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STAGGERED NOMINAL CONTRACTS: TAYLOR vs. CALVO

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This paper compares the Taylor type and the Calvo type staggering schemes in terms of their implications on the behavior of aggregate price and output following a monetary expansion. In the sticky nominal price case, it is shown that Calvo type staggering implies positive autocorrelations (i.e., monotone dampening) in price and output, while Taylor type staggering implies negative autocorrelations (i.e., oscillatory dampening). In the sticky nominal wage case, however, it is found that both types of staggering generate monotonically dampening responses of price and output. In other words, the qualitative implications of wage stickiness are robust to the choice of staggering schemes.

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I. INTRODUCTION

A lot of growing literature has incorporated nominal rigidities into otherwise standard business cycle models using either deterministic

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staggering (Taylor (1979,1980)) or stochastic staggering (Calvo (1983)) setting. Under Taylor type staggering, firms (or households) take turns adjusting prices (or wages), so all firms have opportunities to adjust prices after some deterministic period of price (or wage) fixity. In the Calvo type staggering model, firms (or households) adjust prices (or wages) according to some fixed probability of being chosen to do so, and therefore individual prices (or wages) are stochastically staggered and the duration of fixity is also stochastic. As can be seen in a large volume of literature (e.g., Jeanne (1998)), the two types of staggering have been believed to imply similar dynamics, and therefore many authors have adopted Calvo type staggering, probably for the ease of solving dynamic models. More recent literature, however, put into question such belief. For example, in a small optimizing IS/LM model, Kiley (2002) argues that the two staggering schemes applied to prices have qualitatively as well as quantitatively different implications on the behavior of aggregate price and output after a monetary expansion: Calvo type price staggering can always generate persistent responses, while Taylor type generates oscillatory responses.

The aim of this paper is to compare the two staggering schemes in another dimension: staggered wage contracts. We compare the Taylor type and the Calvo type staggering schemes in terms of their implications on the behavior of aggregate price and output following a monetary expansion. In the sticky nominal price case, it is shown that Calvo type staggering implies positive autocorrelations (i.e., monotone dampening) in price and output, while Taylor type staggering implies negative autocorrelations (i.e., oscillatory dampening). In the sticky nominal wage case, however, it is found that both types of staggering generates monotonically dampening responses of price and output. In other words, the qualitative implications of wage stickiness are robust to the choice of staggering schemes.

Section II provides an introduction to Taylor and Calvo type price and wage setting in a simple dynamic optimizing model. Section III compares the two staggering schemes in a sticky nominal price case. Section IV compares the two staggering schemes in a sticky nominal wage case. Section V gives some more discussion of the results in the two previous sections.

II. THE MODEL

There are three types of agents in the economy: households, firms, and the government. Firms are monopolistic competitors producing differentiated goods using bundles of labor service. Households purchase output for consumption and supply differentiated labor service. The government manages monetary policy. Key features of the model are the rigidities in aggregate price and/or wage. Price and wage settings are staggered.

2.1. Final Goods and Labor Service

We assume that there is a competitive *aggregation* sector, where heterogeneous labor service (supplied by households) and intermediate goods (produced by firms) are transformed into composite goods for consumption and labor service for production, respectively. The transform technologies are

$$Y = (\int Y_j^{\theta_Y} dj)^{\frac{1}{\theta_Y}} \quad and \quad L = (\int L_i^{\theta_L} di)^{\frac{1}{\theta_L}} \ , \ \ 0 < \theta_Y , \theta_L < 1 \tag{2.1}$$

where Y_j and L_i denote differentiated goods and labor service, respectively.

From the cost minimization in the final sector, we obtain the following demand for individual goods and labor service

$$Y_{jt} = (\frac{P_{jt}}{P_t})^{\frac{1}{\theta_Y - 1}} Y_t \quad and \quad L_{it} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t \tag{2.2}$$

where P_j is the price of firm j 's product, and W_i is the wage rate household i sets for its labor service.

The aggregate price and wage index, P and W , are given as below:

$$P_t = (\int P_{jt}^{\frac{\theta_Y}{\theta_Y - 1}} dj)^{\frac{\theta_Y - 1}{\theta_Y}} \quad and \quad W_t = (\int W_{it}^{\frac{\theta_L}{\theta_L - 1}} di)^{\frac{\theta_L - 1}{\theta_L}} \tag{2.3}$$

2.2. Firm's Problem

There is a continuum of firms, indexed by j, distributed on the unit interval [0,1]. They have access to the identical production technology

$$Y_{jt} = L_{jt}^{\alpha} \ , \quad \alpha > 0 \tag{2.4}$$

where L_{jt} is the quantity of composite labor used by firm j, and α governs returns to scale. Cost minimization yields

$$W_t = \alpha\, MC_{jt} Y_{jt} / L_{jt} \tag{2.5}$$

where MC_{jt} is the (nominal) marginal cost corresponding to the Lagrange multiplier on the production function in cost minimization.

