the larger the *EAP* is, the lower the chance of executing a sell limit order. We expect the coefficient of *EAP* to be negative. The higher the absolute value of the coefficient, the more sensitive the hazard rate is to a unit change in *EAP*, denoting a higher degree of liquidity. The absolute value of the coefficient of *EAP* is positively related with the slope of the relationship depicted in Figure 1. Hereinafter, we propose the absolute value of the coefficient estimate of *EAP* as our liquidity measure.[10]

---

[8] δ_i measures semi-elasticity of the hazard rate with respect to x_i. If x_i itself is in log transformation, say $x_i = \log(z_i)$, then δ_i is the elasticity of the hazard rate with respect to z_i.

[9] When the hazard rate function is not a constant, we say that the duration process exhibits duration dependence. With positive (negative) duration dependence, the intensity of sale execution increases (decreases) with the elapsed time since sale posting.

[10] As will be shown later in this paper, the coefficient estimate of *EAP* turns out negative for each (asset, trading day) combination. As a result, we can use its absolute value as a liquidity measure without confusing its sign.

3. Covariates and the Baseline Hazard Function

The excess asking price, *EAP*, observed at the time of sale posting is the most important covariate in our model. Other covariates include: offered wealth ($Wealth =$ volume×asking price), the bid-ask spread ($Spread$), and the outstanding value of the highest bid ($HBid$).

These other covariates are also expected to affect the sale transaction hazard rate. The bigger the wealth of the sell order is, the longer it takes for the entire order to be absorbed in the market. The coefficient of $Wealth$ is thus expected to be negative. The coefficient of the $Spread$, which itself is a measure of liquidity, is expected to be negative. The outstanding value of the highest bid reflects demand for immediacy or buying pressure. So we expect the coefficient of $HBid$ to be positive. Note that we measure all covariates at the time of sale announcement, resulting in time-constant covariate paths:[11]

$$EAP_s \equiv \frac{P_s^A - P_s}{P_s} \times 100 \qquad Wealth_s \equiv \ln(P_s^A \times Q_s^A)$$

$$Spread_s \equiv \frac{P_{iow,s}^a - P_{high,s}^b}{P_s} \times 100 \qquad HBid_s \equiv \ln(Q_{high,s}^b \times P_{high,s}^b)$$

where, s : time of sale announcement

P^A : asking price, Q^A : offered volume

P_{low}^a : lowest asking price, P_{high}^b : highest bid price

Q_{high}^b : outstanding volume of highest bid

P : market price defined as $\dfrac{P_{high}^b + P_{low}^a}{2}$.

We regard as right-censored data those uncompleted sell orders such as orders cancelled before execution and orders not executed by time of market close.

[11] In fact, most covariates are conceptually time varying with the values changing over time. For example, *EAP* changes as the market price changes over time. In this paper, for convenience, we approximate the time varying covariate path as a constant path with its value fixed at the time of sale announcement. We do not think generalization to time varying covariates would change the main results of this paper.

Regarding the baseline hazard function, $h_0(t)$, we do not have any priori restriction on its pattern. To save on the number of parameters to estimate, we choose a step function with four steps: the first step up to one minute, the second step between one and five minutes, the third step between five and ten minutes, and the final step longer than ten minutes. We assume that the baseline hazard function is constant within each step but varies across steps. We define three dummy variables D_1, D_2, and D_3 such that $D_1(t)=1$ if t falls in the first interval (0,1], and zero otherwise; $D_2(t)=1$ if t falls in the second interval (1,5], and zero otherwise; and finally $D_3(t)=1$ if t falls in the third interval (5,10], and zero otherwise. The baseline hazard function can be represented as follows:

$$\begin{aligned} h_0(t) &= \exp[\gamma_0 + \gamma_1 D_1(t) + \gamma_2 D_2(t) + \gamma_3 D_3(t)] \\ &= \exp[D(t)'\gamma], \end{aligned} \tag{6}$$

where $D(t)$ denotes a vector comprised of a constant and three dummy variables, and γ a vector comprised of all four coefficients.

Using the covariates and the baseline hazard specification, equation (5) can be written explicitly as

$$h(t\,|\,x) = \exp[D(t)'\gamma + x'\delta]. \tag{7}$$

where $x'\delta = \delta_0 \cdot EAP_s + \delta_1 \cdot Wealth_s + \delta_2 \cdot Spread_s + \delta_3 \cdot HBid_s$.

The coefficient δ_0 indicates responsiveness of the execution hazard rate in response to unit change in *EAP*. We measure the liquidity of an asset through the absolute value of δ_0, and estimate it using econometric duration techniques.

The log survival function can be written as follows:

$$\ln S(t\,|\,x) = -\exp(x'\delta)\int_0^t \exp[D(w)'\gamma]dw. \tag{8}$$

Since the baseline hazard function is specified as a step function, carrying out the integration appearing in equation (8) is straightforward.

The dependent variable t (duration) is defined as the elapsed time from the point of order (s) till the point of its execution (s_k):

$$t = s_k - s\,. \tag{9}$$

When a sale order is completed not all at once but in several stages, we define the sale duration as the volume weighted average of the component durations:

$$t = \sum_{j=1}^{m} (s_j - s) \frac{Q_j^A}{Q^A} \tag{10}$$

where m is the number of stages and the s_j are those time points when the volumes Q_j^A are traded, with $Q^A = \sum_{j=1}^{m} Q_j^A$ and $s < s_1 < \ldots < s_m$.

III. DATA AND PRELIMINARY ANALYSIS

1. Data

Data used in this paper consist of high-frequency order-level transaction information (intra-day, so called tick data) for sixty stocks listed on the Korea Stock Exchange (KSE) during the period between January 1, 2003 and December 30, 2003. These data are extracted from the IFB/KSE database, which is maintained by the Institute of Securities and Banking at Seoul National University. These sixty sample firms are 1st through 60th firms in terms of market capitalization in KSE. Financial statements for these firms are readily available as public information.

Before launching our duration analyses, we first would like to examine the statistical features of the durations as well as the excess asking prices, which are the two most important variables in this paper.

2. Excess Asking Price

Table 1 shows the distribution of excess asking price (*EAP*) for daily sell limit orders of six major stocks. The daily average number of sell limit orders varies across stocks, ranging from a high of 4,971 for Samsung Electronics to a low of 1,773 for POSCO. However, the *EAP* distribution looks similar across stocks, with about two thirds of the sell limit orders having *EAP* less than 0.5%, and about one fifth having *EAP* between 0.5% and 2.0%. Sell limit orders of which the *EAP* exceeds 10%, indicating that the sellers do not have active intention to sell, come out to be less than 2%.

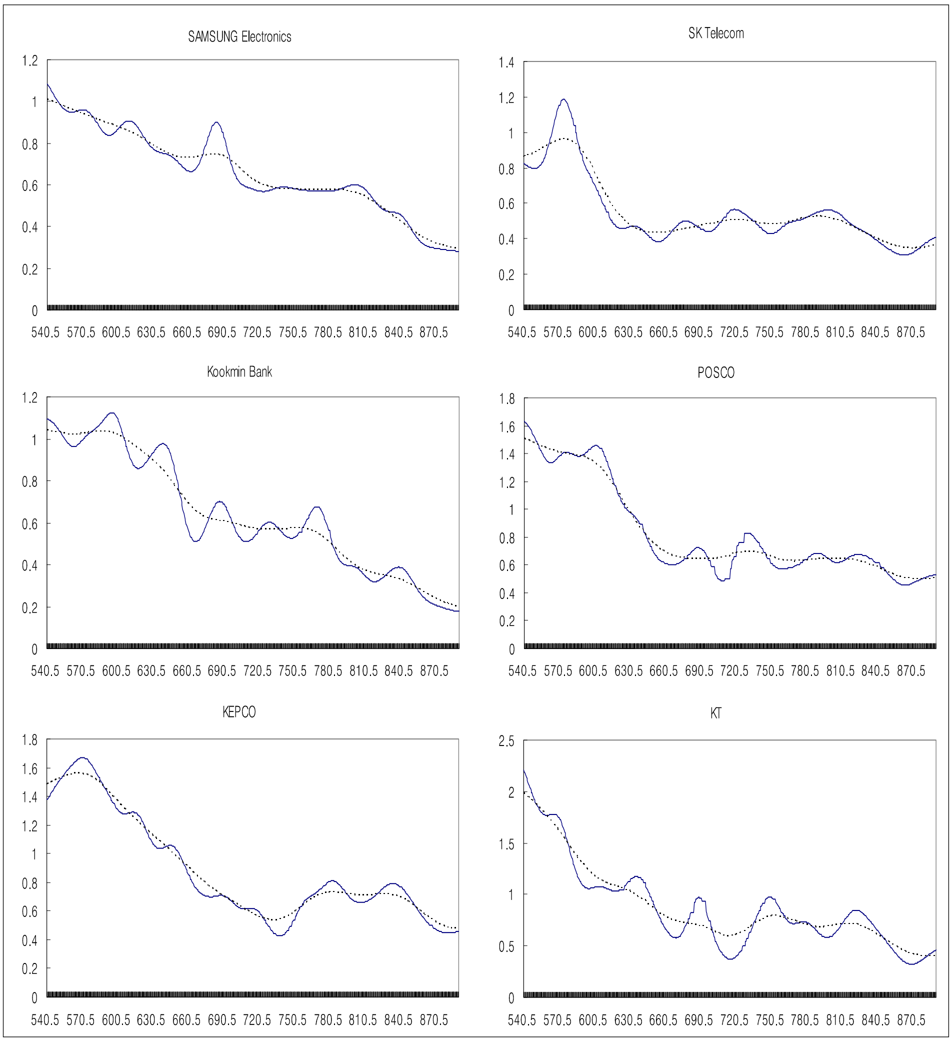
[Table 1] *EAP* Distribution of Sell Limit Orders for Six Major Stocks

