THE KOREAN ECONOMIC REVIEW
Volume 6, Number 1, Summer 1990.

THE WELFARE COST OF SOCIAL SECURITY DUE TO
CHANGES IN PRIVATE SAVING:
The Case of Capital Income Taxation

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It is shown that under the existence of capital income taxation, the welfare cost of social security would be occurred more than the case without capital income taxation if part of the change in tax revenue by social security has been used for investments instead of consumption expenditures. When we look at only the side of welfare cost, it is better to abandon to implement social security if the capital stock for the public is not formulated so that the rate of return on government investment is high.

1. INTRODUCTION

The purpose of this paper is to examine the welfare cost of social security via observing the change in private savings. If the combination of social security tax and social security benefit has no income effect, then, social security tax reduces savings by enough to leave consumption unchanged during the retirement period. We call the substitution of savings by social security tax the wealth replacement effect of social security. Besides the wealth replacement effect, we can consider the induced retirement effect and the bequest effect. However, this paper will analyze only the wealth replacement effect of social security which affects the private savings.

Studies aimed at determining the welfare cost of social security that results from a change in private savings have been rarely undertaken. The most recent analysis dealing with this topic was done by Martin Feldstein (1987). However, this paper was restricted to analysis of the wealth replacement effect without capital income taxation.

*Department of Economics, Kon-Kuk University. This paper is originated from a part of my Ph.D. dissertation at Texas A & M University. I would like to thank Edgar K. Browing, Timothy J. Gronberg, Dann R. Deere, and the members of the Public Finance Workshop at Texas A & M for their helpful comments.

1The set of programs popularly known as “social security” actually has a multiple purpose designation; Old age, Survivors, Disability, and Health Insurance, or OASDHI for short. In this paper the concept of social security will be limited to social retirement pension. The change in saving induced by social security implies a change in the welfare cost.
Feldstein (1987) obtained the welfare cost of social security by summing up the present values of welfare cost which each generation bore, when it was assumed that the social security program would be on going forever. The present paper limits the analysis to obtaining only one generation’s welfare cost that is not belonging to both of the start-up and the ending generations covered by the program. To expand the span of the analysis can be left for the next step of this paper.

In expanding upon Feldstein’s (1987) analysis, this paper will emphasize how the welfare cost of social security is related to the existence of capital income taxation. In the second section, we measure the wealth replacement effect of social security on social welfare under the assumption that there is no capital income taxation. The third section of this paper consists of examining how the welfare cost is affected by social security when there is capital income taxation. The fourth section will show illustrative numerical calculations of the welfare cost of social security, and will examine how the welfare cost is changed by varying the capital income tax rate under the given related parameters. In concluding, we will briefly discuss the potential for extending this paper into future studies.

II. THE WELFARE COST WITHOUT TAXATION OF CAPITAL INCOME

In this section, the welfare cost of social security will be examined for the case of non-capital income taxation. First of all, we assume that each individual lives for two periods, the first period being the working life of individuals and the second period being the retired life of individuals. All individuals are assumed to be identical and to earn a wage rate \( W_t \) if they work in the \( t \) period. The labor force grows at rate \( n \) per period, and the real wage rate grows at rate \( g \) per period.\(^2\)

If the social security tax substitutes private savings for investment in real capital, there may be an increase in the marginal product of capital by the reduction of capital stock according to the classical production theory. This also leads to less capital per worker and lower productivity per worker, which reduces wage rates. However, we assume that the marginal product of capital and the wage rate remain constant regardless of the social security program. If individuals save and invest the social security taxes, their savings will earn the marginal product of capital, \( p \). If there were no tax on capital income, then the individuals would also receive a rate of return of \( p \).

The number of individuals retiring in each period \( = A_t \) is assumed to be equal to the number of workers in the previous period \( = L_{t-1} \)

\(^2\)Paul Samuelson (1958) used this kind of framework to analyze the effect of social security by developing an explicit overlapping generations model. Samuelson assumed an economy without productive capital stock or other durable store of value. Feldstein (1987) expanded the economy to include productive capital. Feldstein assumed that the marginal product of capital remained constant at rate \( p \) per period, however, to avoid the complexities of an endogenous and time varying rate of return. See Martin Feldstein (1987), pp. 2-3.
WON SHIK KIM: WELFARE COST OF SOCIAL SECURITY

(1) \[ L^t = (1 + n) L_{t-1} \text{ and } L_t = (1 + n)A_t. \]

Assume that a pay-as-you-go social security system imposes a tax at rate \( q \) on wage income in each period. The workers in period \( t \) pay taxes of \( T_t (= qW_tL_t) \) and receive benefits of \( B_{t+1} \) which is equal to the taxes paid by the next generation when they retire.