2.2. A. Price Adjustment Rules 1: Stochastic Staggering

Under a staggered price setting a la Calvo (i.e, stochastic staggering), firms set their individual prices in the following way: at each period t, a randomly selected ϕ_Y fraction of firms maintain their prices from the previous period. The remaining $1-\phi_Y$ fraction of firms choose the optimal price so as to maximize their expected present discounted stream of real profits by solving

$$\max E_t [\sum_{\tau=t}^{\infty} \frac{\beta^{\tau-t}\Lambda_\tau}{\Lambda_t} \phi_Y^{\tau-t} \frac{P_{jt}Y_{j\tau} - W_\tau L_{j\tau}}{P_\tau}] \tag{2.6}$$

where $\frac{\beta^{\tau-t}\Lambda_\tau}{\Lambda_t}$ is the discounting factor for firm j's real profit in the period τ.

Suppose that firm j can optimize on its price in the current period t. Then, with probability $\phi_Y^{\tau-t}$, the demand it faces from the period t on evolves as

$$Y_{i\tau} = [\frac{P_{jt}}{P_\tau}]^{\frac{1}{\theta_Y - 1}} Y_\tau \ , \quad \tau \geq t \ . \tag{2.7}$$

The first order condition, given the demand function (2.7) and cost minimization condition (2.5), yields

$$P_t^* = \frac{1}{\theta_Y} \frac{E_t[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_\tau \phi_Y^{\tau-t} MC_{j\tau} P_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau]}{E_t[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \Lambda_\tau \phi_Y^{\tau-t} P_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau]} \ . \tag{2.8}$$

Equation (2.8) has an interpretation that an optimizing firm j determines its nominal price as a weighted average of its future expected marginal costs, scaled up by the constant markup factor $1/\theta_Y$. It should be noted that in the case of fully flexible individual prices, (i.e., $\phi_Y = 0$), (2.8) boils down to the static pricing rule

$$P_t^* = \frac{1}{\theta_Y} MC_{jt} \ . \tag{2.8)'$$

The equation for the aggregate price P_t is given by the combination of outstanding prices:

$$P_t = \phi_Y P_{t-1} + (1-\phi_Y) P_t^* \ . \tag{2.9}$$

2.2. B. Price Adjustment Rules 2: Deterministic Staggering

Under the staggered price setting in the spirit of Taylor, all firms are divided into N cohorts based on the timing of their price decisions, and a fraction $1/N$ of firms can set new prices in each period. A firm j in the cohort that can set new prices in period t determines P_{jt} by solving

$$\max E_t[\sum_{\tau=t}^{t+N-1} \frac{\beta^{\tau-t}\Lambda_\tau}{\Lambda_t} \frac{P_{jt}Y_{j\tau} - W_\tau L_{j\tau}}{P_\tau}] \tag{2.10}$$

subject to (2.5) and (2.7). Once a new price is set, it remains fixed for N periods.

The corresponding first order condition yields

$$P_t^* = \frac{1}{\theta_Y} \frac{E_t[\sum_{\tau=t}^{t+N-1} \beta^{\tau-t}\Lambda_\tau MC_{j\tau} P_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau]}{E_t[\sum_{\tau=t}^{t+N} \beta^{\tau-t}\Lambda_\tau P_\tau^{\frac{\theta_Y}{1-\theta_Y}} Y_\tau]} \tag{2.11}$$

which boils down to the static pricing rule (2.8’) for N=1 (i.e., no staggering).

The aggregate price level P_t evolves according to

$$P_t = \frac{P_t^* + P_{t-1}^* + \cdots\cdots + P_{t-N+1}^*}{N} \quad . \tag{2.12}$$

2.3. The Household’s Problem

A typical household $i \in [0,1]$ chooses consumption (C_i) and labor hours (L_i) to solve

$$\max E_t[\sum_{\tau=t}^{\infty} \beta^\tau U(C_{i\tau}, 1 - L_{i\tau})] \tag{2.13}$$

subject to the budget constraint

$$C_{i\tau} + \frac{B_{i\tau}}{P_\tau} - \frac{B_{i,\tau-1}}{P_\tau} \le \frac{W_{i\tau}L_{i\tau}}{P_\tau} + \frac{\int s_{ij}\Pi_{j\tau}dj}{P_\tau} + r_{\tau-1}\frac{B_{i,\tau-1}}{P_\tau} \tag{2.14}$$

where B_{it} is the nominal bond holding, r_t is the net nominal interest rate, and $\frac{\int s_{ij}\Pi_{j\tau}dj}{P_\tau}$ is the dividend income. We assume that s_{ij}, household i's share of the firm j, is fixed beyond the control of the household.