EAP range	Average[1]	Samsung Electro.	SK Telecom	Kookmin Bank	POSCO	KEPCO	KT
EAP < 0.5%	No. of orders (S.D.)	3,289 (855)	1,311 (450)	3,196 (1,105)	1,141 (311)	1,354 (411)	1,573 (505)
	Composition	66.2	63.1	66.8	64.3	58.4	65.9
0.5%≤ *EAP* < 2.0%	No. of orders (S.D.)	1,099 (509)	420 (184)	917 (517)	337 (143)	577 (216)	489 (218)
	Composition	22.1	20.2	19.2	19.0	24.9	20.5
2.0%≤ *EAP* < 4.0%	No. of orders (S.D.)	365 (155)	196 (88)	365 (158)	176 (66)	232 (72)	180 (57)
	Composition	7.3	9.4	7.6	9.9	10.0	7.5
4.0%≤ *EAP* < 6.0%	No. of orders (S.D.)	104 (49)	68 (35)	139 (57)	63 (30)	71 (28)	59 (23)
	Composition	2.1	3.3	2.9	3.6	3.0	2.5
6.0%≤ *EAP* < 8.0%	No. of orders (S.D.)	43 (24)	31 (19)	66 (37)	20 (11)	28 (14)	26 (13)
	Composition	0.9	1.5	1.4	1.1	1.2	1.1
8.0%≤ *EAP* < 10.0%	No. of orders (S.D.)	24 (15)	18 (12)	37 (21)	13 (11)	17 (10)	17 (11)
	Composition	0.5	0.8	0.8	0.7	0.7	0.7
10.0%≤ *EAP*	No. of orders (S.D.)	47 (41)	35 (32)	65 (44)	23 (26)	41 (38)	45 (37)
	Composition	0.9	1.7	1.4	1.3	1.8	1.9
Total	No. of orders	4,971	2,079	4,785	1,773	2,320	2,389
	Composition	100.0	100.0	100.0	100.0	100.0	100.0

Note: 1) "No. of orders" is daily average number of sell limit orders over 247 trading days in year 2003. Figures in parenthesis denote standard deviation over those 247 trading days.

The six charts in Figure 3 display the intra day movements of *EAP* for each of 6 major stocks on January 6, 2003. These smoothed paths are estimated by the Nadaraya-Watson nonparametric kernel regression method. We observe that the average *EAP* never exceeds 2% for any stock. What is noteworthy is that with the lapse of time, *EAP* is clearly decreasing for each stock, consistent with decreasing duration over time. This phenomenon may be due to sellers' strong motivation to increase the chance of sale execution as the market approaches to closing.

[Figure 3] Intra Day Movement of *EAP*

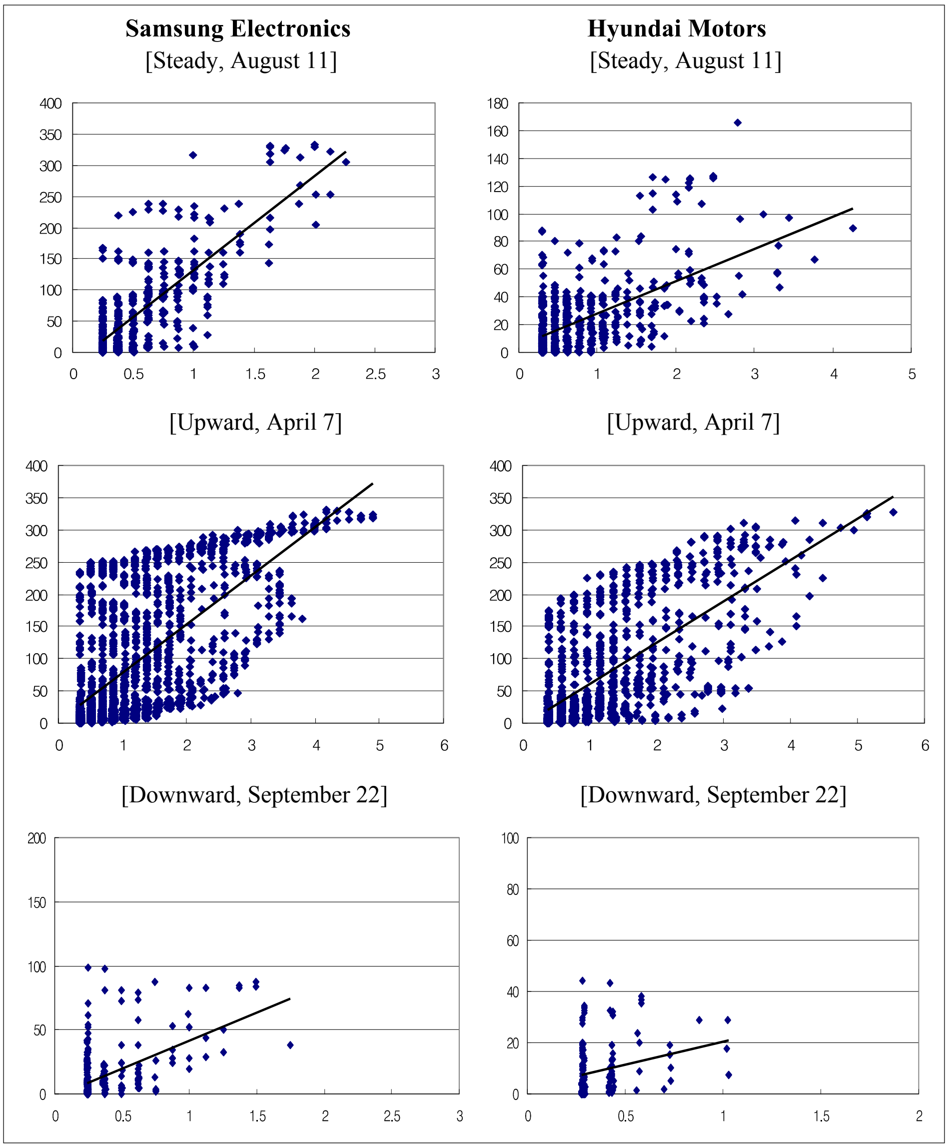


Note: The x-axis denotes trading time in minutes from market opening at 9:00 a.m. (=540th minute of a day) to market closing at 2:50 p.m. (=890th minute of a day), and the y-axis denotes the excess asking price measured in ratio of the market price (%). The solid and dotted lines are the estimated paths when the smoothing parameter (h) takes a value of ten minutes and twenty minutes, respectively.

3. Relationship between EAP and Duration

Earlier, in Figure 1, we argued that an equilibrium relation between the pair $(EAP,\ E(T))$ would be positive. In order to verify existence of that relationship we examine intra day relationship between EAP and durations for Samsung Electronics and Hyundai Motors, the two key

[Figure 4] Relationship between *EAP* and Duration



Note: The x-axis and y-axis depict *EAP* (in %) and duration (in minutes), respectively.

stocks on the Korea Stock Exchange. We select three different trading days - steady, moved upward, and moved downward - to understand the differences in intra day relationship between *EAP* and duration under different market conditions.[12]

Figure 4 confirms the assertion. We observe that on the day of steady or upward movements, the positive slope is clearly noticeable, whereas on the day of downward movement the slope is less marked. The slope seems to increase as market conditions improve, which is consistent with the conventional wisdom that liquidity increases with market conditions.

IV. ESTIMATION RESULTS

In this chapter, we report and interpret the estimation results of the duration model introduced in Chapter II.