(2) \[ B_{t+1} = b_{t+1}A_{t+1} = qW_{t+1}L_{t+1} \]

where \( b_t \) is the benefit per individual retired early in the \( t \) period.

The implicit rate of return that individuals earn on their tax contributions, \( r \), is defined by the ratio of the benefits that they received \( (= B_{t+1}) \) to the taxes that have previously paid \( (= T_t) \):

(3) \[ B_{t+1}/T_t + 1 = (qW_{t+1}L_{t+1})/(qW_tL_t) = (1 + g)(1 + n) = 1 + r \]

where \( g \) is the growth rate of wage rate, \( n \) is the growth rate of population, and \( r \) is the rate of return on social security tax \( (r = g + n + ng) \).

If the marginal product of capital, \( p \), is greater than \( r \), then the individual’s wealth will be decreased, since the individual is forced to substitute the private investment (or savings) by the social security “investment” offering an inferior rate of return. The difference between \( p \) and \( r \) is the welfare cost per dollar of the sacrificed savings by social security.

The wealth replacement effect measures the welfare cost from the reduction of private savings of one dollar by the increase of each dollar of social security tax. For the convenience of analysis, we assume that pre-retirement labor supply behavior and retirement age of an individual are determined exogenously. In the absence of social security, individuals save part of their income during their working period to finance the consumption during their retirement period. Social security, by promising pensions to retired workers, alleviates the need to save for the retirement period and may lead to a reduction in private saving. Under these conditions, this can be shown, by (Fig. 1), where \( CW \) and \( CR \) denote the total consumption of an individual during the working and the retirement periods, respectively. We can measure the impact of social security on private savings in the figure. The individual’s total income is \( ON \) over his working period. The individual’s initial equilibrium is at \( E \), consuming \( OC_1 \) and saving \( C_1 \) during the working period.

\[ ^3 \text{In deriving the rate of return on social security taxes, the value of } gn \text{ becomes negligible since it is a product of small numbers. In general, } r = g + n. \]
The imposition of the social security tax of \( N'N \) will shift the after-tax-budget constraint from \( MN \) to \( M_1N_1 \). The individual will receive social security benefits in his retirement period as a return for the payment of social security tax. It is assumed that the social security benefits can not serve as a collateral for a consumption loan during the working period. When the rate of return on social security tax is the same as the rate of return on private investment, his after-tax-and-pension budget line will be \( MSN_1 \). This means that the maximum consumption during
working period would be ON₁ instead of ON.

If the rate of return on the social security tax (= N₁R) is smaller than that on private investment (= N₁S), then his after-tax-and-pension budget line becomes M₂ RN, because it is assumed that individuals cannot borrow against social security benefits during their working period. At the new equilibrium E₁, both present and future consumptions have been decreased, under the assumption that consumption in each period is a normal good.⁴ On the other hand, gross savings including the social security tax increases from C₁N to C₁N. In this case, the social security tax not only replaces the private saving, but it also induces additional private savings (= C₁C₁) by reducing current consumption. The relation between the social security tax and savings before social security does not become perfectly substitutes, since the pre-retirement saving behavior is changed by the social security’s negative income effect. However, we assume that the social security tax and savings are perfect substitutes. This implies that individuals respond to social security as if they were not faced with the income effect of social security. The negative income effect upon the individual’s lifetime wealth leads to the welfare cost of social security.

As was mentioned previously, individuals substitute the private savings whose rate of return is the marginal product of capital (= p) by the social security tax. The social security program thereby reduces the lifetime budget of the individual by the amount of (p − r)qWₜLₜ, which is measured in the t + 1 period. We regard this as the welfare cost of social security caused by the wealth replacement effect. To get the present value of the welfare cost caused by the wealth replacement effect (= WCₜ), (p − r)qWₜLₜ should be discounted by an appropriate rate since it is occurred in the next period. It then becomes

\[ WCₜ = (p-r)Tₜ/(1+d) = (p-r)qWₜLₜ/(1+d) \]

Where d is the discount rate on the welfare cost of social security.

The question then becomes: what is the appropriate discount rate for the welfare cost of social security?