2.3. A. Wage Adjustment Rules 1: Stochastic Staggering

Under staggered wage setting a la Calvo, households set individual wages as follows: at each period, a randomly chosen ϕ_L fraction of households maintain their wages from the previous period. The other $1-\phi_L$ fraction of households chooses W_{it} to maximize their lifetime utility (2.13). Suppose that household *i* optimizes on its wage rate in the current period *t*. Then with probability $\phi_L^{\tau-t}$, the demand it faces from the period t on evolves as

$$L_{i\tau} = [\frac{W_{it}}{W_\tau}]^{\frac{1}{\theta_L - 1}} L_\tau \ , \ \tau \geq t \, . \tag{2.15}$$

Using equation (2.15), we get the first order condition for household i to determine its optimal wage:

$$W_t^* = -\frac{1}{\theta_L} \frac{E_t[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_L^{\tau-t} \frac{\partial U_{i\tau}}{\partial L_{i\tau}} W_\tau^{\frac{1}{1-\theta_L}} L_\tau]}{E_t[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \phi_L^{\tau-t} \frac{\Lambda_{i\tau}}{P_\tau} W_\tau^{\frac{1}{1-\theta_L}} L_\tau]} \tag{2.16}$$

where $\Lambda_{it} = \frac{\partial U_{it}}{\partial C_{it}}$ is the Lagrangian multiplier on the budget constraint (2.14). Equation (2.16) says that the optimal wage is set as a weighted average of its future expected marginal disutility from working, scaled up by a "markup" factor $\frac{1}{\theta_L}$. It should be noted that in the absence of wage staggering (i.e., $\phi_L = 0$), equation (2.16) boils down to the static condition in a flexible wage model

$$\Lambda_{it}\frac{W_t}{P_t} = -\frac{1}{\theta_L}\frac{\partial U_{it}}{\partial L_{it}}. \tag{2.16'}$$

The equation for the aggregate wage W_t is given by the combination of outstanding wages:

$$W_t = \phi_L W_{t-1} + (1-\phi_L)W_t^*. \tag{2.17}$$

2.3. B. Wage Setting Rules 2: Deterministic Staggering

Under the staggered wage setting in the spirit of Taylor, all households are divided into N cohorts based on the timing of their wage decisions, and a fraction $1/N$ of households can set new wages in each period. For household i, its optimal wage is given by

$$W_t^* = -\frac{1}{\theta_L}\frac{E_t[\sum_{\tau=t}^{t+N-1}\beta^{\tau-t}\frac{\partial U_{i\tau}}{\partial L_{i\tau}}W_\tau^{\frac{1}{1-\theta_L}}L_\tau]}{E_t[\sum_{\tau=t}^{t+N-1}\beta^{\tau-t}\frac{\Lambda_{i\tau}}{P_\tau}W_\tau^{\frac{1}{1-\theta_L}}L_\tau]} \tag{2.18}$$

which remains fixed for N periods. Note that (2.18) boils down to (2.16)' if $N=1$ (i.e., no staggering).

The aggregate wage W_t evolves according to

$$W_t = \frac{W_t^* + \cdots\cdots + W_{t-N+1}^*}{N}. \tag{2.19}$$

2.4. Money

To close the model and examine the behavior of the economy in response to monetary shocks, we need to incorporate money in the model. We specify money demand by the simple quantity equation

$$M_t\bar{V} = P_t Y_t. \tag{2.20}$$

The nominal money supply is assumed to be logarithmic random walk:

$$\log M_t = \log M_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim ii(0,\sigma^2). \tag{2.21}$$

III. CALVO vs. TAYLOR: STAGGERED PRICE SETTING

We compare two types of staggered price schemes in terms of their dynamic implications on output and aggregate price. To facilitate intuitive comparison, we assume that i) the production function is CRS in labor, ii) β is equal to be 1, and iii) the instantaneous utility function is given by $U(C,L) = \log C + \eta \log(1-L)$, for some $\eta > 0$.[1]

3.1. Calvo Type Staggering

Suppose that staggered price contracts a la Calvo are the sole source of nominal rigidities.

We first define the log deviations of some variables:

$$x_t = d\log P_t^*,\ p_t = d\log P_t,\ w_t = d\log W_t,\ m_t = d\log M_t, and\ y_t = d\log Y_t.$$

Then the price equation (2.8) is log-linearized into

$$x_t = (1-\phi_Y)E_t[\sum_{\tau=t}^{\infty}\phi_Y^{\tau-t}w_t] \tag{3.1}$$

which in turn implies

$$x_t - \phi_Y E_t x_{t+1} = (1-\phi_Y)w_t. \tag{3.2}$$

Log-linearization of the wage equation (2.12) with $\phi_L = 0$ gives

[1] The simplified model we consider is similar to that in Kiley (2002), except that he uses $u(C,L) = \log C - \frac{L^{1+z}}{1+z},\ z > 0$. We prefer our specification because i) ours ensures the existence of balanced growth as in King et al. (1998), and ii) Chari et al. (2002) similarly specified utility function, whose results will be extended to the case of price *and* wage rigidities in the appendix.

