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ECONOMIC TURNING POINT FORECASTING USING THE FUZZY NEURAL NETWORK AND NON-OVERLAP AREA DISTRIBUTION MEASUREMENT METHOD

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This paper proposes a new forecasting model based on the neural network with weighted fuzzy membership functions (NEWFM) concerning forecasting of turning points in the business cycle by the composite index. NEWFM is a new model of neural networks to improve forecasting accuracy by using self adaptive weighted fuzzy membership functions. The locations and weights of the membership functions are adaptively trained, and then the fuzzy membership functions are combined by the bounded sum. To simplify the forecasting processes, the non-overlap area distribution measurement method is applied to select important features by deleting less important inputs. The *implementation of the NEWFM demonstrates an excellent capability in the field of business cycle analysis.*

JEL Classification: E3, C8 Keywords: neural network, rule extraction, business forecasting, business cycle, turning point

I. INTRODUCTION

A new forecasting model (Lim, 2005a) based on the neural network with weighted fuzzy membership functions (NEWFM) concerning forecasting of turning points in business cycle by the composite index is

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implemented in this paper. Fuzzy neural network (FNN) is a combination of the neural network and the fuzzy set theory, and provides the interpretation capability of hidden layers using knowledge based on the fuzzy set theory (Lim, 2002; Lim, 2004). Various FNN models with different algorithms such as learning, adaptation, rule extraction were proposed as an adaptive decision support tool in the field of pattern recognition, classification and forecasting (Ishibuchi, 1999; Nauk, 1997; Setnes, 2000).

The NEWFM comprises three layers, namely the input, hyperbox, and class layers. The hyperbox layer consists of *m* hyperbox nodes. Each hyperbox node to be connected to a class node contains *n* fuzzy sets for *n* input nodes. One fuzzy set consists of three small, medium, and large membership functions. NEWFM trains the *n* fuzzy set in the hyperbox nodes using the n input nodes. After the learning¹, small, medium, and large membership functions in all the fuzzy sets are synthesized into one weighted fuzzy membership function by the bounded sum (BS). Therefore each node in the hyperbox is simplified into an *n* weighted fuzzy membership function. The simplified weighted fuzzy membership function reduces the input features using the non-overlap area distribution measurement method by Lim (2005b), which enables the efficient forecasting using the minimum business forecasting indexes.

II. NEURAL NETWORK WITH WEIGHTED FUZZY MEMBERSHIP FUNCTIONS (NEWFM)

2.1 The Structure of NEWFM

The NEWFM (Lim, 2005a) is a supervised fuzzy neural network that classifies using trained weighted fuzzy membership functions. The structure of NEWFM is illustrated in Figure 1. The NEWFM comprises three layers, namely input, hyperbox, and class layer. The input layer contains *n* input nodes for *n* featured input patterns². The hyperbox

 $¹$ The learning in neural network implies the process of refining weights so as to generate the</sup> correct output, i.e. correct estimation.

 $\frac{1}{2}$ In pattern recognition, pattern means input, output data

layer consists of m hyperbox nodes. Each hyperbox node B_i to be connected to a class node contains *n* fuzzy sets for *n* input nodes. The output layer is composed of *p* class nodes. Each class node is connected to one or more hyperbox nodes. An *h*th pattern can be recorded as I_h = ${A_h=(a_1, a_2, \ldots, a_n)}$, *class*}, where *class* is the result of classification and A_h is the pattern on *n* different features³. The *i*th fuzzy set of B_l , denoted by B_i^i , has three weighted membership functions. After learning, the weighted membership functions for classification are located in the hyperbox layer.

[Figure 1] Structure of NEWFM

2.2 Learning Algorithm for NEWFM

This section describes the learning algorithm for $NEWFM⁴$. The algorithm uses the *Learning*(B_l , C_l) procedure to adjust the locations of vertices and weights, and to connect the hyperbox nodes to class nodes.

Algorithm *NEWFM*;

- 1 **While** (result is satisfied)
- 1.1 **for** $l = 1$ **to** $m \neq m$ is number of hyperboxes, usually start // from number input

 3 Features are the individual measurable variables or explanatory variables of the phenomena being observed.

⁴ Programmed using Microsoft .NET.