1. Parameters

The expected sign of each coefficient is as follows:

$$\underset{(-)}{\delta_0 \cdot EAP} + \underset{(-)}{\delta_1 \cdot Wealth} + \underset{(-)}{\delta_2 \cdot Spread} + \underset{(+)}{\delta_3 \cdot HBid} \, .$$

The estimation results are summarized in Table 2 for each of ten major stocks.[13] The results for the remaining 50 stocks are fairly similar as reported in Appendix 1.[14]

[12] Daily rate of returns of KOSPI on August 11(day of steady movement), April 7(day of upward movement) and September 22(day of downward movement) are +0.06%, +5.00%, and -4.46% respectively. Daily rate of returns are calculated with reference to closing price of the previous trading day.

[13] Note that we use right-censored as well as complete data for estimating the model. Between the two types of right-censored data, one may think that right-censored data due to order cancellation would be different in nature from the ones due to market closing. To account for this, we also estimated the model excluding the right censored data due to order cancellation and obtained the basically the same results. The results are available from the authors upon request.

[14] Since the focus of this duration analysis lies on understanding the responsiveness of duration to changes in each covariate, the estimation results for the baseline hazard rate $h_0(t)$, that is, the results for $\hat{\gamma}_i\,(i=0,1,2,3)$, are not reported here. The estimation results for the baseline hazard function, are available upon request though.

Coefficient estimates show the following. First, the key parameter in this paper, $\hat{\delta}_0$, turns out to be negative without exception, clearly demonstrating the relationship that when the *EAP* is higher, the hazard rate of sale execution gets lower (implying that the expected duration gets longer). The results are statistically significant.[15]

Not to carry the negative sign, we will use its absolute value, $|\hat{\delta}_0|$, as a liquidity measure in the sequel and term it '*liquidity delta*'. Liquidity delta measures the responsiveness of duration to the changes in *EAP*.

Second, $\hat{\delta}_1$ and $\hat{\delta}_2$ indicate the influences of offered wealth and spread, respectively, on the hazard rate of sale execution. We conjecture that when wealth and spread are large the hazard rate is low, suggesting negative coefficients. The estimation results, however, are not consistent with the expectation. These contradictory results might be due to so called reverse causality in that as market conditions and liquidity improve, both volume and asking price of a sell limit order increase, generating spurious positive relationship between the pair (wealth, liquidity) and also between the pair (spread, liquidity). This kind of reverse causality would dampen out as one controls for market conditions in our model specification. We do not explore this line of research in the current paper, and leave it as a future research. As of now, one may regard "wealth" and "spread" as playing a role of proxy variables for market conditions in addition to their own respective roles.

Lastly, $\hat{\delta}_3$ captures the degrees of change in the hazard rate in response to change in priority stand-by buy orders (a measure of buying pressure). In most cases, $\hat{\delta}_3$ comes out to be positive, supporting the

---

[15] To interpret the *EAP* coefficient estimate $\hat{\delta}_0$, let us consider the equation "$h(t \mid EAP = e+1$, other controls$)/h(t \mid EAP = e$, other controls$) = \exp(\hat{\delta}_0)$." This equation allows one to quantify the coefficient estimate in terms of increase in sale execution hazard rate. In relative terms, the coefficient estimate of *EAP* for Samsung Electronics (-3.85) is bigger (in absolute value) than that of Kookmin Bank (-2.94), which means that the increase in sale execution hazard rate arising from unit decrease in EAP is bigger for Samsung Electronics than for the Kookmin Bank.

In absolute terms, the interpretation for the coefficient estimate of Samsung Electronics (-3.85) is as follows. A unit increase in *EAP* leads to a 385 percent (ie. 3.85 times) decrease in hazard rate, which results in an 3.85 times increased duration. But a unit increase in *EAP* is unrealistic in Korean stock market. A reasonable variation in *EAP* in the Korean stock market is no more than 5 percent (ie., 0.05) as shown in Table 1. Therefore, in the case of Samsung Electronics, a sell limit order posted at a 5% price margin over the market price suffers from about 19 percent decrease in its sale execution hazard rate, which results in about 19 percent increase in the expected time till sale execution.

claim that the higher the buying pressure is, the higher is the sale execution hazard rate. The statistical significance, however, is less pronounced.

[Table 2] Estimation Results of the Duration Model

		$\hat{\delta}_0$	(*t*-stat.)	$\hat{\delta}_1$	(*t*-stat.)	$\hat{\delta}_2$	(*t*-stat.)	$\hat{\delta}_3$	(*t*-stat.)
Samsung Electronics	Mean	-3.85	(13.83)	0.01	(1.15)	2.37	(2.49)	0.07	(2.09)
	S.D.	2.13	(4.09)	0.08	(0.97)	3.66	(3.14)	0.16	(1.65)
SK Telecom	Mean	-3.52	(13.04)	-0.01	(1.49)	-0.09	(2.53)	0.17	(3.56)
	S.D.	2.04	(3.71)	0.08	(1.21)	2.08	(2.90)	0.15	(2.51)
Kookmin Bank	Mean	-2.94	(13.09)	0.01	(1.08)	3.02	(3.64)	0.05	(1.84)
	S.D.	1.55	(3.65)	0.06	(0.87)	3.20	(3.38)	0.12	(1.44)
POSCO	Mean	-3.08	(13.64)	-0.01	(1.55)	0.69	(2.83)	0.23	(4.29)
	S.D.	1.39	(4.43)	0.08	(1.23)	8.68	(3.15)	0.20	(2.93)
KEPCO	Mean	-3.88	(14.68)	0.01	(1.63)	1.09	(2.54)	0.19	(3.95)
	S.D.	1.91	(4.35)	0.07	(1.25)	2.08	(2.78)	0.21	(2.87)
KT	Mean	-3.94	(11.92)	0.00	(7.60)	3.12	(3.71)	0.08	(1.99)
	S.D.	2.54	(4.06)	0.08	(9.15)	4.50	(3.54)	0.18	(1.51)
Hyundai Motor	Mean	-2.60	(12.85)	0.01	(1.16)	2.49	(3.35)	0.06	(2.02)
	S.D.	1.38	(3.55)	0.06	(0.91)	2.90	(3.15)	0.13	(1.45)
LG Elecronics	Mean	-3.13	(12.96)	0.01	(1.12)	3.09	(3.38)	0.04	(1.85)
	S.D.	1.67	(3.65)	0.07	(0.92)	3.29	(3.26)	0.13	(1.39)
Samsung SDI	Mean	-2.90	(12.20)	-0.00	(1.21)	2.26	(3.32)	0.12	(2.75)
	S.D.	1.52	(4.03)	0.07	(0.98)	2.71	(2.91)	0.17	(2.16)
Shinhan Bank	Mean	-2.56	(13.01)	-0.01	(1.64)	1.16	(3.07)	0.17	(3.51)
	S.D.	1.22	(3.99)	0.06	(1.44)	1.92	(2.89)	0.16	(2.37)

Note: Mean and S.D. (standard deviation) are taken over 247 trading days in year 2003.

We conclude that *EAP* is the single most important factor determining the hazard rate of sale execution, suggesting the use of liquidity delta as a liquidity measure.

We admit that the asking price is potentially endogenous. For example, if sellers ask more as market conditions and liquidity improve, the asking price becomes an endogenous variable due to reverse causality. To minimize this endogeneity problem, we adopt *EAP* (excess asking price) instead of *AP* (asking price) as a covariate in our duration model. If market conditions and liquidity are commonly reflected in both the ask and the bid prices, *EAP* is less problematic than either of ask or bid price.

If we had addressed any remaining endogeneity (reverse causality) in estimating the model, we should have found even stronger trade-off (negative relationship) between *EAP* and the sale hazard rate by stripping off "spurious positive correlation" between EAP and the sale hazard rate.

Noting that time of day is systematically correlated with *EAP* as shown in Figure 3, we may use IV type estimation or joint maximum likelihood estimation to sort out the exogenous variation in *EAP* and its effect on liquidity.[16]

2. Parameter Estimates from Alternative Model Specifications

To check robustness of our estimation results, we also estimate liquidity delta from several other models in which the model specification is slightly modified from the basic model. The results are summarized in Table 3.

The first modification is to narrow the range of *EAP* from $0.2\% \leq EAP \leq 10.0\%$ to $0.2\% \leq EAP \leq 3.0\%$, It is to make the data set more homogeneous in terms of *EAP*. The results show that the estimated liquidity delta stays similar to that of the basic model.

A high value exceeding 0.96 is obtained for the correlation between the pair of liquidity deltas, one estimated from the basic model and the other from the modified model. This high correlation is no surprise considering that most sell limit orders are concentrated in the *EAP* range below 3.0%.

In the second and third modified models, to alleviate potential multi-collinearity problems among covariates, we drop spread and (spread, wealth) from the list of covariates, respectively. The estimation results are again basically the same as those of the basic model. Correlation coefficients between alternative liquidity measures are high as before.