$$w_t = (\frac{L}{1-L}+1)y_t + p_t = (s+1)y_t + p_t \ , \ s = \frac{L}{1-L} . \tag{3.3}$$

We impose the static money demand equation:

$$m_t = y_t + p_t . \tag{3.4}$$

Finally, the price level p_t is a weighted average of p_{t-1} and x_t:

$$p_t = \phi_Y p_{t-1} + (1-\phi_Y)x_t . \tag{3.5}$$

The system of equations (3.1) -(3.5) can be solved to determine how monetary shocks affect prices and output. Substituting for y_t and w_t into (3.2) and using equation (3.5), we obtain

$$E_t x_{t+1} - \Psi_Y x_t + x_{t-1} = -\frac{1}{\phi_Y}(1-\phi_Y)(1+s)(m_t - \phi_Y m_{t-1}) \tag{3.6}$$

where $\Psi_Y = (1+\phi_Y^2 + (1-\phi_Y)^2 s)/\phi_Y$.

Applying standard methods for solving second order stochastic difference equations we can write x_t as

$$\begin{aligned} x_t &= \lambda_y x_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)(1+s)E_t[\sum_{i=0}^{\infty}\lambda_y^i(m_{t+i} - \phi_Y m_{t-1+i})] \\ &= \lambda_y x_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)(1+s)[\frac{1-\phi_Y}{1-\lambda_y}m_t + \phi_Y \Delta m_t] \end{aligned} \tag{3.7}$$

where λ_y is the root (with absolute value less than one) which solves the quadratic equation $\lambda^2 - \Psi_Y \lambda + 1 = 0$. This root is given by

$$\lambda_y = \frac{1}{2}[\Psi_Y - (\Psi_Y^2 - 4)^{\frac{1}{2}}]. \tag{3.8}$$

Then the price level and output are shown to follow

$$\begin{aligned} p_t &= \lambda_y p_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)^2(1+s)E_t[\sum_{i=0}^{\infty}\lambda_y^i m_{t+i}] \\ &= \lambda_y p_{t-1} + \frac{\lambda_y}{\phi_Y}(1-\phi_Y)^2(1+s)\frac{m_t}{1-\lambda_y} \end{aligned} , \tag{3.9}$$

$$y_t = \lambda_y y_{t-1} + m_t - \lambda_y m_{t-1} - \frac{\lambda_y(1-\phi_Y)^2(1+s)}{\phi_Y(1-\lambda_y)} m_t . \tag{3.10}$$

3.2. Taylor Style Staggering

For the ease of solving the model, we assume that only individual prices are set optimally every two periods while individual wages are reset every period.[2]

We first log-linearize the pricing equation around the deterministic steady state. We let

$x_t = d\log P_t^*$, $p_t = d\log P_t$, $w_t = d\log W_t$, $mc_{jt} = d\log MC_{jt}$, and $y_t = d\log Y_t$

Then the price equation is log-linearized into

$$x_t = \frac{1}{2}[mc_{jt} + E_t mc_{j,t+1}], \quad j \in [0, 1/2]. \tag{3.11}$$

Since marginal cost is equal to wage rate, the price equation becomes

$$x_t = \frac{1}{2}[w_t + E_t w_{t+1}]. \tag{3.12}$$

Log-linearizing the wage equation, we get

[2] This case of $N = 2$ is analogous to $(\phi_Y, \phi_L) = (1/2, 0)$ of section 3.2.

$$w_t - p_t = \frac{L}{1-L} l_t + y_t = (s+1) y_t \,. \tag{3.13}$$

We impose the static money demand equation:

$$m_t = y_t + p_t \,. \tag{3.14}$$

The price level p_t is an average of the individual prices:

$$p_t = \frac{1}{2}(x_t + x_{t-1}) \tag{3.15}$$

and $\{m_t\}$ is a random walk process.

The system of equations (3.11)-(3.15) can be solved to determine how money shocks affect prices and output. Substituting for y_t and p_t, we obtain

$$E_t x_{t+1} - 2\frac{1-\theta_Y + \Psi_Y}{1-\theta_Y - \Psi_Y} x_t + x_{t-1} = -\frac{2\Psi_Y}{1-\theta_Y - \Psi_Y} E_t(m_t + m_{t+1}) \tag{3.16}$$

where $\Psi_Y = (s+1)(1-\theta_Y)$.

