NETWORK BENEFIT FUNCTION
AND INDIRECT NETWORK EXTERNALITIES

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In most theoretical and empirical studies of network externalities, the consumer's benefit from network externalities is represented by the network benefit function which is a function of the number of users (the network size) and is well taken as intuitive in the case of direct network externalities. In this paper, we explicitly model the mechanism of indirect network externalities as a positive feedback effect of the demands for a hardware product via an increased variety of available software, providing conditions in which the network benefit function can be derived in the presence of indirect network externalities. Moreover, it will be shown that a linear network benefit function can be obtained under a specific utility function of a hardware/software system.

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I. INTRODUCTION

Network externalities can be understood as positive consumption externalities. In the presence of network externalities, an increased number of users of a product raises the consumer's utility level and hence the demands for that product. In the literature on network externalities, the number of users is called a network size, and the user's benefit from the network size is called a network benefit.\(^1\)

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\(^1\) For surveys of network externalities, see David and Greenstein (1990) and Katz and Shapiro
Network externalities are categorized as either direct or indirect. In the presence of direct network externalities, the benefit to each user increases directly with the network size. A good example is a communications network, such as the network of e-mail users, the network of fax machines, or the network of people who exchange MS-Word files. Indirect network externalities are caused by the linkage of some strongly complementary products, whose collection is called a hardware/software system. The bundle of a PC (such as a MS-Windows-equipped PC or Macintosh) and various applications software programs is an example of this traditional hardware/software paradigm of indirect network externalities. When people choose between MS-Windows-equipped PC and Macintosh, they consider not only prices and product characteristics but also the variety of available applications software. Software programmers will make more applications software products available for a PC if more people use that particular PC. Hence, an increased demand for a PC will increase the variety of available software, which in return raises demands for that PC. These positive feedback effects are called indirect network externalities. Hence, in the presence of indirect network externalities, a consumer purchasing a hardware product raises the value of this hardware product for the other consumers indirectly since an increase in demands for the hardware product (e.g., a PC) typically induces a greater variety of available software products (e.g., applications software programs). In this sense, indirect network externalities are understood as positive consumption externalities.

Indirect network externalities are significant in many industries, such as the computer industry, the broadcasting industry and some consumer electronics industries.

The consumer's benefit from network externalities, therefore, is typically represented by a function of the number of users (the network size) as a part of the consumer's utility function. See, for example, Katz and Shapiro (1985, 1986), Farrell and Saloner (1986), and Park (2004, 2005). This function is called

(1994).

2 A system of components without network externalities differs from a hardware/software system in that the amount and the number of components of the system are technically fixed (see Matutes and Regibeau, 1988; and Economides, 1989).

3 Some researchers consider indirect network externalities as positive production externalities (Liebowitz and Margolis, 1995). In this case, an increase in demands for a hardware product causes a decline in the prices of software products and thus input costs of that hardware product.

4 Examples include home video game systems and video game cartridges, DVD players and DVD movie titles, HDTV sets and HDTV programs, and 3G mobile phones and applications programs.
the network benefit function in the literature of network externalities. The idea of the network benefit function has been well taken as intuitive in the case of direct network externalities. In the presence of indirect network externalities, however, the consumer’s utility depends on a variety of software products which are related to the number of users. In this paper, we provide a set of conditions in which the network benefit function is obtained in the mechanism of the positive feedback effect between the demands for a hardware product and a variety of available software. Furthermore, it will be shown that a linear network benefit function can be obtained under a specific utility function.

The set of conditions provided in the paper will indicate that increasing returns to scale in the software products industry is not sufficient for the network benefit to increase with the network size. In other words, the claim in Chou and Shy (1990) is not complete. Indeed, some restrictions on the consumer’s utility function are necessary to guarantee that more users of a hardware product will induce a greater variety of available software. In addition, it will be shown that the existence of independent software producers plays a key role to generate a functional relationship of network benefit and a network size in the presence of indirect network externalities.