Through alternative model specifications, we have become confident that the proposed liquidity delta, which indicates the responsiveness of duration to changes in *EAP*, is not only conceptually useful but also statistically robust.

---

[16] According to our estimation, the joint maximum likelihood yields basically the same pattern, although the estimates are less significant and less robust. Details of the model are deferred to Appendix 2.

[Table 3] Robustness Check for Liquidity Delta ($|\hat{\delta}_0|$)

		Basic model		Alternative models					
EAP range Covariates		0.2%~10.0% ① $\delta_0 \cdot EAP$ ② $\delta_1 \cdot Wealth$ ③ $\delta_2 \cdot Spread$ ④ $\delta_3 \cdot HBid$		0.2%~ 3.0% ① $\delta_0 \cdot EAP$ ② $\delta_1 \cdot Wealth$ ③ $\delta_2 \cdot Spread$ ④ $\delta_3 \cdot HBid$		0.2%~10.0% ① $\delta_0 \cdot EAP$ ② $\delta_1 \cdot Wealth$ ④ $\delta_3 \cdot HBid$		0.2%~ 10.0% ① $\delta_0 \cdot EAP$ ④ $\delta_3 \cdot HBid$	
		$\|\hat{\delta}_0\|$	(*t*-stat.)	$\|\hat{\delta}_0\|$	(*t*-stat.)	$\|\hat{\delta}_0\|$	(*t*-stat.)	$\|\hat{\delta}_0\|$	(*t*-stat.)
Samsung Electronics	Mean	3.85	(13.83)	4.01	(13.85)	3.92	(14.05)	3.95	(12.35)
	S.D.	2.13	(4.09)	2.23	(4.08)	2.04	(4.05)	2.19	(6.04)
	Correl.			0.97		0.96		0.95	
SK Telecom	Mean	3.52	(13.04)	3.73	(13.08)	3.70	(13.18)	3.67	(12.73)
	S.D.	2.04	(3.71)	2.06	(3.90)	2.04	(3.64)	2.08	(3.97)
	Correl.			0.97		0.97		0.96	
Kookmin Bank	Mean	2.95	(13.09)	3.06	(13.17)	3.01	(13.17)	2.83	(12.35)
	S.D.	1.55	(3.65)	1.59	(3.59)	1.55	(3.71)	1.53	(9.33)
	Correl.			0.99		0.99		0.91	
POSCO	Mean	3.08	(13.64)	3.20	(13.73)	3.12	(13.76)	3.17	(13.49)
	S.D.	1.39	(4.43)	1.38	(4.25)	1.29	(4.35)	1.50	(4.25)
	Correl.			0.99		0.96		0.94	
KEPCO	Mean	3.88	(14.68)	4.13	(15.04)	4.07	(15.19)	4.01	(14.48)
	S.D.	1.91	(4.35)	2.05	(4.15)	1.99	(4.30)	2.07	(4.25)
	Correl.			0.96		0.95		0.93	
KT	Mean	3.94	(11.92)	4.06	(11.89)	3.95	(11.92)	3.88	(10.09)
	S.D.	2.54	(4.06)	2.53	(3.98)	2.52	(3.80)	2.52	(5.41)
	Correl.			0.99		0.99		0.97	
Hyundai Motor	Mean	2.60	(12.85)	2.75	(12.91)	2.71	(13.02)	2.67	(12.38)
	S.D.	1.38	(3.55)	1.40	(3.38)	1.34	(3.20)	1.38	(4.59)
	Correl.			0.97		0.96		0.97	
LG Electronics	Mean	3.13	(12.96)	3.25	(13.17)	3.21	(13.11)	3.17	(12.00)
	S.D.	1.67	(3.65)	1.78	(3.60)	1.75	(3.58)	1.76	(5.34)
	Correl.			0.99		0.98		0.98	
Samsung SDI	Mean	2.90	(12.20)	3.02	(12.52)	2.95	(12.63)	2.89	(12.24)
	S.D.	1.52	(4.03)	1.65	(3.91)	1.56	(3.96)	1.59	(4.25)
	Correl.			0.99		0.99		0.98	
Shinhan Bank	Mean	2.52	(13.01)	2.75	(12.83)	2.70	(12.99)	2.64	(12.76)
	S.D.	1.22	(3.99)	1.24	(3.77)	1.19	(3.97)	1.24	(4.04)
	Correl.			0.97		0.96		0.96	

Note: Mean and S.D. (standard deviation) are taken over 247 trading days in year 2003. Correlation is the correlation coefficient between each pair of liquidity deltas, one estimated from the basic model and the other from each alternative model, across the 247 trading days in year 2003.

As a final attempt to check the robustness of liquidity delta, we add unobserved heterogeneity in our model specification. Unobserved heterogeneity arises from insufficient control for market conditions and/or behaviors of market participants, which potentially affect the hazard rate of sale execution. We specify the hazard rate function containing unobserved heterogeneity, say u, as follows:

$$h(t \mid x) = h_0(t)\exp(x'\delta + u)\,. \tag{11}$$

[Table 4] Liquidity Deltas from a Model with Unobserved Heterogeneity

		Basic model		Unobserved Heterogeneity			Basic model		Unobserved heterogeneity	
		$\lvert\hat{\delta}_0\rvert$	(t-stat.)	$\lvert\hat{\delta}_0\rvert$	(t-stat.)		$\lvert\hat{\delta}_0\rvert$	(t-stat.)	$\lvert\hat{\delta}_0\rvert$	(t-stat.)
Samsung Electronics	Mean	3.85	(13.83)	4.29	(13.46)	KT	3.94	(11.92)	3.88	(9.76)
	S.D.	2.13	(4.09)	2.70	(5.73)		2.54	(4.06)	2.24	(3.60)
	Correl.			0.86					0.87	
SK Telecom	Mean	3.52	(13.04)	3.55	(14.14)	Hyundai Motor	2.60	(12.85)	3.56	(9.75)
	S.D.	2.04	(3.71)	1.70	(4.34)		1.38	(3.55)	2.10	(3.35)
	Correl.			0.77					0.81	
Kookmin Bank	Mean	2.95	(13.09)	3.67	(11.57)	LG Electronics	3.13	(12.96)	4.06	(9.67)
	S.D.	1.55	(3.65)	2.11	(4.43)		1.67	(3.65)	2.59	(3.47)
	Correl.			0.93					0.91	
POSCO	Mean	3.08	(13.64)	3.51	(13.40)	Samsung SDI	2.90	(12.20)	4.10	(8.37)
	S.D.	1.39	(4.43)	1.58	(5.20)		1.52	(4.03)	2.59	(3.52)
	Correl.			0.81					0.75	
KEPCO	Mean	3.88	(14.68)	4.71	(13.69)	Shinhan Bank	2.52	(13.01)	3.46	(10.05)
	S.D.	1.91	(4.35)	2.23	(5.44)		1.22	(3.99)	1.58	(3.48)
	Correl.			0.86					0.84	

Note: Mean and S.D. (standard deviation) are taken over 247 trading days in year 2003. Correlation is the correlation coefficient between the pair of liquidity deltas, one estimated from the basic model and the other from the model with unobserved heterogeneity, across the 247 trading days in year 2003.

The results in Table 4 show that the correlation coefficients between the pair of liquidity deltas, one estimated from the basic model and the other from the model with unobserved heterogeneity, are rather high ranging from a low of 0.75 for the Samsung SDI to a high of 0.93 for the

Kookmin Bank. This could be interpreted as indicating that, even after unobserved heterogeneity is allowed for, the estimated coefficients of *EAP* do not change much.[17]

3. Alternative Liquidity Measures and Rate of Return

In this section, we first analyze the relationships between our new liquidity measure and other conventional liquidity measures such as turnover, bid-ask spread, and then analyze the relationship between the rate of return and alternative liquidity measures.

Using a panel data set for each of liquidity delta, spread, turnover across 60 stocks and over 247 business days in year 2003, we run a panel regression of liquidity delta on stock fixed effects and other liquidity measures as follows.

$$Liquidity\delta_{i,d} = a_i + b_1 Spread_{i,d} + b_2 Turnover_{i,d} + e_{i,d} \tag{12}$$

where i indexes stocks ($i = 1$, ..., 60) and d trading days ($d = 1$, ..., 247). Spread is defined as the daily average of (lowest sell limit price - highest buy limit price) / (highest buy limit price), and turnover is defined as (daily traded volume) / (total number of listed shares).[18]

If there exists coherency among three alternative liquidity measures, we expect the coefficient of spread in (12) to be negative and that of turnover to be positive. The estimation results are reported in table 5. As shown in the table, the coefficient estimates of spread are significantly negative as expected, whereas the coefficient estimates of turnover are also significantly negative against our a priori expectation. These estimation

[17] Typical set of assumptions adopted in most models that incorporate unobserved heterogeneity includes: (i) the heterogeneity is independent of the observed covariates, as well as the starting and censoring times; (ii) the heterogeneity has a distribution known up to a finite number of parameters; and (iii) the heterogeneity enters the hazard function multiplicatively. We have also adopted these assumptions. In order to describe heterogeneity, Gamma distribution or discrete distribution is commonly used in the literature. In this paper, we use a discrete distribution as advocated by Heckman-Singer (1984). Two points of support are used. Details of the model and the resulting likelihood function are deferred to Appendix 3.