The above second order stochastic difference equations can be solved for x_t:

$$x_t = a_y x_{t-1} + (1-a_y) m_t \tag{3.17}$$

where a_y is the root (with an absolute value of less than one) which solves the quadratic equation $a^2 - \Omega_Y a + 1 = 0$, with $\Omega_y = 2(1-\theta_Y + \Psi_Y) / (1-\theta_Y - \Psi_Y)$. This root is given by

$$a_y = \frac{\sqrt{1-\theta_Y} - \sqrt{\Psi_Y}}{\sqrt{1-\theta_Y} + \sqrt{\Psi_Y}} \,. \tag{3.18}$$

Using (3.14) and (3.15), we obtain

$$p_t = a_y p_{t-1} + \frac{1}{2}(1 - a_Y)(m_t + m_{t-1}), \tag{3.19}$$

$$y_t = a_y y_{t-1} + \frac{1}{2}(1 + a_y)(m_t + m_{t-1}). \tag{3.20}$$

3.3. Comparison[3]

We first note that λ_y and a_y determine the degree of sluggishness in the responses of price and output. Examination of the formulas for λ_y and a_y reveals an important *qualitative* difference between the two types of price staggering : for all values of $\phi_Y > 0$, λ_y is greater than zero, whereas a_y is less than zero. In other words, Calvo type staggering implies positive autocorrelations (i.e., monotone dampening) in price and output, while Taylor type staggering implies negative autocorrelations (i.e., oscillatory dampening). In fact, it is exactly in this sense that Chari et al. (2000) conclude that Taylor type staggered price setting embedded in a standard DSGE model *cannot* deliver persistent real responses to monetary shocks. Comparison of λ_y and a_y, however, exhibits that Calvo type price staggering can generate real persistency in typical DSGE models, as shown in Rotemberg and Woodford (1999) and Kim (2003).

This qualitative difference between the two types of staggering is more conspicuous when we change the value of s. Wage equation (3.3) or (3.13) implies that s determines the degree of *real* rigidities, as in the sense of Ball and Romer (1990), controlling the degree by which the increases in output cause those in real wage. Intuitively, lower degree of real rigidity reflected in higher values of s will tend to decrease the degree of persistency in price and output following a monetary shock. As can be seen from equations (3.8) and (3.17), this intuition clearly holds for λ_y. For the case of Taylor type price staggering, however, higher values of s decrease a_y (further below 0), causing higher degree of oscillations in price and output responses.

In the context of a model similar to ours, Kiley (2002) summarizes the

[3] To make two staggering schemes comparable, we set $\phi_Y = \frac{1}{2}$ so that the *average* duration of new price is 2 under Calvo type staggering.

intuition why the two staggering schemes impart qualitatively different implications on persistence. Under Calvo type staggering, there exists a fraction of 'tail' firms that do not adjust prices for longer periods than the average duration of price fixity. Also, since the aggregate price level is the average of new price and all prices set previously, there also exists a fraction of 'tail' firms whose prices set in the distant past are reflected in the aggregate price level. The presence of such 'tail' prices under Calvo staggering leads to more sluggish responses of aggregate price than under Taylor type staggering, under which the 'tail' prices set N-1 and farther periods earlier do not enter the horizon of firm's price decision.

IV. CALVO vs. TAYLOR: STAGGERED WAGE SETTING

We now compare two types of staggered contracts applied to nominal wage setting. We maintain the assumptions i)-iii) in section 3.

4.1. Calvo Type Staggering

Suppose that staggered wage contracts a la Calvo are the sole origin of nominal rigidities.

For notational simplicity, we let

$$x_t = d\log W_t^* .$$

Log-linearization of the wage equation (2.12) implies

$$\begin{aligned}(\frac{1-\theta_L+s}{1-\theta_L})[x_t-\phi_L E_t x_{t+1}] &= (1-\phi_L)[(1+s)m_t + s\frac{\theta_L}{1-\theta_L}w_t] \\ &= (1-\phi_L)(1+s)m_t + (1-\phi_L)^2\frac{s\theta_L}{1-\theta_L}(1-\phi_L B)^{-1}x_t \end{aligned} \tag{4.1}$$

where B is the lag operator. The second equality in equation (4.1) comes from the equation for aggregate wage rate:

$$w_t = \phi_L w_{t-1} + (1-\phi_L)x_t . \tag{4.2}$$

Rearranging terms in equation (4.1), we get

$$E_t x_{t+1} - \Psi_L x_t + x_{t-1} = -\frac{1}{A\phi_Y}(1-\phi_L)(1+s)(1-\phi_L)m_t \tag{4.3}$$

where

$$\Psi_L = \frac{D}{A\phi_L},\ A = \frac{1-\theta_L+s}{1-\theta_L} \quad and\ D = A + A\phi_L^2 - (1-\phi_L)^2 \frac{s\theta_L}{1-\theta_L}.$$