The major gain of the structural network benefit function is that it enables us to analyze the welfare consequences of the coordination between hardware and software producers as in Park (2005). The previous studies of the hardware producer’s control over software provisions have usually postulated the consumer’s network benefit as an increasing (and concave) function of the variety of available software (see Chou and Shy, 1996; Church and Gandal, 1992a, 1992b, 1993, 1996). However, this specification cannot fully reflect that the hardware producer can benefit further from the positive feedback effects induced by independent software producers’ provision of a greater variety of available software. Indeed, Kende (1998) showed that due to indirect network externalities, the increased profits in the hardware product market induced by a greater variety supplied under the open system (or in the existence of independent software producers in our term) can dominate the foregone monopoly profits in available software products markets. Park (2005) went further to show that the hardware producer even has an incentive to subsidize software producers to further internalize this indirect network externality under

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5 The integration between hardware and software producers was a major issue in the historical antitrust case of the Microsoft (See Economides, 2000).
the open system. The mechanism of the indirect network externality in the paper enables us to derive both a hardware producer's marginal benefit from subsidization and a subsidy function necessary to compensate for software producers' losses from producing an excessive variety of software products.

The paper is organized as follows. Section II posits a two-stage model in the presence of indirect network externalities. Section III derives a network benefit function based on this model. Section IV concludes the paper.

II. THE MODEL

We posit a two-stage framework for the linkage between hardware and software products markets: in the first stage, the consumer chooses one of the incompatible hardware products, say \( j = 1, \ldots, J \), or the outside alternative, say \( j = 0 \); and in the second stage, the consumer who has a hardware product purchases available software products, and each software maker produces a differentiated software product with the same constant marginal cost and the same fixed cost in a monopolistically competitive market with free entry. Note that we impose no restriction on the market structure of hardware products. Note also that since software production is characterized by a constant marginal cost with a fixed cost, the software industry has increasing returns to scale as in Chou and Shy (1990). We also assume the independence between hardware and software producers.

Under this two-stage framework, we aim to model the mechanism of indirect network externalities as a positive feedback effect between the demands for a hardware product and a variety of available software. To reflect this positive feedback effect, we begin by specifying the consumer's utility function for a hardware/software system (the bundle of a hardware product and available software products) as the sum of the hardware product's stand-alone benefit and the utility from the consumption of software products. In what follows, let \( V_{ij} \)

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6 Microsoft has also engaged in explicit subsidization to independent software vendors for the development of applications software for MS Windows. One of Microsoft's rebuttals to the applications barrier to entry at the trial was that the huge number of applications software programs for MS Windows was partly due to the effort Microsoft had made. R. L. Schmalensee testified in favor of Microsoft: "Microsoft spent $630 million in fiscal year 1998 to help software developers write applications for MS Windows family of operating system products".

7 The outside alternative is a composite good other than the products (the hardware and software products in this case) under consideration.
denote consumer $i$'s stand-alone benefit of hardware product $j$; $B_j$ denote the expected network size of hardware product $j$ in the first stage; $p_j$ denote the price of hardware product $j$; $y_i$ denote consumer $i$'s income; $z_{ik}$ denote the quantity consumed by consumer $i$ of software product $k$ available for hardware product $j$ (for notational simplicity, we let $z_{ik}$ abbreviate $z_{ik(j)}$); $K_j$ denote the variety (or the number) of software products available for hardware product $j$; and $q_{ij}$ denote the quantity consumed by consumer $i$ of the outside alternative.

ASSUMPTION 1: The expected utility level of consumer $i$ for hardware/software system $j$ in the first stage is given by the scalar value:

$$U_i = V_i + E[\xi(Z_{ij}) + q_{ij} | I_1],$$

where $Z_{ij} = (\sum_{k=1}^{K_j} z_{ik}^{1/\beta})^\beta$ with $\beta \geq 1$; $f$ is a real function such that $g = f' > 0$ and $g' = f'' < 0$; and $I_1$ is the information set available in the first stage.