[18] Concerning the definition and/or calculation of turnover, it is desirable to take into account non-floating shares. We do not think, however, it would make much difference to exclude those non-floating shares in defining the turnover measure.

results imply that at least in our data either liquidity delta or turnover is not a good proxy for true liquidity.

[Table 5] Relationship b/w Liquidity Delta and Other Conventional Liquidity Measures

Dependent Var. \ Explanatory Var.	Spread ($\hat{b}_1$)	Turnover ($\hat{b}_2$)
Liquidity delta	-1.990 (-16.4)***	-0.297 (-26.9)***
	-1.654 (-13.7)***	
		-0.277 (-27.3)***

Note: 1) Spread is calculated as daily average of (lowest sell limit price - highest buy limit price) / (highest buy limit price), and turnover rate is calculated as (daily traded volume) / (total number of listed shares).

2) Figures in parentheses are t-statistics. ***, ** and * respectively denote significance at 1%, 5% and 10% significance levels

By studying the relationship between the rate of return and each alternative liquidity measure, we can infer which liquidity measure is better in terms of capturing illiquidity premium. In the literature, it has been widely believed that expected market illiquidity positively affects ex ante excess return, suggesting existence of *illiquidity premium*. Since Amihud and Mendelson (1986) first attempted to empirically confirm this belief, many researchers have succeeded in corroborating the belief.[19]

In order to identify which liquidity measure is most useful among liquidity delta, spread, and the turnover rate, we separately estimate the relationship between the rate of return and each alternative liquidity measure. The regression models are as follows.

$$\begin{aligned} Rate\ \ of\ \ Return_{i,d} &= c_1 + \beta_1 Spread_{i,d} + v_{1i,d} \\ Rate\ \ of\ \ Return_{i,d} &= c_2 + \beta_2 Turnover_{i,d} + v_{2i,d} \\ Rate\ \ of\ \ Return_{i,d} &= c_3 + \beta_3 Liquidity\delta_{i,d} + v_{3i,d} \end{aligned} \tag{13}$$

[19] See, among others, Hasbrouck and Schwartz (1988), George, Kaul, and Nimalendran (1991), Brennan and Subrahmanyam (1996), Haugen and Baker (1996), Chalmers and Kadlec (1998), Datar, Naik, and Radcliffe (1998), Vayanos and Vila (1999), Chordia, Subrahmanyam, and Anshuman (2001), Amihud (2002), Marshal and Young (2003), Pastor and Stambaugh (2003), Acharya and Pedersen (2003).

Table 6 shows the estimation results. As expected, the liquidity delta shows a negative relationship with the rate of return for each sub period as well as for the entire period. Against expectation, however, the turnover shows a positive relationship with the rate of return. The spread shows a negative, rather than the expected positive, relationship. Among the three liquidity measures, it is only the liquidity delta which reveals the expected relationship with the rate of return. Results from the fixed effect panel models are basically the same.

[Table 6] Comparision of Relationship b/w Rate of Return and Liquidity Measures

Dependent Var. \ Explanatory Var.		Spread ($\hat{\beta}_1$)	Turnover ($\hat{\beta}_2$)	Liquidity delta ($\hat{\beta}_3$)
Rate of Return	Whole year	-0.253(-2.20)**	0.305(18.85)***	-0.533(-46.19)***
	1st Q(62 days)	-0.046(-0.23)	0.284(4.76)***	-0.512(-20.23)***
	2nd Q(61 days)	-0.230(-1.04)	0.400(12.35)***	-0.660(-26.41)***
	3rd Q(61 days)	-0.810(-3.57)***	0.246(6.84)***	-0.468(-25.33)***
	4th Q(63 days)	0.130 (0.49)	0.464(11.61)***	-0.518(-24.75)***

Note: 1) Turnover is calculated as (daily traded volume)/(total number of listed shares), spread is calculated as the daily average of (lowest sell limit price - highest buy limit price)/(highest buy limit price), and the rate of return is calculated with reference to the closing price of the previous trading day.

2) Figures in parentheses are *t*-statistics. ***, ** and * respectively denote significance at 1%, 5% and 10% significance levels

[Table 7] Correlation b/w Alternative Liquidity Measures and Rate of Return

	Spread vs. Rate of return	Turnover vs. Rate of return	Liquidity delta vs. Rate of return
Mean	-0.01	0.10	-0.37
Range of Correlation	-0.39~0.46	-0.77~0.76	-0.11~-0.68

Note: Each entry in the table is calculated in two steps. First, for each of 247 trading days in 2003, we calculate cross sectional correlation coefficients using data covering 60 stocks. Second, each statistic in the first column is calculated across the 247 trading days in 2003.

Table 7, based on time series of cross-sectional correlations, corroborates our findings in Table 6. Liquidity delta shows the expected negative correlation with the rate of return in a most stable manner, whereas turnover and spread do not.

We would like to point out that the suggested liquidity delta, in spite of its conceptual usefulness, is harder to measure than the traditional liquidity measures like turnover and spread. Results in Tables 6 and 7, though, suggest that it is worth to go through the trouble of computing the liquidity delta as a liquidity measure.

Finally, we would like to carry out extended CAPM type analysis regressing rates of return on a constant, asset beta, liquidity delta (in terms of characteristic as well as risk factor), using a cross-section of assets. Through this analysis we can test whether the illiquidity premium and liquidity risk premium prevail even after controlling for asset beta and other characteristics like size and book to market ratio.[20] We constructed the panel regression model as follows;

$$R_{i,t} = \theta_{0,i} + \theta_1 \hat{\beta}_{i,t} + \theta_2 \hat{\phi}_{i,t} + \theta_3 M(L)_{i,t} + v_{i,t}. \tag{14}$$

$i =$ sixty stocks, $t =$ twelve months, $L =$ liquidity delta,

where $R_{i,t}$ is monthly excess return relative to the risk-free rate for month t. $\hat{\beta}_{i,t}$ and $\hat{\phi}_{i,t}$ are factor loadings estimated from the two factor return generating process $R_{i,d} = a_i + \beta_i R_{m,d} + \phi_i LMH_d + \varepsilon_{i,d}$ fitted to the daily (d) returns in month t (R_m is the excess rate of return of market portfolio and LMH is the risk factor related to common liquidity).[21] $\varepsilon_{i,d}$ is an idiosyncratic zero-mean disturbance. $M(L)_{i,t}$ is monthly average of daily liquidity deltas calculated for each month. Note that $M(L)_{i,t}$ is a proxies for the level of liquidity as a characteristic like size and B/M ratio of each stock as done in Fama-French (1992).

From the estimation results (Table 8), we confirm the existence of *illiquidity premium* (negative sign of $M(L)$) and *liquidity risk premium*

---

[20] Equation (14) may suffer from the potential measurement error because the independent variables are measured with errors. Measurement error arising from generated regressors, causes a well-known attenuation bias (biasing the estimate toward zero). We are observing significant, positive effect of liquidity even faced with the attenuation bias. In estimating liquidity delta, we have used so many transaction records, thus the measurement error arising from the estimated nature would not be that big.

[21] LMH is calculated as $R(\textit{Low Liquidity}) - R(\textit{High Liquidity})$ just like SMB and HML of the three factor model suggested in the Fama and French (1993). $R(\textit{Low Liquidity})$ and $R(\textit{High Liquidity})$ are weighted average rate of return of ten stocks of which liquidity level are low and high ranking respectively.

(positive sign of $\hat{\phi}_{i,t}$).[22] In terms of the significance of coefficients, however, the degree of liquidity risk premium is weak in comparison with the degree of illiquidity premium.

[Table 8] Extended CAPM Type Regression Results

	Model 1	Model 2	Model 3
$\hat{\beta}$	1.86(1.78)*	-0.86(0.85)	-1.12(1.08)
$\hat{\phi}$	0.93(1.04)		0.98(1.15)
$M(L)$		-6.14(9.60)***	-6.14(9.63)***
R^2	0.08	0.20	0.20
Obs.	720	720	720

Note: Figures in parentheses are t-statistics. ***, ** and * respectively denote significance at 1%, 5% and 10% significance levels.