Applying standard methods for solving second order stochastic difference equations, we can write x_t as

$$\begin{aligned} x_t &= \lambda_l x_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)(1+s)(1-\phi_L)E_t[\sum_{i=0}^{\infty}\lambda_l^i m_{t+i}] \\ &= \lambda_l x_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)(1+s)[\frac{1-\phi_L}{1-\lambda_l}m_t + \phi_L \mu_t] \end{aligned} \tag{4.4}$$

where λ_l is the root (with absolute value less than one) solving the quadratic equation $\lambda^2 - \Psi_L \lambda + 1 = 0$. This root is given by

$$\lambda_l = \frac{1}{2}[\Psi_L - (\Psi_L^2 - 4)^{\frac{1}{2}}].$$

Then the price level and output are shown to follow

$$\begin{aligned} p_t &= \lambda_l p_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)^2(1+s)E_t[\sum_{i=0}^{\infty}\lambda_l^i m_{t+i}] \\ &= \lambda_l p_{t-1} + \frac{\lambda_l}{A\phi_L}(1-\phi_L)^2(1+s)\frac{m_t}{1-\lambda_l} \end{aligned} \tag{4.5}$$

$$y_t = \lambda_l y_{t-1} + m_t - \lambda_l m_{t-1} - \frac{\lambda_l(1-\phi_Y)^2(1+s)}{A\phi_Y(1-\lambda_l)}m_t \qquad (4.6).$$

4.2. Taylor Type Staggering

Now we consider the situation where individual wages are set optimally every two periods a la Taylor. We first log-linearize the wage equation around the deterministic steady state. Let

$$x_t = d\log W_t^*, \quad and \quad l_{1t} = d\log L_{1t}$$

where L_{1t} is the demand for labor of the cohort of households setting their wages in the current period t. Then the wage equation is log-linearized into

$$x_t = \frac{1}{2}s[l_{1t} + E_t l_{1,t+1}] + \frac{1}{2}[y_t + p_t + E_t(y_{t+1} + p_{t+1}). \tag{4.7}$$

Individual labor demand function implies

$$\begin{aligned} l_{1t} &= \frac{1}{\theta_L - 1}[x_t - w_t] + l_t \\ l_{1,t+1} &= \frac{1}{\theta_L - 1}[x_t - w_{t+1}] + l_{t+1} \end{aligned}, \tag{4.8}$$

Therefore the wage equation becomes

$$\begin{aligned} [1 + \frac{1}{1-\theta_L}s]x_t = {} & \frac{1}{2}s[\frac{1}{1-\theta_L}(w_t + E_t w_{t+1}) + l_t + E_t l_{t+1}] \\ & + \frac{1}{2}[m_t + E_t m_{t+1}]. \end{aligned} \tag{4.9}$$

Using the production function, money demand, and price equation, we have

$$l_t = y_t = m_t - w_t \tag{4.10}$$

The aggregate wage rate is an average of the individual wage rates

$$w_t = \frac{1}{2}(x_t + x_{t-1}) . \tag{4.11}$$

After some manipulations, it can be shown that the log-linearized version of the wage equation is

$$E_t x_{t+1} - 2\frac{\Psi_L + s\theta_L + \Psi_L}{\Psi_L + s\theta_L - \Psi_L} x_t + x_{t-1} \\ = -\frac{2\Psi_L}{\Psi_L + s\theta_L - \Psi_L} E_t(m_t + m_{t+1}) \tag{4.12}$$

where $\Psi_L = (s+1)(1-\theta_L)$.

Applying standard methods for solving second order stochastic difference equations, we can write x_t as

$$x_t = a_l x_{t-1} + (1-a_l)m_t \tag{4.13}$$

where a_l is the root (with absolute value less than one) solving the quadratic equation $a^2 - \Omega_l a + 1 = 0$, with $\Omega_l = 2(\Psi_L + s\theta_L + \Psi_L)$ / $(\Psi_L + s\theta_L - \Psi_L)$. This root is given by

$$a_l = \frac{\sqrt{\Psi_L + s\theta_L} - \sqrt{\Psi_L}}{\sqrt{\Psi_L + s\theta_L} + \sqrt{\Psi_L}} .$$

Finally, price and output are shown to follow

$$p_t = w_t = a_l w_{t-1} + \frac{1}{2}(1-a_l)(m_t + m_{t-1}) , \tag{4.14}$$

$$y_t = a_l y_{t-1} + \frac{1+a_l}{2}(m_t - m_{t-1}) . \tag{4.15}$$

4.3. Comparison[4]

λ_l and a_l determines the degree of sluggishness in the responses of price and output. From the formulas for λ_l and a_l, we first find that both types of staggering, if applied to nominal wages, can generate positive autocorrelation (i.e., monotone dampening) in the responses of price following a monetary shock. For example, if we set $\phi_L = 1/2$ an $s = 1/2$, $\lambda_l = 0.7035$ and $a_l = 0.3333$. The qualitative similarity of Taylor and Calvo type wage staggering is in sharp contrast with the results in section 3. In particular, Taylor type staggering has totally different implications on the dynamics of the model, depending on whether it is applied to prices or wages. We will discuss this finding below in more detail

Another finding is that λ_l is greater than a_l for all plausible levels of s: for example, (λ_l , a_l)=(0.6574, 0.2519) for $s = 0.25$, and (λ_l, a_l)=(0.7265, 0.3758) for $s = 0.75$. This can be explained by the same intuition as in the case of price staggering: under Calvo type staggering, there are some 'tail' households whose wages are set in the distant past, leading to more sluggish responses of price and output.