The information set $I_1$ includes $(B_j, p_j, y_i)$ and the marginal and fixed costs of software production.

In the literature of a multi-stage budgeting procedure, $Z_{ij}$ is called the quantity index (Green, 1964). The utility function for various available software products and the outside alternative, $\xi(Z_{ij}) + q_{ij}$, in the second stage (we will call it the second-stage utility function) is specified as a (special form of the) two-sector utility function in Dixit and Stiglitz (1977) in order to reflect the consumer's preference for the variety of available software products. "$\beta > 1$" guarantees concavity of the utility function in the number of software products, and is sufficient to ensure that the marginal rate of substitution between $q_{ij}$ and $Z_{ij}$ is decreasing.

In order to reflect indirect network externalities as positive consumption externalities, we need to impose a restriction on the functional form of $f$ as follows.

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8 $V_i$ may be treated as a function of the attributes of product $j$ and consumer $i$'s idiosyncratic taste for product $j$ as in Park (2004) in the case of differentiated hardware products.
ASSUMPTION 2: \[-\frac{\partial Z_{ij}}{\partial Y} Y^{\beta-1} < g^{-1}(Y)\], where \(Z_{ij} = g^{-1}(Y)\).

Note that since \(g\) is a decreasing function, \(g^{-1}\) exists and is also a decreasing function. Since \(Y = g(Z_{ij})\) is the marginal utility of \(Z_{ij}\), \(Y\) is proportional to the price of \(Z_{ij}\). Hence the left-hand side of Assumption 2 means the elasticity of the quantity index of software products. The right-hand side of Assumption 2 is, on the other hand, the elasticity of demand for each software product (see equation (3) in section III). Hence, Assumption 2 implies that elasticity of the quantity index \(Z_{ij}\) is less than that of each individual software products.

As discussed above, the intuition of indirect network externalities is that the consumer prefers a greater variety of available software and more users of a hardware product induce a greater variety of available software. The second-stage utility function satisfying Assumption 1 and Assumption 2 will imply that the optimal consumption level of \(Z_{ij}\) increases with a variety of available software but the optimal consumption level of each software product, \(z_{ij}\), decreases with this variety.\(^9\) Hence, a shift of the demand for software products induced by increased network size will cause more entries of differentiated software products with less consumptions of the each software product but with more overall consumptions of the software products in equilibrium.

Note that Assumptions 1 and 2 indicate that increasing returns to scale (due to a fixed cost and a constant marginal production cost) of the software production alone is not sufficient to obtain the network benefit as an increasing function of the network size in the presence of indirect network externalities.

III. NETWORK BENEFIT FUNCTION

We will assume that consumers form rational expectations for network sizes of hardware products. Since there is no uncertainty in our model, the expected network size, \(B_i\), of hardware product \(j\) in the first stage will be the equilibrium network size. To obtain a network benefit function, we begin by solving the expected second-stage utility function in Assumption 1. In the second stage, consumer \(i\) who purchased hardware product \(j\) splits the remainder of

\(^9\) See (5) and (9) in section III and (A3) and (A4) in Appendix A.
his/her income between available software products, \( z_{ik} \), and the outside alternative, \( q_B \), to maximize:

\[
f(Z_{ij}) + q_B \quad \text{subject to} \quad q_B + \sum_{k=1}^{K_j} \rho_k z_{ik} = y_i - p_j,
\]

(1)

where \( \rho_k \) denotes the price of software product \( k \). Recall that \( y_i \) is consumer \( i \)'s income and \( p_j \) denotes the price of hardware product \( j \).

