EXCHANGE RATES AND PRICING-TO-MARKETS (PTM)

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This paper sets up a model where firms, which sell their products both in domestic and foreign markets, pre-set price in terms of destination country currency, and where firms in the market produce differentiated products. Using Spence-Dixit-Stiglitz (S-D-S) type substitutability function and Green's theorem the paper derives the demand for the firm's brand (domestic and export). The paper shows how PTM results when the changes of the exchange rate are uncertain if firms price their products following Leland (1972)'s concepts of PIU and PDU.

1. INTRODUCTION

To the firm facing exchange risks in international markets advance planning in production, market selection, and pricing strategy is important since, if the firm fixes prices in the firm's domestic currency, export market prices will continuously change as exchange rate changes. As S. Grassman's [1973] study shows, with around two-thirds of the export from the small open economies denominated in the exporter's currencies, the distributor in the importing country has to adjust their profit margins to keep local currency prices constant and to conform to the exporter's prices they pay. If the distributors keep their profit margins constant, then consumers have to face continuous price changes as exchange rate changes. In addition to that, if the exporting firm changes prices frequently and distributors keep profit margins constant, consumers may face big swings in their prices to pay as well as frequent price changes. As Gottfries [1986] shows, the existence of consumer's search cost induces inelastic 'customers flow' adjustment to the change of prices. But large search cost per period by frequent price changes, meanwhile, leads downward shift in the demand curve which the firm faces. So, without price planning in terms of price change interval and the magnitude of

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1 See Barro [1972] for this point.
price changes, customers, especially those who invest in equipments, plants, machinery, and other fixed assets which functions well with the products from the exporting firm, may feel insecure and change the supplier to other exporting or domestic producers. Decisions of firms in period $t$ which maximize the expected utility of profit, with demand as a function of only prices and income, and cost as a function of quantity produced and variable inputs, have no impact on the future profits beyond period $t+1$. Thus, firms maximize expected profits by choosing prices once in every period. When the exchange rate changes are uncertain, with exporter currency denomination and price pre-setting at the end of the period $t-1$, what is uncertain to the exporting firms is the quantity demanded for the current period and hence the nominal revenue in its own currency. In general functional form the firm faces demand $q$ in its export market as follows:

$$q = q(p, e),$$

where $e \in E$ denotes the realized exchange rates between the currencies of the exporting and the importing country. When the home country currency depreciates the firm faces higher demand when the firm pre-set price $p^*$ at the end of the last period. When the home country currency appreciates the firm faces lower demand. Since exchange rate changes are uncertain, and hence the demand is, risk preferences of the firm determine the pricing decision of that firm. The paper consists of 5 sections. This introduction section is followed by the derivation of the demand of the brand $k$ of firm $k$, using Green's 2 stage budgeting procedure, in section II. The causes of PTM in relation with mark-up pricing are discussed in section III. Section IV shows how PTM results when the manager of a firm, facing uncertain exchange rate changes, prices their product using Leland's principles of PIU and PDU. And section V concludes with some discussions on the implications of the results which are shown in the paper.

II. DEMAND FOR DIFFERENTIATED PRODUCTS

Assume a customer consumes in many varieties. Then his utility from the brands of the product of firms in industry $I$, where $m$ foreign and domestic firms produce differentiated products, can be represented by Spence-Dixit-Stiglitz (S-D-S) type subutility function

$$u_i(q_1, \ldots, q_m) = \left\{ \sum_{i=1}^{m} q_i^{g_i} \right\}^{1/g_i},$$

(1)

Shapiro[1989] cites the example of U.S. bicycle manufacturers who were unprepared for the 1971 currency realignment and hence failed to expand its domestic market share through comparatively advantageous pricing latitude. For the example of the need for advance planning, see also Shapiro(p. 332)
where $\beta=\frac{1}{1+1/\sigma}$, and $\sigma$ is the elasticity of substitution. There are $J$ industries of which a customer consumes the products. Assume that the consumer’s total utility $U$ be in the weakly separable form, i.e.,

$$U = U[u(\cdot), \ldots, u(\cdot), \ldots, u(\cdot)]$$

Then, by Green’s theorem, consumer’s utility maximization problem can be solved in two stages. Given the allocation of income $E_i$ for the industry $I$ the demand for variety $k$ of firm $k$ can be solved in the second stage by