1.1.1 *Random*(*Bl*); 1.1.2 **for** $j = 1$ **to** $p \text{ // } p$ is number of class nodes 1.1.2.1 $w_{lj} = 0$; // initial connection weight⁵ between B_l and C_j 1.2 **for** $k = 1$ **to** h // h is number of input patterns 1.2.1 find B_l that has the maximum value of $\mathbf{EnhOutput}(B_l)$ among *m* hyperbox nodes from the input A_k ; // input vector: $I_k = \{A_k = (a_1, a_2, ..., a_n), \text{diagnostis}\}$ 1.2.2 *Learning* (B_l, C_l) ; // C_l is a *diagnosis* in I_k

Procedure *Learning*(*Bl*,*Ci*);

// *m* is number of hyperboxes

Procedure *Adjust*(B₁);

2.4 Fuzzy Rule Extraction

The learned NEWFM can be used for fuzzy rule extraction in *if-then* form to classify input patterns. After learning, each of *n* fuzzy sets in

⁵ A connection weight in neural network is the value or the coefficient that a connection between two neurons has to the overall neural network.

hyperbox node *Bl* contains three *weighted fuzzy membership functions* (*WFMs*, grey membership functions in Figure 2), where *n* is the number of input nodes.

[Figure 2] Example of Bounded Sum of the 3 Weighted Fuzzy Membership Functions (BSWFM, Bold Line)

The rules can be extracted directly from the WFMs. We suggest a rule extraction strategy as described below.

The *bounded sum* (one of operations on fuzzy set) of WFMs (BSWFM) in the *i*th fuzzy set of $B_l^i(x)$, denoted as $\mu_b^i(x)$ (bold line in Figure 2), is defined by

$$
\mu_b^i(x) = \sum_{j=1}^3 B_l^i(\mu_j(x)).
$$
 (1)

The BSWFM combines the fuzzy characteristics of three WFMs, which simplifies fuzzy rules and inference process. The rules for a class C_i is the fuzzy membership functions represented by enhanced BSWFMs in *Bl.* The visualization of the BSWFMs in B_l proffers interpretability and finding significance of the rules.

The overlapped BSWFMs of lower phase and upper phase of machinery orders received index, one of input features of CI, are shown in Figure 3, which is an example of a non-overlap area distribution. When machinery orders received index is defined as an *i* th input feature, the black area of lower phase, (A_c^i) , represents the larger fuzzy values of lower phase, and the grey area of upper phase, (A_{α}^{i}) , represents the larger fuzzy values of upper phase. If the area of $A_c^i + A_a^i$ is large and each area is equally divided, two classes are more easily separable. The non-overlap area distribution heuristic function of *i* th input feature is shown in Equation (2).

$$
f(i) = (A_c^i + A_e^i)^2 / \text{Max}(A_c^i, A_e^i)
$$
 (2)

[Figure 3] An Example of Non-Overlap Area Distribution of Upper and Lower Phase of Enhanced BSWFMs

III. METHODOLOGY OF BUSINESS FORECASTING

The business cycle, which was used to be a central part of economic theory before the Second World War, is a cyclic phenomenon of expansion and contraction phase of an aggregate economic activity in a country such as productions, prices, employment etc. These business cycles were traditionally divided into three types, i.e. Kitchin cycle, Juglar cycle and Kondratiev wave as pioneered by J. Schumpeter, and much interest was concentrated on describing the Juglar cycle which was the medium-term cycle with periodicity of 8 to 10 years. Although the opinions concerning the cause of business cycle was divided into various doctrines, it fell behind the mainstream of economics after Keynsians proved that financial and banking policy can overcome those phenomena (Cho, 1990).

However, the study of the business cycle started to gain interest by the start of a recession due to the oil crisis in the 70's after the unusual

economic boom in the 60's. Recently much interest is focused on it because of the long recession in the world economy.

The study of the business cycle has been based mainly on the conventional econometrics approach (The Bank of Korea, 2004). However, a new approach based on the information technology has been applied to it and much research work has been published recently. One of the conventional methodologies is the forecasting of the Reference Cycle Dates using the GDP or other economic indicators which are closely related to the national economic activities, or economic situations. Composite indexes which closely reflect the economic situation are often used to distinguish economic turning points by analyzing coincident index or leading index. The sequential signaling method by Zarnowitz and Moore (1982) predicts the pattern of business cycle by analyzing 6 different signal patterns using both the changes of increased rates in leading index and coincident index.

There are various models such as Neftci model (1982) and state-space Markov switching model which can analyze the probability of occurrence of transfer in the phase of business cycle statistically and eliminate false signals. Neftci model predicts the trough of the business cycle by analyzing the probability of economic turning point occurrence which is estimated from the theoretical probability density of the leading index both in the contraction and the expansion phase. Space-state Markov model is the combination of the Hamilton model (1989) and Markov transfer model based on the econometrical approach assuming that the phases of the business cycle are the phenomena of the nonlinear transfer between two cycles with different characteristics.