Following a two-stage budgeting procedure as in Dixit and Stiglitz (1977), we derive a consumer's demand function for an available software product as follows:

\[
z_{ik} = g^{-1}(P_j)(P_j/\rho_k)^{\beta/(\beta-1)}, \quad \text{where} \quad P_j = (\sum_{k=1}^{K_j} \rho_k^{1/(\beta-1)})^{-(\beta-1)}.
\]

(2)

In the literature, \( P_j \) is called the price index (Green, 1964). A consumer’s demand function for an available software product \( z_{ik} \) in (2) does not depend on the consumer’s income level since the utility function is specified as linear in the outside alternative. In other words, this specification of utility function may be justified if income effect on the demand for an available software product is negligible (see Spence, 1976). A change in \( \rho_k \) alone affects \( z_{ik} \) directly, and also through \( P_j \). As in Dixit and Stiglitz (1977), we assume that \( K_j \) is reasonably large and accordingly neglect the effect of each \( \rho_k \) on \( P_j \).

This leaves us with the price elasticity of demand for software product \( k \):

\[
- (\frac{\partial z_{ik}}{\partial \rho_k})/(z_{ik}/\rho_k) = \beta/(\beta-1).
\]

(3)

Each software maker is assumed to produce a differentiated software product with the same marginal cost, say \( s \), and the same fixed cost, say \( F \). Then, the first order condition of the software producer's profit maximization leads to:

\[
\rho_k = \beta s \equiv \rho, \quad \text{for all} \quad k.
\]

(4)

Substituting the prices of software products in (4) into the consumer's demand function in (2) yields:

\[\text{For example, there are 70,000 or so applications software programs that now run on MS Windows (see “Microsoft and the future,” The Economist, November 13, 1999).}\]
\[ z_{ik} = g^{-1}(\rho K_j^{-(\beta-1)})K_j^{-\beta}, \text{ for } k = 1, \ldots, K_j. \]  

Substituting the consumer's demand functions for software products in (5) into the budget constraint in (1), we derive a consumer's demand function for the outside alternative as follows:

\[ q_k = y_i - p_j \rho g^{-1}(\rho K_j^{-(\beta-1)})K_j^{1(\beta-1)}. \]  

Substituting both the consumer's demand functions for available software products in (5) and the consumer's demand function for the outside alternative in (6) into the second-stage utility function in (1), we derive the following indirect utility function:

\[ \max \{ f(Z_{i0}) + q_k \} = y_i p_j + x^{\star}(K_j), \]  

where

\[ x^{\star}(K_j) = -\rho g^{-1}(\rho K_j^{-(\beta-1)})K_j^{1-(\beta-1)} + f(g^{-1}(\rho K_j^{-(\beta-1)})). \]  

Due to the utility function specification in Assumption 1, \( x^{\star}(\cdot) \) is increasing in \( K_j \). In other words, a greater variety of available software raises the consumer's utility level (see (A3) in Appendix A for the proof).

A variety of available software, \( K_j \), however, is endogenously determined by the zero-profit condition imposed by free entry of independent software producers. The zero-profit condition implies: \( (\rho - s)z_k - F = 0 \), where \( z_k = \sum_{i=1}^{B_j} Z_{ik} \). Using the consumer's demand function in (5), we can rewrite the zero-profit condition as follows:

\[ \frac{K_j^\beta}{g^{-1}(\rho K_j^{-(\beta-1)})} = B_j \frac{s(\beta-1)}{F}. \]  

As discussed in section II, Assumption 2 implies that an increase in network size, \( B_j \), raises the variety of available software, \( K_j \) (see (A4) in Appendix A for the proof). Hence, the equilibrium variety in (9) can be solved in such a way that \( K_j = h(B_j) \), where \( h \) is an increasing function. Since \( x^{\star} \) in (8) is
also an increasing function, we can define an increasing function \( x \) as follows:

\[
x(B_j) = x^*(h(B_j)).
\]  

We call \( x(B_j) \) the network benefit function in the paper. Note that the existence of independent software producers and the zero-profit condition of (9) play a key role to derive a functional relationship between the equilibrium variety and the network size and thus a functional relationship between the network benefit and the network size in the presence of indirect network externalities.