$$\max u_i(q_i, \ldots, q_m) = \left\{ \sum_{j=1}^{m} q_j^{\sigma_i} \right\}^{1/\sigma_i},$$

$$\text{s.t. } \sum_{j=1}^{m} p_j q_i \leq E_i$$

From the first stage budgeting problem allocation of income across products$(E_1, E_2, \cdots, E_J)$ is determined. Then the demand function $q_i$ for variety $j$ is:

$$q_j = \frac{p_j^{\sigma_j}}{\sum_{i=1}^{m} p_i^{\sigma_i}} E_i \quad (2)$$

Define the price index for the varieties of the industry $I$ in the importing country as

$$\bar{P} = \left\{ \sum_{j=1}^{m} p_j^{\sigma_j} \right\}^{1/\sigma_i}$$

Then equation (2) can be written as follows:

$$q_j = \frac{p_j^{\sigma_j}}{\bar{P}^{1-\sigma_i}} E_i \quad (3)$$

If a foreign firm $k$ invoices export contract in its home currency with unit price $p_k^*$, then the customer faces $e p_k^*$ in the local (importing country) currency. Here $e$ is the exchange rate between the two country currencies (dollars per unit of foreign currency). The demand for the variety of foreign firm $k$ is:

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3 Green's [1964] theorem 4 states that the necessary and sufficient conditions for the two stage maximization problem are that each quantity index be a function of homogeneous of degree one in its elementary commodities or inputs.
\[ q_e = \frac{(eP^*_e)^{-\epsilon}}{P^{(1-\epsilon)}} E_i \]  

(4)

So, the demand for foreign firm \( k \)'s variety is a function of importing country income, domestic prices, and the exchange rates as well as its own price. Since we study the behavior of a foreign firm that sells its product both at home and abroad the firm's domestic and export market demand can be written as follows:

\[ q_e = \frac{p^{*\epsilon}_{e}}{P^{(1-\epsilon)}} E_i^* \]  

(4a)

\[ q^*_k = \frac{(eP^*_k)^{-\epsilon}}{P^{*\epsilon(1-\epsilon)}} E_2 \]  

(4b)

where \( \overline{P^*} = \left\{ \sum_{i=1}^{n} p^*_{i}^{(1-\epsilon)} \right\} \) is the exporting firm's home price index, \( E_i^* \) is domestic income, \( E_2 \) is the income in the export market and \( \sigma_i, i = 1, 2 \), is the elasticity of substitution between goods in domestic and export markets, respectively.

III. MARK-UP, DEPRECIATION, AND PRICING-TO-MARKETS

Having confirmed that the demand for the variety \( k \) is a function of exchange rates, domestic price level and income, let's denote the domestic and export demand for the variety of the firm by the following general functions:

\[ s(p, \overline{P}, E_i), \quad q(e, p^*_e, \overline{P^*_e}, E_2) \]  

(5)

where \( s(\cdot) \) is domestic, \( q(\cdot) \) is foreign demand. The total cost is a function of total demand and factor costs.

\[ C(q(\cdot) + s(\cdot), w, m) \]  

(6)

Here, \( w \) and \( m \) are wage and raw materials price in the domestic country. Then the foreign exporting firm's domestic currency profit in period \( t \) is

\[ \pi = p_1 s(p, \overline{P}, E_i) + p^*_e q(e, p^*_e, \overline{P^*_e}, E_2) - C(q(\cdot) + s(\cdot), w, m) \]  

(7)

\[ \text{Marston[1989] studies in detail the determinants of the 'exchange rate pass-through' coefficient when the exchange rates depreciate under certainty. This section is a variant of his model with taking into consideration of currency denomination practices, and focuses on PTM with its determinants.} \]
Maximizing the profit function with respect to prices for the two markets gives the first-order conditions:

\[ s(\cdot) + p_i s_i - C_i s_i = 0 \quad \text{(domestic)} \tag{8a} \]

\[ q(\cdot) + p^{*}_i e_i q_i - e_i q_i C_i = 0 \quad \text{(export)} \tag{8b} \]

where \( C_i \) is the common marginal cost of production\(^5\) in period \( t \) and the second-order condition is assumed to be satisfied. Equation (8) can be written as follows:

\[ p_i + \frac{s(\cdot)}{s_i} = C_i \tag{8a} \]

\[ p^{*}_i + \frac{q(\cdot)}{e_i q_i} = C_i \tag{8b} \]