V. DISCUSSION

In recent literature focusing on generating real effects of monetary shocks, wage staggering is favored to price staggering in two senses. First, as in Huang and Liu (2002), the former always produces monotone-dampened responses of output and price while the latter necessarily produces damped oscillations. Our finding of $a_y < 0$ and $a_l > 0$ squares with this view. Second, as in Andersen (1998), the responses of output are substantially longer lives under wage staggering than under price staggering. We find that $\lambda_y = 0.4312$ for $(\phi_Y, \theta_Y, s) = (\frac{1}{2}, 0.9, \frac{1}{2})$ and $\lambda_l = 0.7035$ for $(\phi_L, \theta_L, s) = (\frac{1}{2}, 0.9, \frac{1}{2})$, which is also consistent

[4] To make two staggering schemes comparable, we set $\phi_L = \frac{1}{2}$ so that the *average* duration of new wage is 2 under Calvo type staggering.

with this view.

Our finding that both λ_l and a_l are positive, however, provides another virtue of wage staggering: the qualitative implications of wage stickiness are *robust* to the choice of staggering schemes. If nominal stickiness is introduced in the model via price, the responses of the economy to monetary shocks are very sensitive to which staggering scheme to assume. This implies that results of estimating key parameters or welfare levels in the context of sticky-price business cycle models may also be highly sensitive to the choice of staggering scheme. That being the case, it is likely that the comparison of results from models using different price staggering schemes is genuinely nonsensical. Generating qualitatively identical responses of the variables in models, on the other hand, sticky-wage models are relatively free from the issue of what staggering scheme to choose.[5]

We still need to explain why output and price show hump shaped responses under Taylor type wage staggering, while their responses are dampened oscillations under Taylor type price staggering. In section III, we resort to Kiley (2002) to argue that the absence of 'tail' prices under Taylor type price staggering is the reason why the responses of price and output are dampened oscillations. In fact, the absence of 'tail' prices under Taylor type price staggering allows the cohort of firms adjusting first following monetary expansion to raise their prices *too much*, rendering the next cohort of firms to *lower* their optimal prices in the next period.[6] When wage-setting decisions are staggered, the same intuition applies as well, rendering λ_l (under Calvo staggering) larger than a_l. One critical difference here is that a_l is positive, which implies that the first cohort of household adjusting wages following monetary expansion raise their wages *moderately*. We discuss why it is the case below.[7]

Under staggered wage setting, the optimal nominal wage of adjusting cohort is so determined to balance the expected marginal utility of leisure and marginal utility of wage income. If there occurs an expansionary

[5] Another dimension of robustness check by an anonymous referee is to see if the oscillatory (or hump-shaped) responses of output in the case of Taylor-type price (or wage) stickiness are robust to N, the number of periods which price (or wage) is fixed. Our conclusion is that the qualitative features are preserved even if we consider the case of N=4, as discussed in the appendix.

[6] This can be seen from equation (3.17).

[7] Our discussion is in a large part based on Huang and Liu (2002).

monetary shock, the increase in real aggregate demand raises both households' income and the demand for their individual labor services. The higher income reduces the households' marginal utility of income and the higher labor demand raises their marginal utility of leisure. Utility maximization requires that households who can renew contracts raise wages to re-balance their marginal utility of income and of leisure. Wage decisions being staggered, however, an increase in a household's nominal wage leads to an increase in its relative wage and a higher relative wage reduces both the demand for the corresponding type of labor services and the associated wage income. Therefore, the marginal utility of leisure will decrease and the marginal utility of income will increase, both of which serve to restore the balance between the marginal utility of income and of leisure. That being the case, the increase in relative wages of the first movers tends to be small.

It is worth noting that, in the case of Taylor type price staggering, there is no mechanism that can introduce 'substitution' and 'income' effects as above. In other words, the real marginal cost being exogenous to each firm, promotes no incentive for the first movers to moderate the increase in their relative (and absolute as well) prices, which renders *excessive* adjustment in prices.

VI. CONCLUSION

In this paper, we compare two types of staggering schemes, Taylor type and Calvo type, in terms of their implications on the behavior of aggregate price and output. In a staggered price setting, Calvo type staggering implies positive autocorrelations (i.e., monotone dampening) in price and output, while Taylor type staggering implies negative autocorrelations (i.e., oscillatory dampening). In staggered wage setting, however, it is found that both types of staggering schemes can generate monotonically dampening responses. Therefore, the qualitative implications of wage stickiness are *robust* to the choice of staggering schemes.