Hence, for given available information including \( (B_j, p_j, y_i) \), in the first stage, consumer \( i \) will expect the utility level for hardware/software system \( j \) to be:

\[
U_{ij} = V_{ij} + E[f(Z_{ij}) + q_{ij} \mid I_i] = V_{ij} + y_i - p_j + x(B_j).
\]  

Then the consumer will choose a hardware product to maximize his/her expected utility in (11) among incompatible hardware products (and the outside alternative). The expected utility level for a hardware/software system obtained in (11) is essentially identical to the utility functions assumed in Katz and Shapiro (1985; 1986) and Farrell and Saloner (1986). Hence, the conclusions drawn in Katz and Shapiro (1985; 1986) and Farrell and Saloner (1986) can be applied to indirect network externalities as well.

Furthermore, in many theoretical and empirical studies of network externalities, a linear network benefit function has been specified to facilitate not only analytical solutions of static models but also dynamic modeling of network externalities (see Farrell and Saloner, 1986; Katz and Shapiro, 1992; Park, 2004, 2005). The network benefit function \( x(\cdot) \) will be linear in \( B_j \), if \( f(Z_{ij}) \) in Assumptions 1 and 2 has a specific form as follows.

PROPOSITION: Suppose that \( f(Z_{ij}) = Z_{ij}^{1/(2\beta-1)} \). Then

(i) \( z_{ik} = (2\beta - 1) \beta s \) \( -(2\beta-1)/(2\beta-1) \) \( K_j^{-1/2} \),

(ii) \( K_j = B_j^2 \left( \frac{2(\beta-1)}{F} \right)^2 \) \( (2\beta - 1) \beta s \) \( -(2\beta-1)/(\beta-1) \), and

(iii) \( x(B_j) = xB_j \) with \( x > 0 \).
Refer to Appendix B for the proof of Proposition. Proposition-(i) implies that the quantities consumed of each software product decrease in the variety, but the aggregate consumption level of available software products increases in the variety. Proposition-(ii) indicates that even in the case that the network benefit is linear in the network size, the network benefit is concave in the variety of available software. Note also that the coefficient, $x$, of the linear network benefit function in Proposition-(iii) depends on the software producer's marginal and fixed costs as well as the parameter of the consumer's utility function (see Appendix B).

IV. CONCLUDING REMARKS

We have explicitly modeled the mechanism of indirect network externalities as positive consumption externalities and derived the network benefit function as typically posited in the literature. A specific two-sector utility function and the monopolistic competition of independent software producers are sufficient conditions to derive a functional relationship between the network benefit and the network size in the presence of indirect network externalities.
From the definition of the quantity index, $Z_u = (\sum_{k=1}^{K_i} z_{ik}^{1/\beta})^\beta$, and the consumer’s demand functions for available software products in (5), we have:

$$Z_u = g^{-1}(\rho K_j^{-(\beta-1)}).$$  \hspace{1cm} (A1)

Let $Y = g(Z_u)$. Assumption 1 posits: $\partial g^{-1}/\partial Y < 0$ and $\beta > 1$. Hence,

$$\frac{\partial Z_u}{\partial K_j} = -\frac{\partial g^{-1}}{\partial Y} \rho(\beta - 1) K_j^{-\beta} > 0.$$  \hspace{1cm} (A2)

Using (A1), we can rewrite $x^*(K_j)$ in (8) as:

$$x^*(K_j) = -g(Z_u)Z_u + f(Z_u).$$

Keeping in mind that $g = f'$ in Assumption 1, we obtain:

$$\frac{\partial x^*(K_j)}{\partial K_j} = -g(Z_u) \frac{\partial Z_u}{\partial K_j} - Z_u \frac{\partial g}{\partial Z_u} \frac{\partial Z_u}{\partial K_j} + g(Z_u) \frac{\partial Z_u}{\partial K_j}$$

$$= -Z_u \frac{\partial g}{\partial Z_u} \frac{\partial Z_u}{\partial K_j} > 0$$  \hspace{1cm} (A3)

since $\partial g/\partial Z_u < 0$ in Assumption 1 and $\partial Z_u/\partial K_j > 0$ in (A2).