In terms of mark-up rate \( p_i/C_i \) and demand elasticities in two markets,

\[ p_i = C_i \left[ \frac{1}{1 - \frac{1}{e(p)}} \right] \tag{9a} \]

\[ p^{*}_i = C_i \left[ \frac{1}{1 - \frac{1}{e^{*}(e, p^{*})}} \right]. \tag{9b} \]

Notice that the mark-up in each market depends on the price elasticities of demand on those markets respectively and \( e^{*} \) is a function of \( e, p^{*} \). In the case where the number of export markets is large (\( n \)), this condition can be written as:

\[ C_i = MR_i = MR_1 = \cdots = MR_n = \cdots = MR_x, \text{ where } MR_i = \frac{q_i}{e_i} \frac{1}{q_i}. \]

Here, \( e_i \) is the bilateral exchange rates between the exporting country currency and importing country \( i \)'s currency. The equation (9b) can be written as follows:

\[ p_i' \left( 1 - \frac{1}{|e'_i|} \right) = p_i' \left( 1 - \frac{1}{|e'_1|} \right) = \cdots = p^{*}_i \left( 1 - \frac{1}{|e^{*}_i|} \right) \]

\(^5\) We assume the exporting firm produces both exporting and domestic goods in domestic country. So, marginal cost is same to both destination products.
So, the smaller the value of $|e'(e^p)|$, i.e., when the demand is inelastic with respect to price, the higher the price charged in that destination market $i$ will be. Also, since we assume that all production occurs at home here, the price in one market is not independent of the price in the other market.\(^6\) Depreciation of the destination (importing) country currency has the effects on the export and domestic prices of a good through two channels: demand decreases (direct effect) in the destination market and the changes in marginal cost $C_i$ as demand changes. If $C_{ii} \geq 0$ (non-decreasing marginal cost), then the foreign currency depreciation decreases the export prices in terms of exporting country currency $p^*$. If $C_{ii} < 0$ (marginal cost decreasing), then the effect of depreciation on the export price $p^*$ is indeterminate.\(^7\) The effect on home price depends only on the changes in marginal cost. For the usual non-decreasing marginal cost the effect of foreign currency depreciation will be negative (home price will decrease). When the indirect marginal cost effect is negligible, however, the effect of depreciation on export price will always be negative and the effect on domestic price will be none. 'Pricing-to-Market'(PTM) effect of depreciation, termed by Krugman[1987], which is defined as

$$PTM \equiv \frac{p^*}{p}$$

will be positive, i.e., exporting firm will increase the difference between the prices charged in domestic and foreign market.\(^8\)

**IV. PRICING UNDER UNCERTAIN EXCHANGE RATE CHANGES**

Assume that the manager of a firm has an increasing concave utility function $u$, i.e.,

$$u'(\pi) > 0, \quad u''(\pi) < 0$$

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\(^6\) Changes in the price of one market induces changes in the price of other markets via changes in demand and hence changes in the marginal cost. But this link seems to be small if the marginal cost curve is constant or if the firm neglects this indirect effect and changes mark-up rates instead of changing prices for price stability. The rationale for price stability and price planning is given in section I.

\(^7\) The effect depends upon the relative changes in the marginal cost to the magnitude of foreign currency depreciation. If the foreign currency depreciates much relative to the change in marginal cost, then the export price in terms of exporting country currency will decrease. See equation (8b)\(^9\) for the relative effect of marginal cost and exchange rates on the export price.

\(^8\) Here we assume that, initially (before depreciation), there is no PTM. This is the case of the risk-neutral firm, the behavior of which will be discussed later.