According to Feldstein (1987) the appropriate rate should be equal to the consumer’s marginal rate of substitution between present and future consumptions, the amount of income required in the t period to compensate for a decrease in income in the t + 1 period.⁵ If there are perfect capital markets, this then will be the consumer rate of interest, or in other words, the real rate at which individuals

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⁴ This model will consider only the case of interior solution in determining the equilibrium of consumptions. If the initial equilibrium is located on SN of the initial budget constraint MN, individual’s consumptions after social security would be placed at R as a corner solution. In this case, individuals are forced to save more by the difference between savings before social security and social security tax regardless of their preferred choice.

can borrow and lend. If the social security tax is transferred from private investment, however, the marginal rate of substitution does not measure the relevant alternative yield. In this case, the marginal rate of transformation becomes the opportunity cost of the social security tax. If firms can borrow and lend at the rate, \( p \), then, we refer to \( p \) as the opportunity cost rate.\(^6\) If there exist perfect capital market without capital income taxation, the time preference rate is equal to the opportunity cost rate. Feldstein (1987) used the marginal product of capital, \( p \), as the rate of time preference.

As far as the social security tax is transferred from the fund for private investment, the social rate of discount (or the discount rate) should equal the opportunity cost rate in the case without capital income taxation. However, if it is from the sacrifice of present consumption for future consumption, the discount rate should be the time preference rate, which may be different from the opportunity cost rate with the existence of capital income taxation. However, if it is from the sacrifice of present consumption for future consumption, the discount rate should be time preference rate, which may be different from the opportunity cost rate with the existence of capital income taxation. In the presence of the perfect capital markets without capital income taxation, both of these rates are the same.

In the above analysis, we can see that the change in the welfare cost which is determined by the wealth replacement effect depends upon the size of the marginal product of capital and also upon the rate of return on the social security tax, which is determined by the population growth rate and the GNP growth rate as well as the amount of social security tax revenue. Presently, the marginal product of capital is greater than the rate of return on the social security tax in the U.S.\(^7\) Moreover, the decrease in the population growth rate will increase the welfare cost of social security by the wealth replacement effect.

III. THE WELFARE COST WITH TAXATION OF CAPITAL INCOME

In this section, the present paper examines how the existence of capital income taxation affects the welfare cost of social security. Social security may affect the amount of tax revenue collected by capital income taxes by changing private savings, which is the tax base in capital income, or income from capital stocks. The wealth replacement effect will reduce private saving for private investment, and decrease capital income to be taxed by the government. Therefore, we can trace the decrease in tax revenue by the wealth replacement effect of social security.

\(^6\)Joseph Stiglitz (1982) referred to it as the "producer's rate of interest". Also see Edgar Browning and Jacqueline Browning (1987) p. 119.

\(^7\)According to Feldstein et al. (1983), the marginal product of capital was 11.4 percent annually during the period from 1950 to 1980. On the other hand, the calculated rate of return on social security tax from the rates of population growth and wage growth was 2.87 percent yearly which was much lower than the marginal product of capital. This will be reconsidered in section IV.
The implication from the change of the tax revenue can vary with the usage of tax revenue. If all tax revenue is used for government investment which yields the rate of return $p_g$ (which is greater than $p(1-t)$ per period forever,⁴ then the net present value of government investments becomes positive. The reduction of private savings by social security cause lead the tax revenue from capital income taxation to decrease. Therefore, government investment will decline, yielding a decrease in the welfare of society. However, if all tax revenue is used for consumption expenditure, then the existence of capital income taxation will not change the welfare of society. Here, it is implicitly assumed that all tax revenue is returned to the existing generation at the time it is collected.

In the analysis with capital income taxation, Feldstein (1987) considered only the case of consumption expenditure in the usage of the tax revenue by assuming that the benefits of that tax revenue accrues to the generation of savers who own the capital.⁹

In this paper, we assume that there are only two extreme possibilities for the usage of tax revenue: one is for consumption expenditures that benefit the existing generation, and the other is for government investments, which yields an unending and a constant rate of return per period, $p_g$. However, by weighing these two extreme cases, we can determine the welfare cost of social security under intermediate conditions. Given this situation, it is important to consider what factors will determine the rate of return on government investment.