With the help of recent developments in information technology, the new research attempt using fuzzy theory, neural network and wavelet is made in the wide range of economic field, such as money and banking, exchange rate, stock price, credit standing of companies etc. Among them are new models proposed in the field of business forecasting.

Hoptroff (1991) used the long-term and short-term leading indexes as main indicators to forecast economic turning points in the UK based on the backpropagation algorithm, and submitted the research results to the UK government. Freisleben (1995) proved the superiority of the backpropagation algorithm compared to the ARIMA model by publishing the result of business forecasting in Germany using the data such as GDP, unemployment rate, and unemployees based on the backpropagation algorithm.

There are many other algorithms used for business forecasting application, and one of the examples is the business forecasting model in the US by Vishwakarm (1994). Manufacturing products, unemployees and personal income data are used for the main indexes and they are processed using the Kalman filtering-based state space neural network.

In addition, new approaches to analyzing and predicting the business cycle based on the wavelet de-noising and multi-resolution analysis, which are one of the signal processing tools, have recently been published. Mitra (2004) proposed a new approach to the business forecasting in India based on the wavelet filtering-based neural network using industrial production indexes. Raihan (2005) analyzed the characteristics of business cycle in the US using the increased rates of the real GDP during 1960-1996 based on the wavelet-based time-frequency transform and published the result as a report for the Federal Reserve Bank of St. Louis.

IV. EXPERIMENTAL RESULT

The data set of CI components and GDP (Gross Domestic Products) are used to generate the fuzzy rules (BSWFMs), thereby to forecast the business cycle. The final 7 BSWFMs are generated from the leading indexes and 6 BSWFMs from the coincident indexes, respectively. The forecasting results by the NEWFM are presented to evaluate the effectiveness of the overall business cycle analysis.

4.1 Used Indicators

The division of the data into the training data and test data is normally performed by trial and error. The training data should be selected to cover the entire region where the network is expected to operate. The neural network can be trained in an open-loop or in a closed-loop fashion. Vishwakarama (1994), Palit (2005) and Hines (1997) suggest that the latter option is preferable because it embodies assimilation of successive observation data in a time series analysis. The aim here is to extract fuzzy rules, thereby to integrate the different nonlinear characteristics of multiple time series of CI components by employing the neural network and to formulate a single time series representing the business cycle. The training data used here is designed to cover the latest data as far as possible in a closed-loop fashion.

The total number of 192 component samples of the composite index including a leading index, a coincident index, and a lagging index published monthly by the National Statistical Office (NSO) is used from Jan 1991 to Dec 2006. The target class is chosen to be the GDP which represents the aggregate activity of national economy and the monthly data of GDP is interpolated for classification. The summaries of input and test data are shown in Table 1.

Data set	Index (No)	Detail Index	Time Series		
Training	Leading	employment1, manufacturing1, consumption1,	180		
data	Index (10)	investment3, money & banking3, trade1	$(91.1 \sim 05.12)$		
Training	Coincident	employment1, manufacturing4, consumption2,	180		
data	Index (8)	trade1	$(91.1 - 05.12)$		
Test	Leading	employment1, manufacturing1, consumption1,	192		
data	Index (10)	investment3, money & banking3, trade1	$(91.1 \sim 06.12)$		
Test	Coincident	employment1, manufacturing4, consumption2,	192		
data	Index (8)	trade1	$(91.1 \sim 06.12)$		
class 0: lower phase (less than 5.5% growth rate of GDP)					
class 1: upper phase (higher than 5.5% growth rate of GDP)					

[Table 1] Used Indicators for the NEWFM Business Forecasting Model

4.2 Generation of Fuzzy Rule for Business Forecasting

Ten leading indexes and eight coincident indexes are used to generate the fuzzy rules for business forecasting and the minimum fuzzy rules among them are extracted using the non-overlap area distribution measurement method, thereby to carry out the final business forecasting.

In the case of the leading index, 10 indexes including open-toapplication ratio from Jan 1991 to Dec 2005 are used as input features for the training. The GDP average increase rate since 1970 is chosen to be the threshold value which decides class 0 and class 1. The threshold value is adjusted according to the declining tendency of potential growth rate (The Bank of Korea, 2005). In case of coincident index, 8 indexes including the number of employed persons of non-farming households covering the same time series are used as input features for the training.