\(^9\) Pass-through coefficient is $\frac{d(e^p)}{de^p}$.\(^{10\)}
To concentrate on the effect of pure exchange rate uncertainty on export prices, let's assume domestic and foreign price levels, and domestic and foreign income levels are uncorrelated with the expected exchange rate changes. And also demand elasticities are assumed to be the same in domestic and foreign markets: i.e.,

\[ s_i = e q_i, \]

where \( s(\cdot) \) is domestic and \( q(\cdot) \) is an export demand. \( e \) is destination currency price of a unit of exporter's currency, and subscripts are the partial derivatives with respect to the number-th argument.

Then, with uncertain exchange rates, the firm's problem is to maximize expected utility of profits, i.e.,

\[ E u(\pi) = \int u(\pi) dF \]

where \( e \in E \) denotes the realized value of exchange rates in period \( t \) and \( F \) is the distribution function of \( E \). Differentiating with respect to prices for the two markets gives the first-order conditions:

\[ \int u'(\pi) s(\cdot) + p s_i - s_i C_i | \ dF = 0 \quad \text{(home)} \tag{11a} \]

\[ \int u'(\pi) q(\cdot) + p^* e q_i - e C_i q_i | \ dF = 0 \quad \text{(export)} \tag{11b} \]

where \( C_i \) in each market is the common marginal cost of production.\(^{10}\) Equation (11) can be written as

\[ \int u'(\pi) s(\cdot) + p s_i | \ dF = \int u'(\pi) s_i C_i | \ dF \quad \text{(home)} \]

\[ \int u'(\pi) q(\cdot) + p^* e q_i | \ dF = \int u'(\pi) e q_i C_i | \ dF \quad \text{(export)} \]

Or equivalently,

\[ E u'(\pi) (p^* e q_i + q(\cdot)) | = E u'(\pi) e q_i C_i \tag{12} \]

\(^{10}\) Giovannini [1988] shows with his empirical results that this restriction do not affect the conclusion.

\(^{11}\) Here, \( s_i C_i \) and \( e q_i C_i \) are total cost effects by the change in prices in each market. Aspects of exporting firm's long-run adjustments under volatile exchange rate changes, e.g. firm's strategic production management such as input-mix, selection of plant locations are not considered here.
Since Cov(x, y) = Exy – ExEy, we have

\[
Eu'(\pi) E(p^*eq_i + q(\cdot)) + Cov(u', p^*eq_i + q(\cdot))
\]
\[
= Eu'(\pi) E(eC,q_i) + Cov(u', eC,q_i)
\]

(13)

where \( mr = q + p^*eq_i \) is the marginal revenue from the sale to the export market and \( mc = eC,q_i \) is marginal cost. Using the same procedure as above we can finally get the following result\(^{17}\):

\[
E(mr) = E(mc) - \frac{Cov(u', mr - mc)}{Eu'(\pi)}
\]

(14)

Equation (14) shows that the firm sets prices such that the expected marginal revenue equals marginal cost plus some exchange risk premium. To sign this (marginal) risk premium we assume that the riskiness of the firm's profits increases with the activity of the firm.\(^{11}\) Here, for the price setting firm, it should follow that \( q_i = p^*q_i > 0 \) and \( q_{i'} = q_{i'} > 0 \) with given \( p \). Notice that

\[
\pi_i = \frac{\partial \pi_i}{\partial e} = p^*q_i + p^* - q_i + p^*C_i
\]
\[
= q_i p^*(p^* - C_i)
\]
\[
= q_i (p^* - C_i) > 0
\]

if \( p^* > C_i \), which should hold when firms have some degree of market power either because of the differentiated products or of monopolistic market structure, and

\[
\pi_{i'} = \frac{\partial \pi_{i'}}{\partial p^*} = q_{i'} + p^*q_{i'} - q_{i'} + C_i q_{i'} > 0
\]

if \( p^* > C_i \) and \( C_{i'} > 0 \); and PIU holds. Thus, the increase in price increases the

\(^{17}\) See Appendix for derivation.

\(^{11}\) Leland[1972] introduces the notions of the Principle of Increasing(Decreasing) Uncertainty(PIU, PDU). There he assumes that when the profits are a function of prices and random factor \( u \), i.e., \( \pi(p, u, \pi_i > O(\cdot)) \) and \( \pi_{i'} > O(\cdot) \) for PIU (and \( \pi_i > O(\cdot) \) and \( \pi_{i'} > O(\cdot) \) for PDU) should be satisfied for the above assumption. See MacMinn and Holtman[1983] for an interpretation of PIU and PDU.