The determination of $p_g$ can be thought as being related to the interest rate on government bonds. When the government issues bonds for government investment, it must compete with other borrowers when it sells its bonds. Therefore, the decision concerning a project which will be financed by issuing bonds depends on the net return which private sector competitors will pay to capital owners. Because of property taxes and the corporate income tax, the net return to owners of capital is well below its gross (or real) rate of productivity. Therefore, in the presence of taxes levied on the return to capital, the average level of the rate of return on government investment would be greater than the net rate of return from private investment, $p(1-t)$ (where, $t$ is the capital income tax rate), which is the interest rate paid by the government on its bonds.¹⁰

In reality, the interest rate paid by the government on its bonds is far below the real return sacrificed in the private sector.¹¹ In issuing bonds, the government competes only with the net return to private investment, and can therefore divert resources away from projects that have a real rate of return substantially above

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⁴We assume that the investment of the government has a characteristic like a console.
¹⁰The determination of the rate of return on government investment will be discussed again in section IV.
the interest rate paid on government bonds. The project with bond financing is likely to impose a substantial cost, which comes in the form of reduced future productivity. However, the project funded by the additional tax revenue, which is collected by the increase in private savings, will not impose this cost because it does not compete with other investments in the private and the public sectors.

In general, it is well known that the capital income tax distorts factor markets. To avoid the loss caused by capital income taxation, owners of capital reduce the effort to accumulate capital. This will decrease the capital stock, which increases the rate of return on capital and decreases the wage rate. However, we assume that both prices are unchanged by the social security and the capital income taxation.

The retirement benefits provided by social security are not affected by the existence of capital income taxation because the social security tax is levied on labor income. Consequently, the pension benefits for individuals in the $t+1$ period can be expressed as follows:

$$B_{t+1} = T_{t+1} = (1 + r)T_t$$

where $T_t = qW_tL_t$

On the other hand, capital income decreases since private savings will fall in response to the social security tax and expected retirement benefit. The government loses tax revenue by the amount of $tpqW_tL_t$ in the $t+1$ period, resulting from the reduction in the return on private savings due to the social security program ($=pqW_tL_t$) times the capital income tax rate ($=t$). This change in tax revenue would have been used for consumption expenditures or government investments if social security did not exist. That is, the welfare gain from consumption expenditures or government investments should be considered in measuring the cost of social security.

If the loss in tax revenue would have been used for consumption expenditures, then the additional cost to society is the amount of the decreased consumption expenditures. This amount is the same as the amount of the decrease in tax revenue ($=tpT_t$, where $T_t = qW_tL_t$). On the other hand, if the loss in tax revenue would have been used for government investments, the total present value of the yield, which would be produced afterward, ($=p\bar{g}pT_t/d$) should be added to the welfare cost of social security.\(^{12}\) It will be obtained in the following manner:

\(^{12}\)We assume that consumption expenditures in the $t+1$ period would be done with the fund which is the collected tax revenue at the $t+1$ period fiscally. Then, the consumption expenditures and the collection of the tax revenue would occur simultaneously. Government investments is assumed to take time to produce their outputs since it needs time to plan and to finish them. Therefore, the benefit of government investments would be generated from the $t+2$ period, even though the spending on government investment was done in the $t+1$ period.
(6) (The present value of the yield from government investment in the \( t + 1 \) period)

\[
= t p T_i \{ \frac{p_g}{(1 + d)} + \frac{p_g}{(1 + d)^2} + \frac{p_g}{(1 + d)^3} + \ldots \} \\
= \frac{p g T_i}{d}
\]

The gross cost to the society — from the decreased private saving due to the wealth replacement effect in the \( t + 1 \) period — \( ( = C_{t+1} ) \) becomes:

(7) \[ C_{t+1} = \{1 + p(1-t)\} T_i + \frac{p g T_i}{d} \]

When we use a conjectural variable, \( g \), this becomes:

(8) \[ C_{t+1} = \{1 + p(1-t)\} T_i + g T_i \]

\( g = 1 \) if the tax revenue is used for consumption expenditure, and

\( g = \frac{p g}{d} \) if the tax revenue is used for government investment.

The first part of the right hand side of the equation indicates the primary burden borne by the \( t \) period generation, and the second part of that is the secondary burden borne by the following generations if social security does exist.