V. CONCLUSIONS

In this paper, we regard the responsiveness of sale execution intensity to changes in excess asking price as an operational measure of liquidity, and we suggest ways of estimating it using econometric duration techniques. The new measure builds on the fact that there is a trade-off between excess asking price and sale execution intensity.

Our new measure is termed “liquidity delta.” For each major 60 stock listed on the Korea Stock Exchange, the new measure is reliably estimated without a single exception. Alternative model specifications do not change the new liquidity measure much. We are confident that the suggested “liquidity delta” is conceptually useful and statistically reliable as a liquidity measure.

We examine several features of the liquidity delta for sixty major stocks in terms of relationship with the traditional liquidity measures such as turnover rate and bid-ask spread, and correlation with the rate of return. The main results and implications can be summarized as follows.

First, there is somewhat consistent relationship between liquidity delta and spread whereas there is no reasonable relationship between liquidity

[22] When we control for the size and B/M ratio of each stock as done in Fama-French (1992), the results are quite similar to the current results. The results are not reported here, but available upon request.

delta and turnover rate.

Second, in terms of the relationship between different liquidity measures and the rate of return, only liquidity delta reveals a steady and meaningful relationship with the rate of return. These results imply that the liquidity delta be a superior proxy for liquidity conditions relative to other existing measures.

We would like to conclude the paper by suggesting four lines of future research. First, using the framework suggested in this paper, we can measure liquidity in other markets such as real estate, used cars, and so forth. Second, our approach suggests a new measure of total liquidity in an economy as a liquidity weighted market values of assets whereas existing measures such as M1, M2, and M3 use "all or nothing" weights. Third, we would like to explore the possibility of using liquidity delta as a tool of optimal portfolio management. We believe that liquidity risks should be taken into account for an optimal portfolio management.[23] Fourth, we would like to estimate purchase hazard rate as a function of excess bid price.

[23] The bank of international settlement (BIS) emphasizes liquidity risk as well as market risk. Bangia, Diebold, Schuermann, and Stroughair (1998) model liquidity risk on top of market risk. They measure liquidity using bid-ask spread.

APPENDIX

Appendix 1: Estimation Results for Fifty Other Stocks

		$\hat{\delta}_0$	(t-stat.)	$\hat{\delta}_1$	(t-stat.)	$\hat{\delta}_2$	(t-stat.)	$\hat{\delta}_3$	(t-stat.)
Hyundai Mobis	Mean	-2.40	(10.86)	-0.02	(0.48)	2.16	(3.55)	0.06	(0.99)
	S.D.	1.26	(3.70)	0.08	(1.74)	1.96	(3.31)	0.12	(2.13)
Woori Bank	Mean	-2.41	(9.64)	-0.01	(0.22)	1.76	(3.52)	0.06	(0.75)
	S.D.	1.34	(3.32)	0.08	(1.45)	1.89	(3.46)	0.12	(1.79)
Hana Bank	Mean	-2.18	(10.98)	-0.02	(0.52)	0.58	(1.56)	0.10	(1.64)
	S.D.	1.08	(3.30)	0.08	(1.95)	1.40	(3.63)	0.14	(2.35)
Shinsegae	Mean	-2.49	(9.99)	-0.08	(1.18)	1.47	(2.78)	0.13	(1.95)
	S.D.	1.28	(3.38)	0.12	(2.19)	1.85	(3.27)	0.16	(2.52)
KIA Motor	Mean	-3.23	(10.17)	-0.02	(0.26)	2.14	(2.87)	0.09	(1.30)
	S.D.	1.89	(3.90)	0.07	(1.36)	3.35	(3.27)	0.17	(2.38)
LG Chemical	Mean	-2.85	(10.11)	-0.03	(0.43)	2.68	(4.37)	0.05	(0.69)
	S.D.	1.62	(3.76)	0.09	(1.63)	2.42	(3.93)	0.13	(2.49)
S-Oil	Mean	-2.20	(7.75)	-0.08	(1.17)	1.18	(2.27)	0.10	(1.08)
	S.D.	1.26	(2.49)	0.11	(1.61)	1.73	(3.24)	0.16	(1.84)
Samsung Electo-Mech	Mean	-2.83	(12.29)	0.01	(0.14)	1.49	(2.38)	0.03	(0.28)
	S.D.	1.46	(3.45)	0.07	(1.30)	2.69	(3.30)	0.11	(2.09)
DSME	Mean	-2.10	(10.54)	-0.04	(0.79)	0.82	(1.84)	0.09	(1.64)
	S.D.	1.22	(3.62)	0.08	(1.86)	1.85	(3.07)	0.12	(2.22)
Hyundai H-Indus.	Mean	-2.40	(10.01)	-0.05	(0.78)	1.89	(3.40)	0.08	(1.13)
	S.D.	1.30	(3.40)	0.10	(1.70)	1.79	(3.27)	0.12	(1.80)
Koram Bank	Mean	-2.67	(8.95)	-0.04	(0.70)	2.00	(3.24)	0.08	(1.07)
	S.D.	1.54	(3.72)	0.09	(1.67)	2.56	(3.54)	0.15	(2.27)
Daewoo E. & C.	Mean	-2.13	(8.53)	-0.04	(0.87)	1.73	(3.14)	0.08	(0.73)
	S.D.	1.49	(3.30)	0.14	(2.04)	3.49	(3.42)	0.16	(1.49)
Samsung Securities	Mean	-3.21	(10.47)	-0.01	(0.08)	3.17	(3.52)	0.09	(0.85)
	S.D.	2.14	(3.71)	0.11	(1.59)	3.01	(3.15)	0.23	(2.18)
Hite Brewery	Mean	-3.15	(6.00)	-0.08	(0.69)	2.41	(3.22)	0.09	(0.81)
	S.D.	2.05	(2.34)	0.19	(1.63)	2.70	(2.87)	0.26	(1.77)
Daewoo H-Industry	Mean	-2.17	(9.69)	-0.04	(0.98)	1.34	(2.84)	0.05	(0.77)
	S.D.	1.20	(3.51)	0.07	(1.49)	1.67	(2.92)	0.11	(1.64)
Amore Pacific	Mean	-2.56	(6.53)	-0.15	(1.37)	1.57	(2.36)	0.09	(0.85)
	S.D.	1.70	(2.45)	0.20	(2.04)	2.30	(2.66)	0.22	(2.04)
Samsung Corp.	Mean	-3.11	(11.10)	-0.01	(0.02)	2.80	(3.55)	0.07	(0.90)
	S.D.	1.96	(3.92)	0.07	(1.32)	2.63	(3.28)	0.13	(1.92)
Samsung H-Indus.	Mean	-3.46	(10.62)	-0.00	(0.04)	1.89	(2.73)	0.07	(0.67)
	S.D.	2.51	(3.58)	0.07	(1.28)	3.97	(3.42)	0.16	(2.01)