\(^{11}\) Strictly, \( C_{i'} \) should be in range of \( 0 < C_{i'} < \frac{1}{q_i} \).
riskieness of the firm's profit distribution. If the firm is risk neutral, then the equation (12) can be written as

\[ E \left( p^* + \frac{q(\cdot)}{eq_i} \right) = E[C_i] \]  

(15)

which states that expected marginal revenue equals expected marginal cost. Since

\[ \frac{\partial u'(\pi)}{\partial e} = u'(\pi) \pi \]
\[ = u'(\pi) q(p^* - C_i) < 0 \]

and \( \frac{\partial (mr - mc)}{\partial e} = \pi_{\rho_s} = \pi_{\rho_s} > 0 \), the covariance is negative. That means an imperfectly competitive market structure and the non-decreasing marginal cost are enough to have positive exchange risk premium. Thus, price is selected so that the expected marginal revenue is larger than marginal cost. And the firm with PIU(PDU) will charge a higher(lower) price and a lower(higher) output than that of a risk neutral firm. Therefore pricing to foreign market results.

\[ \text{V. CONCLUSIONS} \]

What is noteworthy during the dollar cycle in the 1980s is that the trade balance of major trading countries such as Japan, Germany and the U.S. showed different behaviors. Especially, the Japanese trade surplus with the U.S. did not decrease when the dollar depreciates sharply against the Japanese yen after the first quarter of 1985('inertia' results). This paper explains these different responses of trade balances of major trading countries using the usual expected utility maximization framework where the manager of an exporting firm having increasing concave utility with respect to profits. The paper derives explicitly the demand for the brand of the exporting firm's product, where firms in the market produce

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15 The increase in the riskieness of profit distribution means the increase in the risk even after correcting for the change in the mean profits through changes in the price.
16 Most empirical studies show that the range of decreasing marginal cost attains early in the small level of output. Except in the natural monopoly case such as electricity and long-distance telephone lines this is the usual case. Recent development of fiber-optics technology make large investment unnecessary even in the long-distance lines.
17 Baron[1976] cannot get the result of the effect of risk aversion on the optimal price decision. With linear demand curve he concluded that the optimal price with risk aversion is less than that of a risk neutral firm. So the firm with PIU contradicts Baron.
18 PTM \( = \frac{\partial}{p} \) \( 1 \) since, in the domestic market marginal revenue does not change as exchange rate changes, covariance term will vanish.
differentiated products, using Spence-Dixit-Stiglitz(S-D-S) type subutility function and Green's theorem. Having the common non-decreasing marginal costs between the production of the product selling in domestic and foreign markets, the firm with PIU will charge a higher price and a lower output with the uncertain exchange rate changes. Whereas the firm with PDU will charge a lower price and a higher output when the firm faces uncertain exchange rates. Therefore the different behavior in the trade balances between countries can be explained, with some caveats in terms of aggregation problem, by the direction in which the riskiness of profit changes with the price the firm charges. That is, pricing to market(PTM) behavior is determined by the market power which the firms of an exporting country command in the export market. Thus the exchange rate uncertainty has the same effect on the exporting firm with PIU(PDU) as the appreciation(depreciation) of the import country currency under certainty.

APPENDIX

\[ EU'(\pi) = EU'(\pi)E(eC,q_i) + Cov(U', eC,q_i) - Cov(U', p^*eq_i + q) \]
\[ = EU'(\pi)E(eC,q_i) + Cov(U', eC,q_i - p^*eq_i - q) \]

Since \( Cov(x, y) + Cov(x, z) = Cov(x, y + z) \), we have

\[ E(p^*eq_i + q) = E(eC,q_i) + \frac{Cov(U', mc - mr)}{EU'(\pi)} \]
\[ = E(mc) - \frac{Cov(U', mr - mc)}{EU'(\pi)} \]

where \( mr = q + p^*eq_i \) and \( mc = eC,q_i \).
REFERENCES