The net welfare cost to the society by this effect at the \( t + 1 \) period is:

(9) \[ C_{t+1} - B_{t+1} = \{p(1-t)-r\} T_i + g T_i \]

When the tax revenue is used as consumption for the current generation, it becomes \( (p-r) T_i / (1 + d) \) in the \( t \) period. That is, the welfare change by social security is independent of the taxation of the capital income in this case. However, when it is used for government investments, the present value of the welfare cost by the wealth replacement effect \( (= WC_t) \) will be:

(10) \[ WC_t = \{p(1-t) - r + \frac{p g T_i}{d}\} T_i / (1 + d) \]

\[ = \{(p-r) + \frac{p g}{d} - 1\} T_i / (1 + d) \]

where \( T_i = q W_i L_t \).

Consequently, we face the problem of what the appropriate discount rate should be when there is the capital income taxation. This same problem was raised in section II. If a private investment yields a rate of return of \( p \), and the government taxes at a rate \( t \) on this yield in a perfect capital market, \( p(1-t) \) will be left to be paid to investors. The net return to investors becomes \( p(1-t) \). This is contrasted with their previous return of \( p \) in the case without capital income taxation. The
time preference rate is now p(1 - t), and the opportunity cost rate, p. Any change is savings caused by social security results in a sacrifice of present consumption for future consumption, or the reverse. If workers continue to save the same amount, they will receive the net rate of return, p(1 - t). Therefore, the discount rate for the welfare cost brought about by the wealth replacement effect should be the time preference rate, p(1 - t).

We can see that the greater in the difference between p and r, and between p_g and d, the larger is the welfare cost to the society. If p(1 - t) is applied as a discount rate and as a rate of return on government investment, then the welfare cost formula incorporating this effect is the same as the case without capital income taxation. That is, the difference in the welfare cost, due to the wealth replacement effect, between the case of capital income taxation and the case of no capital income taxation depends upon the discount rate in the two cases.

Feldstein (1987) assumed that the benefits of the tax revenue occur to the generation of savers who own the capital. This means that capital income tax revenue is consumed by the existing generation. The formula for the welfare change due to social security is not affected by the existence of a capital income tax. The result obtained with our formula in this section is identical with the one obtained by Feldstein (1987) when the tax revenue is spent for the current generation's consumption.

IV. THE ILLUSTRATION

For the illustrative calculation of the welfare cost of social security, we must first determine parametric values for r, p, q, p_g, and t. In obtaining the parametric values of any given period, Feldstein (1987) assumes that the length of a period or generation is thirty years.

Firstly, the rate of return on social security tax will be decided by the growth rate of real wage rate (= g) and the growth rate of the population (= n). The annual rate of return on the social security tax (= r) will be determined by the formula, r = g + n + gn. Therefore, the implicit rate of return on social security taxes is assumed to be the rate of growth of real income. As a proxy for real income, Feldstein used real personal income from the national income accounts. The average annual rate of growth of real personal income was 0.037 over the period from 1950 to 1980. Its rate of return in a period will be 1.97, which is form (1 + r)30 - 1.

Secondly, we assume that the annual marginal product of capital was 0.114 during the period 1950 to 1980. This figure was calculated by Martine Feldstein, Louis

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14In general, the length of a generation is defined as the period of time between the birth of the parents and the birth of their children. A generation is considered to be about thirty years. See The Encyclopedia Americana, vol. 12 (1982), p. 384. Also, see David Sills (1968), pp. 88-92.
Dicks-Mireaux, and James Poterba (1983). The marginal product of capital for the period is then 24.50 if we assume that all the annual (before tax) gross return of capital is reinvested for thirty years.

Thirdly, we adopt the social security tax rate as 0.143, which is the sum of current social security tax rate ($= q$) for workers and employers in the fiscal year of 1987. The reason to include the employers share of the social security tax is that this share will be borne by workers in the long run.

Fourthly, the capital income tax rate is assumed to be, as Feldstein did in his paper (1987)\(^\text{15}\), 0.5. We should mention that \( t \) is an effective period tax rate which is defined by \( P_n = (1 - t)P \), where \( p_n \) is the rate of return on private investment per period. Then, \( P_n \) will be decided by the following equation.

\[
(11) \quad P_n = \left\{ 1 + (1 - t_c)P \right\}^{y - 1} \tag{11}
\]

where \( t_c \) is the capital income tax rate, and

\( y \) is the number of years in a generation.