APPENDIX

In this appendix, we check if the oscillatory (when aggregate price is sticky) and hump-shaped (when aggregate wage is sticky) responses of output exhibited in sections III and IV under Taylor type staggering scheme are robust to the length of price or wage fixity. For this aim, we extend the model of Chari et al. (2000), so that both prices and wages set by individual firms and households, respectively, are fixed for 4 periods. In doing so, we take all parameters and functional forms from Chari et al. (2000), except that i) individual labor service provided by each household is not a perfect substitute with the elasticity of substitution θ_L being 0.9 as in the good market, ii) reasonable degrees of capital adjustment costs are imposed to prevent too high volatility in investment, and iii) we fix $\sigma = 1$ in the instantaneous utility function of CKM:

$$u(C, 1-L, \frac{M}{P}) = \frac{\left[[C^{\nu} + b(\frac{M}{P})^{\nu}]^{\frac{1}{\nu}} [1-L]^{\psi} \right]^{1-\sigma}}{1-\sigma}.$$

Figure A1 displays the impulse responses of some key variables, which closely replicate the results of CKM. In panel (a), output rises initially as a result of monetary expansion, but there is no endogenous persistence: in the fourth quarter after the shock when all cohorts of households and firms have been able to adjust their wages and prices, output is below normal. Hence, even in the presence of wage stickiness as well (which are expected to enhance the persistence in output), deterministic ally staggered nominal contracts generate oscillatory rather than monotonous dampening of output responses.

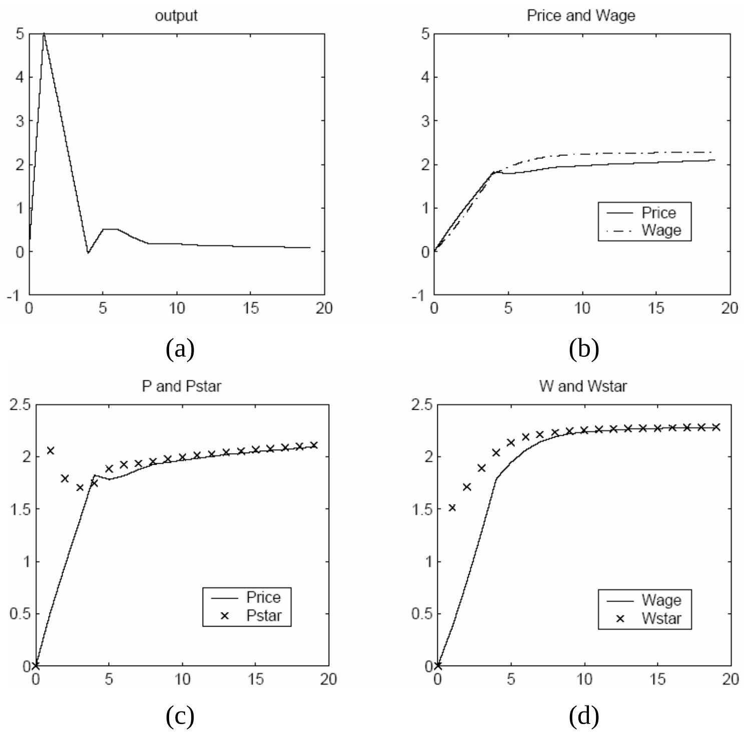
In panel (d), we plot two wage indices together, one for the aggregate wage rate (W) and the other for the wage rate set each period (W^*). The plots show the typical “catch-up” pattern: the aggregate wage rate follows the newly set wages with some lags.

When it comes to the price indices, however, panel (c) shows that the prices reset every period (P^*) display an overshooting and oscillatory pattern - jump up in the price of the first cohort, followed by a considerable decrease in those of the second and third cohorts, and ensuing recovery by the fourth cohorts. As a result, the relative magnitude of P^* with respect to P is reversed in the third period after the shock.

Taylor (1980) gives an intuition for a staggered contract: the idea behind staggered price (or wage) contracts is that smoothed-out adjustment of aggregate price (or wage) will be achieved when firms (or households) look both forward and backward in time to see what other firms (or households) charge during their own contract period, and this causes shocks to be passed on from one contract to another. But the plots in (c) and (d) show this intuition works in the labor market only.

Although the responses of output in Figure A1 show oscillatory dampening, most likely due to the anomalous behavior of P^* and P, we deduce that the general message from Figure A1 is as follows: with higher degrees of Taylor type price and wage fixity, staggered wage contracts can generate sluggish adjustments in aggregate wage and output, while staggered price contracts cannot.

[Figure A1] Impulse Responses under Taylor-type Price and Wage Contracts



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