CJ	Mean	-3.03	(7.34)	-0.04	(0.43)	2.49	(2.89)	0.07	(0.61)
	S.D.	1.85	(2.69)	0.14	(1.52)	2.65	(2.67)	0.19	(1.71)
Hankook Tire	Mean	-2.20	(9.66)	-0.03	(0.62)	1.90	(3.75)	0.06	(0.95)
	S.D.	1.32	(3.22)	0.09	(1.60)	1.64	(3.20)	0.12	(1.61)
KAL	Mean	-2.29	(12.42)	-0.02	(0.49)	1.11	(2.44)	0.17	(3.05)
	S.D.	1.12	(3.55)	0.06	(1.73)	1.89	(2.99)	0.15	(2.52)
LG Petro-chemical	Mean	-2.28	(9.11)	-0.05	(0.93)	1.13	(2.36)	0.10	(1.39)
	S.D.	1.15	(3.34)	0.09	(2.13)	1.74	(3.09)	0.15	(2.31)
Ssangyong Motor	Mean	-2.06	(10.27)	-0.04	(0.77)	1.49	(3.24)	0.06	(0.92)
	S.D.	1.22	(2.94)	0.06	(1.36)	1.63	(3.59)	0.10	(1.70)
INI Steel	Mean	-2.54	(7.60)	-0.05	(0.69)	2.31	(2.95)	0.07	(0.71)
	S.D.	1.64	(3.04)	0.11	(1.41)	5.20	(2.72)	0.16	(1.74)
Samsung Techwin	Mean	-2.29	(10.59)	-0.02	(0.33)	1.29	(2.26)	0.07	(1.09)
	S.D.	1.28	(3.44)	0.07	(1.44)	1.85	(2.62)	0.13	(2.10)
Hanwha Petro.	Mean	-2.05	(9.28)	-0.04	(0.83)	1.52	(3.16)	0.07	(1.12)
	S.D.	1.30	(3.21)	0.08	(1.50)	1.82	(3.27)	0.11	(1.80)
LG Securities	Mean	-2.45	(13.00)	-0.01	(0.13)	0.16	(1.20)	0.21	(3.36)
	S.D.	1.35	(3.15)	0.05	(1.44)	0.75	(2.06)	0.18	(2.60)
Hyundai Corp.	Mean	-2.56	(7.79)	-0.06	(0.90)	2.25	(3.04)	0.09	(0.97)
	S.D.	1.73	(3.24)	0.11	(1.65)	4.76	(2.85)	0.18	(2.06)
S1 Corp.	Mean	-2.51	(6.10)	-0.12	(1.41)	1.87	(2.59)	0.10	(0.96)
	S.D.	1.35	(1.99)	0.14	(1.65)	3.86	(2.71)	0.17	(1.67)
Cheil Industry	Mean	-2.95	(10.51)	-0.02	(0.28)	1.32	(1.93)	0.17	(2.42)
	S.D.	1.61	(3.66)	0.09	(1.78)	2.12	(3.08)	0.16	(2.07)
Daewoo Securities	Mean	-2.77	(9.69)	-0.04	(0.87)	2.31	(3.27)	0.06	(0.88)
	S.D.	1.82	(3.20)	0.08	(1.46)	2.47	(3.04)	0.11	(1.64)
Cheil Com-munication	Mean	-2.57	(6.76)	-0.17	(1.43)	1.26	(1.72)	0.15	(1.45)
	S.D.	1.55	(2.75)	0.21	(1.96)	2.12	(2.47)	0.23	(2.45)
Sindo-Rico	Mean	-3.33	(5.05)	-0.16	(1.03)	2.93	(2.53)	0.08	(0.44)
	S.D.	2.11	(1.70)	0.30	(1.59)	6.03	(2.26)	0.29	(1.72)
Anam Semicond.	Mean	-2.33	(10.45)	-0.04	(0.86)	1.56	(2.83)	0.05	(0.77)
	S.D.	1.52	(3.11)	0.06	(1.41)	2.08	(3.14)	0.10	(1.55)
LG Cable	Mean	-3.36	(6.86)	-0.07	(0.81)	1.53	(1.33)	0.22	(1.75)
	S.D.	1.78	(2.75)	0.14	(1.56)	6.90	(2.36)	0.26	(1.95)
Yuhan Corp.	Mean	-3.70	(5.00)	-0.12	(0.67)	2.57	(2.22)	0.10	(0.60)
	S.D.	2.53	(1.89)	0.32	(1.50)	6.01	(2.01)	0.29	(1.65)
Hyosung Corp.	Mean	-2.32	(8.12)	-0.07	(1.00)	0.58	(1.05)	0.19	(2.02)
	S.D.	1.33	(3.12)	0.13	(1.75)	1.33	(2.41)	0.20	(2.13)
Hanwha Corp.	Mean	-1.98	(7.34)	-0.08	(1.41)	1.51	(2.78)	0.07	(0.81)
	S.D.	1.43	(2.75)	0.10	(1.65)	1.62	(2.43)	0.16	(1.54)
Samsung Fine Chem.	Mean	-3.17	(7.32)	-0.08	(0.85)	1.08	(1.36)	0.17	(1.57)
	S.D.	1.71	(2.76)	0.13	(1.61)	1.97	(2.39)	0.18	(1.74)

Hansol Paper	Mean	-2.71	(7.73)	-0.04	(0.70)	1.94	(3.17)	0.06	(0.60)
	S.D.	1.92	(2.99)	0.11	(1.46)	2.11	(3.11)	0.16	(1.63)
Sambo Computer	Mean	-2.13	(11.60)	-0.02	(0.49)	2.08	(3.25)	0.04	(0.70)
	S.D.	1.29	(2.98)	0.06	(1.32)	1.85	(2.67)	0.09	(1.67)
Han Glas	Mean	-2.90	(4.20)	-0.21	(1.14)	2.73	(1.90)	0.19	(0.76)
	S.D.	2.17	(1.46)	0.37	(1.55)	3.07	(2.38)	0.72	(1.52)
Pulmuwon	Mean	-2.72	(6.14)	-0.10	(0.93)	2.10	(2.45)	0.09	(0.80)
	S.D.	1.96	(2.13)	0.27	(1.50)	4.32	(2.34)	0.26	(1.75)
Mirae Industry	Mean	-2.37	(11.34)	-0.04	(1.22)	1.07	(2.49)	0.17	(2.81)
	S.D.	1.50	(3.03)	0.06	(1.68)	1.78	(3.23)	0.16	(2.01)
Pantech Co.	Mean	-2.42	(10.74)	-0.05	(0.92)	0.92	(1.76)	0.16	(2.59)
	S.D.	1.41	(3.55)	0.08	(1.65)	1.94	(3.01)	0.19	(2.86)
Dong A Phama.	Mean	-3.78	(6.72)	-0.09	(0.95)	3.19	(2.25)	0.16	(1.41)
	S.D.	2.45	(2.67)	0.16	(1.63)	2.93	(2.60)	0.21	(1.87)
Daesang	Mean	-2.14	(7.14)	-0.09	(1.44)	1.44	(2.74)	0.08	(0.88)
	S.D.	1.42	(2.65)	0.11	(1.63)	1.41	(2.63)	0.12	(1.45)
Hankuk Paper	Mean	-3.21	(4.36)	-0.36	(0.80)	6.52	(1.79)	0.15	(0.69)
	S.D.	3.05	(2.06)	3.37	(1.57)	3.85	(2.17)	1.81	(1.83)
Lotte Samgang	Mean	-4.61	(4.48)	-0.47	(1.97)	4.60	(1.73)	0.86	(0.89)
	S.D.	3.56	(2.11)	3.36	(1.38)	5.44	(1.79)	1.31	(1.75)
SK Chemical	Mean	-2.76	(5.66)	-0.10	(1.09)	1.77	(2.09)	0.09	(0.67)
	S.D.	2.15	(2.16)	0.18	(1.52)	5.09	(2.39)	0.27	(1.57)

Note: Mean and S.D. (standard deviation) are taken over 247 trading days in year 2003.

Appendix 2: Model with Endogeneity in EAP

EAP variable might be endogenous. We may use time of the day(T) for each transaction (measured in minutes since market opening) as an instrumental variable for *EAP*. Time of day satisfies both conditions to be qualified for an IV: correlated with *EAP*, but not correlated with any unobservable variables causing changes in sale transaction hazard rate. To utilize this idea, we may apply joint MLE, where
(1) hazard rate is modeled as it is now with an additional term of u_1,
(2) add another equation determining *EAP*, say

$$EAP = \gamma_0 + \gamma_1(T) + \gamma_2(T^2) + u_2 + \varepsilon\ , \quad \varepsilon \sim N(0, \sigma^2)\ ,$$

ε is independent of (u_1, u_2)

Note that the endogeneity nature of *EAP* variable can be picked up through the correlation between u_1 and u_2. We model (u_1, u_2) as a bivariate discrete distribution with two points of support each. Let u_1 take values u_{10} and u_{11}, and let u_2 take values u_{20} and u_{21}. For level identification, we need to impose one restriction on each of u_1 and u_2. Put restrictions of $u_{10} = u_{20} = 0$. Specifically, the joint distribution of (u_1, u_2) is as follows:

$(u_1, u_2) = (0,0)$ with probability of $1/(1+\exp(w_{01})+\exp(w_{10})+\exp(w_{11}))$,
$(u_{11}, 0)$ with probability of $\exp(w_{10})/(1+\exp(w_{01})+\exp(w_{10})+\exp(w_{11}))$,
$(0, u_{21})$ with probability of $\exp(w_{01})/(1+\exp(w_{01})+\exp(w_{10})+\exp(w_{11}))$,
(u_{11}, u_{21}) with probability of $\exp(w_{11})/(1+\exp(w_{01})+\exp(w_{10})+\exp(w_{11}))$

Once modeling is finished, one may apply a joint MLE to the (*T*, *EAP*) pair using the following joint density function calculated in steps:
Step 1: Characterize $f(T, EAP \mid u_1, u_2) = f(EAP \mid u_1, u_2) * f(T \mid EAP, u_1, u_2)$, where $f(EAP \mid u_1, u_2) = f(EAP \mid u_2)$ is just a normal density function and $f(T \mid EAP, u_1, u_2) = f(T \mid EAP, u_1)$ is basically the same as in the text other than that u_1 is added.
Step 2: Once you characterize $f(T, EAP \mid u_1, u_2)$, compute its expected value with respect to (u_1, u_2) to obtain $f(T, EAP) = E_{u_1,u_2} f(T, EAP \mid u_1, u_2)$. Using the joint MLE, you estimate parameters $(w_{10}, w_{01}, w_{11}, u_{11}, u_{21})$ together with other parameters in the text.