Therefore, the effective period tax rate will be:

\[
(12) \quad t = 1 - \left\{ (1 + (1 - t_c)p)^y - 1 \right\} / p \tag{12}
\]

If we assume that \( t_c = 0.50 \) with \( p = 0.114 \), then \( P_n = (1 - t)p = \left\{ 1 + (1 - t)p \right\} \)

\[
= 4.28. \text{ Since the period marginal product of capital is 24.50, the effective period tax rate will be 0.825.}
\]

Finally, we need to determine the rate of return on government investment. When we assume that the rate of return on government investment is equal to the net rate of return on private investment, the total welfare costs of social security, in the cases of government investments and consumption expenditures are the same. However, such an equality is not satisfied in the real world since the capital market is not perfectly competitive so that \( p_g = p(1 - t) \). Therefore, the problem of how to determine the "real world" rate of return on government investment still remains.

Pestieau (1974) has devised a model that addressed the rate of return on government investment. This model is based upon a life-cycle general equilibrium, and assumes that the government aims at an intergenerational maximization of social welfare. Under the assumption that the compensated elasticity of labor supply is equal to zero, the formula for the rate of return on government investment (= \( p_g \)) is expressed as follows when the government raise all the revenue for government investment by taxation. Then, How can we justify the rate of return on govern-

\(^{15}\)See Martin Feldstein (1987), p. 8. During the period from 1950 to 1980, the U.S. nonfinancial corporate sector's shareholders and creditors paid approximately sixty-eight percent of their pre-tax capital income in taxes to federal, state and local governments (Martin Feldstein, Louis Dicks-Mireaux, and James Poterba, 1983).
ment investment which is financed by taxation as $p_e$ in this paper. Joseph Stiglitz (1982) answered this problem by saying that few government have deliberately used debt policy to affect an intergenerational redistribution of income.\(^{16}\)

$$
(13) \quad P_e = p - \theta e_i = p(1 - e_1) + p(1 - t)e_1
$$

where

the capital income tax per unit of capital ($= p_t$), and $e_i$ is the compensated elasticity of retirement consumption with respect to the consumer’s rate of interest.

Since we assume an effective period capital income tax rate equal to 0.825, the amount of period capital income tax per unit of capital ($= \theta$) be 20.213 ($= 24.5 \times 0.825$). Now we encounter with how to measure the value of $e_1$. For the sake of convenience, we assume that an individual has a Cobb-Douglas utility function, which consists of consumptions during the working and the retirements periods. We then obtain the Marshallian demand for retirement consumption, which is a function of the consumer’s rate of interest.

$$
\text{Max } U(C_1, C_2) = C_1 \cdot C_2^b
$$

subject to $(C_1 + C_2)/(1+i) = Y$

where,

$C_1, C_2$ are the consumption levels of the working and the retirement periods, respectively, and

$i$ is the consumer’s rate of interest.

As a solution of the above problem, we can get the value of the consumption during the retirement period.

$$
C_2 = bY(1+i)/(a+b)
$$

Since the Marshallian demand and the compensated demand for retirement consumption are the same in the individual consumption equilibrium, and the values of $a$ and $b$ are unknown, we will use the Marshallian demand for retirement consumption to get $e_i$ as a proxy of the compensated demand for retirement consumption. Then, the value of $e_i$ will be determined as follows:\(^{17}\)

$$
e_i = \frac{d \ln C_2}{d \ln i} = (\frac{d \ln C_2}{d \ln (1+i)}) \times (\frac{d \ln (1+i)}{d \ln i}) = \frac{d \ln (1+i)}{d \ln i}
$$


\(^{17}\)That is, we are taking the uncompensated elasticity of retirement consumption instead of the compensated one, for $e_i$.\)
(Since, \( \frac{d \ln C_2}{d \ln (1+i)} = 1 \))
\[
= (\frac{d \ln(1+i)}{d i}) / (\frac{d \ln i}{d i})
\]
\[
= \frac{i}{1+i}
\]

Since the consumer's rate of interest, \( i \), is \( p(1-t) \), it is easy to derive the value of \( e_1 \). Under the same assumption about \( p \) and \( t \) as before, \( i \) will be 4.29. Thus, \( e_1 = 0.811 \). From the equation for \( P_g \),

\[
(13') P_g = p - \theta e_i = 24.5 - 20.21 \times 0.81 = 8.13
\]

The period rate of return on government investment, 8.13, implies an annual rate of return of 0.072.