From the zero-profit condition in (9) and the result in (A1), we have:

$$\frac{\partial}{\partial K_j} \left( \frac{K_i^\beta}{Z_u^\beta} \right) dK_j = \frac{s(\beta-1)}{F} dB_j.$$  \hspace{1cm} (9)

Using the result in (A1), we derive:

$$\frac{\partial}{\partial K_j} \left( \frac{K_i^\beta}{Z_u^\beta} \right) = \frac{K_i^{\beta-1}}{Z_u^\beta} (\beta Z_u - K_j \frac{\partial Z_u}{\partial K_j})$$

$$= \frac{K_i^{\beta-1}}{Z_u^\beta} \left\{ \beta Z_u + (\beta - 1) \frac{\partial g^{-1}(Y)}{\partial Y} g(Z_u) \right\}.$$  \hspace{1cm} (A4)

Hence, (A4) is greater than zero if Assumption 2 holds. Therefore, under Assumptions 1 and 2, we conclude: $dK_j/dB_j > 0$. 

APPENDIX B

We will first show that \( f(Z_\hat{y}) = Z_\hat{y}^{1/(2\beta - 1)} \) satisfies Assumption 2. By simple calculus rules,

\[
Y = g(Z_\hat{y}) = f'(Z_\hat{y}) = (2\beta - 1)^{-1}Z_\hat{y}^{-\frac{2\beta - 1}{2\beta - 1}} , \text{ and} \\
g^{-1}(Y) = Z_\hat{y} = \left\{ (2\beta - 1) Y \right\}^{-\frac{2\beta - 1}{2\beta - 1}}.
\]

(B1)

Then,

\[
\frac{\partial g^{-1}}{\partial Y} = -\frac{1}{2(\beta - 1)} (2\beta - 1)^{-\frac{1}{2\beta - 1}} g(Z_\hat{y})^{-\frac{4\beta - 3}{2\beta - 1}}.
\]

Therefore, Assumption 2 is satisfied if and only if

\[
\frac{1}{2} (2\beta - 1)^{-\frac{1}{2\beta - 1}} g(Z_\hat{y})^{-\frac{4\beta - 3}{2\beta - 1}} < \frac{Z_\hat{y}}{g(Z_\hat{y})}
\]

\[
\iff \frac{1}{2} (2\beta - 1)^{-\frac{1}{2\beta - 1}} g(Z_\hat{y})^{-\frac{2\beta - 1}{2\beta - 1}} < \beta Z_\hat{y}
\]

\[
\iff \frac{1}{2} (2\beta - 1) \left\{ (2\beta - 1) g(Z_\hat{y}) \right\}^{-\frac{2\beta - 1}{2\beta - 1}} < \beta Z_\hat{y}.
\]

From (B1), we know that \( Z_\hat{y} \left( 2\beta - 1 \right) g(Z_\hat{y}) \) \(\sim\) \( Z_\hat{y}^{-\frac{2\beta - 1}{2\beta - 1}} \). Hence, the last inequality is satisfied since \( \beta > 1 \).

Substituting the results in (B1) into the consumer’s demand functions in (5) yields:

\[
z_{ik} = \left\{ (2\beta - 1) \beta \right\}^{-\frac{2\beta - 1}{2\beta - 1}} K_j^{-\frac{1}{2}}.
\]

(B2)

Then the zero profit condition in (9) can be rewritten as follows:

\[
K_j = B_j^2 \left( \frac{s(\beta - 1)}{F} \right)^2 \left\{ (2\beta - 1) \beta \right\} -\frac{2\beta - 1}{\beta - 1}.
\]

(B3)

Substituting the results in (B1) and (B3) into the network benefit function in (10) yields:
\[ x(B_i) = xB_i, \]

where

\[ x = \frac{\beta - 1}{F} (2 + s) (\beta s) ^{\beta/(\beta - 1)} (2\beta - 1) ^{- (2\beta - 1)/(\beta - 1)}. \]

Note that \[ x > 0 \] since \[ \beta > 1. \]
REFERENCES