In this experiment, two hyperboxes are created for classification. While a hyperbox which contains a set of bold lines (BSWFMs) in Figure 4 and Figure 5 is a rule for class 0 (lower phase), the other hyperbox which contains a set of light lines (BSWFMs) is another rule for class 1(upper phase). Each graph in Figure 4 and Figure 5 shows the difference between the lower phase and the upper phase for each input feature graphically.

The classification result of NEWFM is shown in Table 2 and is compared with Fuzzy ARTMAP (Carpenter, 1992) which is one of the similar supervised fuzzy neural networks. The comparison result of the classification performance between NEWFM and fuzzy ARTMAP is shown in Table 3. The final result of the classification rate is 92.22% in the case of the leading index and 93.88% in the case of the coincident index respectively. In the case of non-trained data during the period from January to December 2006, the final result gives 91.67% and 83.33% respectively, which is lower than that of the trained data. These classification capacities in the order of 92~94% are different from the experiment result of 99.56% from the medical data (Lim, 2005a). This is due to the irregularities and noisy nature of economic data.

[Table 3] Comparison of classification performance

[Figure 4] BSWFMs generated by 10 leading indexes

(X axis: min, max value of annual increase rate of each index during

[Figure 5] BSWFMs generated by 8 coincident indexes

(X axis: min, max value of annual increase rate of each index during the period of $91.1 \sim 05.12$, Y axis: fuzzy mapping value)

4.3 Minimum Fuzzy Rule Extraction by Non-Overlap Area Distribution Measurement Method

The rank of input indexes obtained by the non-overlap area distribution measurement method from the generated fuzzy membership functions is shown in Tables 4 and 5. This rank represents the efficiency of classification, and varies during the re-training. The reduced number of high rank input indexes are used for the re-training, and the optimum combination of minimum input indexes are extracted which maintains a reasonable classification rate. The BSWFMs generated by the final extracted leading and coincident indexes using non-overlap area distribution measurement method are shown in Figure 6 and Figure 7

[Table 4] Final 7 leading indexes extracted using the non-overlap area distribution measurement method

No of leading indicators	10		9		8		7	
	rank	area	rank	area	rank	area	rank	area
Opening-to-application ratio	3	0.134	7	0.062	5	0.059	\mathcal{R}	0.098
Inventory circulation indicator	$\mathbf{1}$	0.186	5	0.069	1	0.275	2	0.163
Consumer expectation index	4	0.071	2	0.208	7	0.044	7	0.011
Machinery orders received	8	0.019	6	0.063	\mathfrak{D}_{1}	0.246	6	0.029
Import of capital goods	5	0.055	8	0.045	6	0.045	5	0.036
Construction orders received	\overline{c}	0.165	1	0.356	3	0.236	1	0.197
Stock price index	7	0.034	3	0.137	8	0.042		
Liquidity aggregates	6	0.047	$\overline{\mathcal{A}}$	0.011	4	0.197	4	0.056
Interest rate spread	10	0.013						
Net barter terms of trade	9	0.014	9	0.035				

[Figure 6] BSWFMs generated by 7 leading indexes

(X axis: min, max value of annual increase rate of each index during the period of $91.1 \sim 05.12$, Y axis: fuzzy mapping value)

No of coincident indicators	8				6	
	rank	area	rank	area	rank	area
Number of employed persons	2	0.165		0.303	2	0.136
Industrial production index		0.020	6	0.027	3	0.060
Operation ratio index	6	0.064	2	0.214	4	0.042
Value of construction completed	4	0.118	5	0.053	6	0.022
Service industry activity index	5	0.106	$\overline{4}$	0.073	5	0.035
Wholesale and retail sale index		0.320	3	0.178		0.181
Shipment index (domestic)	3	0.135	7	0.008		
Imports	8	0.007				

[Table 5] Final 6 coincident indexes extracted using the non-overlap area distribution measurement method

[Figure 7] BSWFMs generated by 6 coincident indexes

(X axis: min, max value of annual increase rate of each index during the period of $91.1 \sim 05.12$, Y axis: fuzzy mapping value)

V. FORECASTING OF ECONOMIC TURING POINT

The forecasting result of NEWFM can be represented by the trend lines using the defuzzyfication of the center of gravity method (Mamdani, 1981). The nonlinear multiple time series of CI components are integrated by NEWFM model and a single time series representing the business cycles is plotted in the space of the upper and lower phase responding to the strength of fuzzy membership functions over time. The forecasting results in Figure 8 from the finally extracted leading indexes (Figure 6) during Jan 1991 and Dec 2006 demonstrate the similar fluctuations with GDP.