With the above assumption about the parametric variables, we can calculate one generation's welfare cost of social security due to changes in private savings. Even though this analysis operates under very restrictive assumptions, it is obvious that the existence of capital income taxation will have a variety of effects on social welfare. We will illustrate the change in the ratio of the welfare cost to the social security tax revenue, calling this ratio the average welfare cost of social security. The average welfare cost of social security in the case of full consumption of the tax revenue (\( = AWCC \)) is:

\[
(14) \quad AWCC_1 = TWCC_1 / T_1 = (p - r) / (1 + d)
\]

where \( T_1 = qW_tL_t \), and
\[
d = p(1-t)
\]

In the case of full investment of the change in tax revenue, the average welfare cost will be calculated as follows:

\[
(15) \quad AWCl_1 = TWCl_1 / T_1 = \left\{ p(1-t) + P_atp/d - r \right\} / (1 + d)
\]

The average welfare cost of the full consumption case will be an increasing function of the capital income tax rate, \( t \). Also, the change in the average welfare cost of the full investment case has positive relation with the change in capital income tax rate. The two cases are depicted in (Fig. 2).

From these two plottings, we can obtain another plot for an intermediate case between the two extreme cases of expenditure. For instance, during the fiscal year 1984, state and local expenditures can be classified as either a consumption ex-
Figure 3: The change of weighted average welfare cost

Assumptions:
- Annual marginal productivity of growth rate = 0.114
- Annual for on the Social Security Tax = 0.0525
- Social security tax rate = 0.103
- $AWCI/AWCC = 6/4$

Plot of $WAWC*TC$ Symbol used is W
[Figure 2] The change of Average welfare cost
(Asumptions)
Annual Marginal Productivity of Growth Rate = 0.114
Annual for on the Social Security Tax = 0.0525
Social security tax rate = 0.143
Plot of AWCC*TC Symbol used is C
Plot of AWCI*TC Symbol used is I
penditure or a government investment. We assume that the change in capital income tax revenue brought about by the effects of social security has been used for government investments and consumption expenditures with a ratio of six to four. Therefore, we can get the weighted average welfare cost of social security (= WAWC_s) with the above weight on the expenditures of tax revenue. The weighted welfare cost of social security is shown in (Fig. 3). At the capital income tax rate of fifty percent, the weighted welfare cost of social security is 6.01 times the collected social security tax.

When the average welfare cost of social security is measured by the same method of Feldstein, which assumed that there is no income taxation, it is 3.8. That is, the existence of income taxation may seriously affect the welfare state of society when social security is imposed. The increase in the welfare cost of social security is accelerated by the increase in capital income tax rate at the range of over fifty percent.

V. CONCLUSION

It is well known that social security leads to a welfare loss for society. The rationale behind this conclusion is based on the observed substitution of private savings by the social security tax. That is, the reduction of private savings will decrease society's production capacity by reducing capital accumulation.

The welfare measure of social security, in the present paper, depends on a change in saving. However, in the present paper, depends on a change in saving. However, we should not fail to perceive the primary purposes of social security: income redistribution, correction of the market failure, and paternalism.

It is shown that under the existence of capital income taxation, the welfare cost of social security would be occured more than the case without capital income taxation if part of the change in tax revenue by social security has been used for investments instead of consumption expenditures. There are two rationales of the larger welfare cost in the case of capital income taxation: one is that the capital income tax decreases the discount rate for individuals; the other one is that the existence of capital income tax reduces the output from the government investments which have been produced since the tax revenue decreases by the implementation of the social security.

One more point should be emphasized in the viewpoint of policy implications. In this paper, the value of the rate of return on government investment becomes

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18See Edgar Browning and Jacqueline Browning (1987), p. 467. When we assume that government investment consists of education, transportation, public safety, and environment and housing, and that the consumption expenditure includes all the other items as well, the ratio of government investments to total government expenditure was about sixty percent.

19See Peter Diamond (1977).
a decisive factor in implementing social security system. When we look at only the side of welfare cost, it is better to abandon to implement social security if the capital stock for the public is not formulated so that the rate of return on government investment is so high.

More extensive research concerning the welfare cost of social security should include the analyses on the induced retirement effect and the bequest effect of social security. In calculating the welfare cost of social security, this paper is limited to only one generation, who does not belong to either a start-up or ending generation. Consequently, it becomes possible to extend this paper to a multiple generation model as Feldstein did. It would improve this paper in evaluating the welfare cost of social security.

REFERENCES


Feldstein (1987) mentioned about the distortion in retirement behavior and in pre-retirement labor supply caused by social security taxes and benefits.