Appendix 3: Model with Unobserved Heterogeneity

The hazard rate function containing unobserved heterogeneity, say u, is specified as follows:

$$h(t \mid x) = h_0(t)\exp(x'\delta + u).$$

In this paper, we model the heterogeneity term u as a discrete random variable having two points of support. Let u_j be those support points, and p_j the corresponding probabilities ($j = 1, 2$). It is convenient to reparametrize p_j as follows:

$$p_j = \frac{\exp(q_j)}{\sum_{j=1}^{2} \exp(q_j)} .$$

Naturally, p_j takes a value between 0 and 1 without any restriction on the q_j except for a restriction, say $q_1 = 0$, which arises from the condition $\sum_{j=1}^{2} p_j = 1$.

Regarding the level normalization of the hazard rate function, we let the baseline hazard function pick up the level, and accordingly put one restriction on each of the covariate and heterogeneity: excluding a constant term from the covariates and imposing the restriction $u_1 = 0$.

From the relationship $f(t \mid x) = h(t \mid x) S(t \mid x)$, a likelihood value for a complete spell "*t*" can be derived as follows:

$$f(t \mid x) = E_u f(t \mid x, u) = \sum_{j=1}^{2} p_j \cdot h(t \mid x, u_j) \exp\left[-\int_0^t h(\tau \mid x, u_j) d\tau \right],$$

where $h(t \mid x, u_j) = \exp[D(t)'\gamma + x'\delta + u_j]$ with $D(t)'\gamma$ and $x'\delta$ being the same as defined earlier. A likelihood value for a right-censored spell "*t*" can be computed similarly as follows:

$$S(t \mid x) = E_u S(t \mid x, u) = \sum_{j=1}^{2} p_j \cdot \exp\left[-\int_0^t h(\tau \mid x, u_j) d\tau \right].$$

References

Acharya, V. V. and L. H. Pedersen (2003), Asset Pricing with Liquidity Risk, Working Paper, London: London Business School.

Amihud, Yakov (2002), Illiquidity and Stock Returns: Cross-Section and Time-Series Effects, *Journal of Financial Markets*, Vol. 5, pp. 31-56.

Amihud, Y. and H. Mendelson (1986), Liquidity and Stock Return, *Financial Analyst Journal*, pp. 43-48.

Bayer, D., M. Borell and U. Moslener (2005), Quantifying Liquidity in Emissions Allowance Markets: Issues and Perspectives, mimeo., Center for European Economic Research.

Brennan, M. J. and A. Subrahmanyam (1996), Market Microstructure and Asset Pricing: On the Compensation for Illiquidity in Stock Returns, *Journal of Financial Economics*, Vol. 41, pp. 441-64.

Chalmers, J. M. R and G. B. Kadlec (1998), An Empirical Examination of the Amortized Spread, *Journal of Financial Economics*, Vol. 48, pp. 159-88.

Cho, Jin-Wan, and E. Nelling (2000), The Probability of Limit Order Execution, *Financial Analyst Journal*, Vol. 56, pp. 28-33.

Chordia, T., R. Roll, and A. Subrahmanyam (2000), Commonality in Liquidity, *Journal of Financial Economics*, Vol. 56, pp. 3-28.

Chordia, T., A. Subrahmanyam, and V. R. Anshuman (2001), Trading Activity and Expected Stock Returns, *Journal of Financial Economics*, Vol. 59, pp. 3-32.

Chordia, T., R. Roll, and A. Subrahmanyam, 2001, Order Imbalance, Liquidity, and Market Returns, *Journal of Financial Economics*, Vol. 65, pp. 111-30.

Chordia, T., A. Sarkarl, and A. Subrahmanyam (2005), The Joint Dynamics of Liquidity, Returns and Volatility Across Small and Large Firms, FRB of New York, Staff Report no. 207. April.

Constantinides, G. (1986), Capital Market Equilibrium with Transaction Costs, *Journal of Political Economy*, Vol. 94, pp. 842-62.

Cooper, K., J. Groth and W. Avera (1985), Liquidity, Exchange Listing and Common Stock Performance, *Journal of Economics and Business*, Vol. 37, pp. 19-33.

Cox, D. R. and D. Oakes (1984), *Analysis of Survival Data*, Chapman & Hall, New York.

Datar, V. T., N. Y. Naik, and R. Radcliffe (1998), Liquidity and Stock Returns: An Alternative Test, *Journal of Financial Markets*, Vol. 1, pp. 203-19.

Fallick, B. and K. Ryu (2000), The Recall and New Job Search of Laid-off

Workers: A Bivariate Proportional Hazard Model with Unobserved Heterogeneity, FRB Working Paper

Fama, E. and K. R. French (1992), The Cross Section of Expected Stock Returns, *Journal of Finance*, Vol. 47, pp. 427-465.

George, T. J., G. Kaul, and M. Nimalendran (1991), Estimation of the Bid-Ask Spread and Its Components: A New Approach, *Review of Financial Studies*, Vol. 4, No. 4, pp. 623-56.

Glosten, L. and L. Harris (1988), Estimating the Components of the Bid-Ask Spread, *Journal of Financial Economics*, Vol. 21, pp. 12-42.

Gourieroux, C., J. Jasiak, and G. L. Fol (1999), Intra-day Market Activity, *Journal of Financial Markets*, Vol. 2, pp. 193-226.

Grossman, S. and M. H. Miller (1988), Liquidity and Market Structure, *Journal of Finance*, Vol. XLIII, No. 3, pp. 617-33.

Hasbrouck, J. and A. Schwartz (1988), Liquidity and Execution Costs in Equity Markets: How to Define, Measure, and Compare Them, *Journal of Portfolio Management*, Spring, pp. 10-16

Hasbrouck, J. and D. J. Seppi (2001), Common Factors in Prices, Order Flows, and Liquidity, *Journal of Financial Economics*, Vol. 59, pp. 383-411.

Haugen, R. A. and N. L. Baker (1996), Commonality in the Determinants of Expected Stock Returns, *Journal of Financial Economics*, Vol. 41, pp. 401-39.

Heckman, J. J. and B. Singer (1984), Econometric Duration Analysis, *Journal of Econometrics*, Amsterdam: Jan/Feb 1984. Vol. 24, Iss. 1, 2, pp. 63-133.

Hershleifer, Jack (1972), Liquidity, Uncertainty, and the Accumulation of Information, in C. F. Carter and J. L. Ford, eds., Essays in Honor of G. L. S. Shackle, Oxford: Basil Blackwell.

Huberman, G. and D. Halka (2001), Systematic Liquidity, *Journal of Financial Research*, Vol. 24, pp. 161-78.

Jacoby, G., D. J. Fowler and A. A. Gottesman (2000), The Capital Asset Pricing Model and the Liquidity Effect: A Theoretical Approach, *Journal of Financial Markets*, Vol. 3, pp. 69-81.

Keynes, J. M. (1936), *The General Theory of Employment, Interest, and Money*.

Lippman, S. A. and J. J. McCall (1986), An Operational Measure of Liquidity, *The American Economic Review*, Vol. 76, No. 1, pp. 43-55.

Makower, H. and J. Marschak (1938), Assets, Prices and Monetary Theory, Economica, August 1938, Vol. 5, pp. 261-87.

Marshall, B. R. and Martin Young (2003), Liquidity and Stock Returns in Pure Order-Driven Markets: Evidence from the Australian Stock Market, *International Review of Financial Analysis*, Vol. 12, pp. 173-88.

Nadaraya, E. (1964), On Estimating Regression, *Theory of Probability and its Applications*, Vol. 10, pp. 186-190.

Novy-Marx, R. (2004), On the Excess Returns to Illiquidity, Working Paper, Graduate School of Business, University of Chicago.

Pagano, Marco (1989), Trading Volume and Asset Liquidity, *The Quarterly Journal of Economics*, Vol. 104, No. 2, pp. 255-74.

Pastor, L. and R. F. Stambaugh (2003), Liquidity Risk and Expected Stock Returns, *Journal of Political Economy*, Vol. 111, No. 31. pp. 642-85.

Roll, R., 1984, A Simple Implicit Measure of the Bid Ask Spread in an Efficient Market, *Journal of Finance*, Vol. 39, pp. 1127-59.

Vayanos, D. (1998), Transaction Costs and Asset Prices: A Dynamic Equilibrium Model, *Review of Financial Studies*, Vol. 11, pp.1-58.

Vayanos, D. and J. L. Vila (1999), Equilibrium Interest Rate and Liquidity Premium with Transaction Costs, *Journal of Economic Theory*, Vol. 13, pp. 509-39.