The Government publishes leading composite indexes (LCI) using the same 10 components and GDP as in NEWFM (Figure 9). Table 6 summarizes the results of the linear regression analysis in order to compare the approximation between NEWFM and LCI against GDP. NEWFM and LCI are used as the independent variables and GDP is used as a dependent variable.The approximation quality of NEWFM measured by the statistical coefficient of determination R^2 is higher than that of LCI which demonstrates the higher explanatory power of the model as shown graphically in Figure 10.

$$
R^{2} = \left(\sum_{i=1}^{t} \left(\hat{y} - \hat{y}\right)^{2}\right) / \left(\sum_{i=1}^{t} \left(y_{i} - \hat{y}_{i}\right)^{2}\right)
$$
(3)

where \hat{y}_i is the estimated dependent variable, and \hat{y}_i and \hat{y}_i are the mean values. R^2 represents the ratio between the variation explained by the variables and total variation of the dependent variables. Consequently, $R²$ lies between 0 and 1; the higher the explanatory power of the model, the closer R^2 is to 1. In addition, the predicting quality of NEWFM

measured by $\sum (\mu_i)^2$ and RMSE demonstrates that NEWFM is superior to LCI. $\sum (\mu_i)^2$ with $\mu_i = y_i - \hat{y}_i$, the sum of the squared residuals in the forecast period is often used as quality criterion.

[Figure 9] The trend of GDP, NEWFM and LCI

[Table 6] The results of the linear regression analysis

	GDP				
	$R^2({\%})$	$(\mu)^2$	RMSE		
NEWFM	79.3	729.7	2.0453		
LCI	69.7	1064.9	2.4863		

The forecasting result of the leading indexes using the NEWFM (Figure 11) shows sensitively the expansion and the contraction phases of the economy with certain time lag compared with the cyclical component of coincident CI, i.e. the Reference Cycle published by the Government.

Table 7 shows the turning point forecasting capability of NEWFM model in detail: During the $5th$ to $8th$ cycle period, forecasting results leads over the Reference Cycle Date by 8 to 14 months in the peak point and by 4 to 8 months in the trough point respectively. The accuracy in the trough point is relatively higher than that in the peak point.

[Figure 11] Comparison between the Reference Cycle Date and forecasting result of the leading indexes using the NEWFM economic forecasting model

[Table 7] Reference Cycle Date and the forecasting result using the NEWFM economic forecasting model

On the other hand, the trend line of the forecasting result from Jan 1991 to Dec 2006 using the finally extracted fuzzy rules (Figure 7) by the coincident indexes is shown in Figure 12. This result shows high correlation with the cyclical component of coincident composite index. However, it shows a little time lag between them during a certain period because NEWFM model used the GDP as a training target.

[Figure 12] Comparison between the Reference Cycle Date and forecasting result of the coincident indexes using the NEWFM economic forecasting model

The irregularities and the noise of the trend lines can be removed using the denoising feature of the wavelet to overview and recognize the economic phase more easily (Walker, 1999). The denoised result is shown in Figure 13. From the recent economic fluctuation after the peak of the $8th$ cycle period, economic cycles are occupied mainly in the lower region. This is due to the use of the only one threshold value in the whole cycle periods even if the threshold value is adjusted according to the declining trend of the recent potential GDP growth rate.

Therefore the more realistic forecasting result can be obtained by using the multiple threshold values corresponding to the periodic changes of the potential GDP growth rate estimated by The Bank of Korea.⁶

⁶ Before CI revision made by NSO, the Cyclical component of coincident composite index had been showed continuously the unrealistic low level (Sangho Nam, "The evaluation of the characteristics of the recent Korean business cycle," The study series of banking & economics, The Bank of Korea, August 2006).

[Figure 13] The wavelet transform result from the forecasting result of the leading indexes using the NEWFM economic forecasting model (denoised signal from level 3 of db4)

VI. CONCLUSIONS

This paper proposes a new forecasting model based on the neural network with weighted fuzzy membership functions (NEWFM). This model provides a considerable potentiality of integrating the different nonlinear characteristics of multiple time series of CI components, thereby to represent the business cycle, although further study will be necessary to improve the accuracy of the economic forecasting capability, especially in case of non-trained data. In addition, it measures the efficiency of the economic indexes, enabling the users to select the simpler fuzzy rules to forecast. Moreover, the automation of business cycle forecasting by business indicators can be used readily for policy making and establishing business plan by responsible authorities, CEOs of banking institutions, enterprises etc.

The accuracy of NEWFM business forecasting model can be improved if the careful selection of input economic data which reflects the real world economy is made and the recent trend of the potential economic growth rate is used to decide the upper and lower threshold values which result in the overall performance of the forecasting model.

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