Optimal Investment in Pollution Control Capital with Debt Financing

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I. Introduction

Recently, Comolli [8, 9] has employed models to address the issue of optimal investment in pollution control capital. His major concern was to analyze the optimal mix between directly productive capital, say private capital, and capital employed in waste treatment, say public capital. He shows that under a perfectly planned economy where the central planner has full control over the resource allocation among consumption, private investment and public investment, there exists an optimal growth path in which resources can be allocated optimally between private and public sectors, Several other works [13, 15, 18, 23, 24] has also looked at this issue under different specifications of the models. Unfortunately, their models did not consider the mechanics of financing the pollution control capital. Under a decentralized economy, however, the issue of financing is important because the pollution control capital is a public good in nature, so that its price as well as its cost cannot be fully determined in the market and thus the problem of obtaining control over resources becomes that of financing. Moreover, if the planner has only a limited collection of policy instruments, it is no longer certain that an optimal growth path can be implemented. If it is not achievable, the planner may need to accomplish the second-best optimal growth path. Arrow and Kurz [1] originally raised this issue under a decentralized economy model which is featured by institutional contraints such as fixed savings ratio and a limited number of financing instruments. They, however, examined the second-best optimal growth path of capital accumulation in nonenvironmental context.

The purpose of this paper is to take up this issue of financing the pollution control capital under a decentralized economy. For this purpose, we incorporate Comolli's planning model [9] into the bond financing model employed by Arrow and Kurz [1].[1] It would be interesting to compare the nature of optimal growth path under a perfectly centralized economy with that of the second-best optimal

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1) Kim [16, 17] attempted to discuss the same issue under the income tax-financing scheme.

growth path under a decentralized economy.

In the following section the model of the decentralized economy will be fully described. As a point of departure, we will amend Comolli's planning formulation of the problem. A general analysis of debt-financing policy will be undertaken in Section III. In this section, we shall state and characterize the second-best optimality conditions and analyze various subpolicies of the hybrid environmental policy considered. Section IV provides an analysis of the issue of a steady state. We will show the existence of a steady state, and characterize it. Section V provides further analysis of the optimal path under a separable production model. Utilizing the various properties of the special production function, we will perform comparative statics analyses.

II. Model

The model to be followed is based on the Comolli's planning model [9], in which, for an economy without technological progress, the representative individual with an infinite life span derives direct utility from the consumption of a physical good and environmental quality. Throughout the paper we shall assume that the planner utilizes direct control over polluting activities as a regulation method. A direct control may refer to giving individual firms permits to pollute within pollution standards specified by legislation, and prohibiting them from exceeding them. Thus if they violate this requirement, whether by a small or large amount, the individual is considered to be a lawbreaker who is subject to punitive action. Concerning investment possibilities, we consider the situation in which the planner operates with a budget deficit, financing public investment in pollution control capital by issuing bonds. More specifically, the planner with a positive stock of debt at the initial time in the planning horizon, issues perpetuity with which to finance public investment and raises income taxes to pay the costs of interest on the public debt.

It is known that all waste residuals have adverse effects on the natural environment, especially air and water. The nature of such effect depends greatly on time, form and the place of wate discharges, and the effect of many discharges are poorly understood. In simple form, however, such effect can be reasonably represented by the equation, called the environmental quality function

$$q = Q(\mu)^{2)} \tag{1}$$

where q is an index of environmental quality, $e \equiv E/K_g = e/k_g$ signifies the economy's utilization rate of assimilative capacity, and $e \equiv E/N$ and $k_g \equiv K_g/N$ denote effluents and assimilative capcity, say public capital, respectively, in per capita terms.[3] Here, N stands for the size of population, and at a point of time t, $N(t) = e^{nt}N(o)$, where n is the population growth rate. We shall restrict q to satisfy the following assumptions:

A. 1. $Q'(\mu) < o$
A. 2. Q is concave
A. 3. $Q(o) = \bar{q} < \infty$
A. 4. $Q(\bar{\mu}) = o$, $Q'(\bar{\mu}) < o$, where $\bar{\mu} < \infty$

A.1 implies that q is nonincreasing in effluents for a given assimilative capacty. A.2 permits the threshold nature of the environmental quality, which implies that the environmental quality would be degraded very sharply beyond some level of effluents. A.3 is intended to specify the natural state of environmental quality to guarantee the upper bound of (1). A.4 guarantees the existence of a unique utilization rate that results in a zero level of environmental quality, viz a lower bound of it. The choice of zero is arbitrary, but may be determined on the grounds of human health.

It is now assumed that this environmental quality plays an important role for the level of human satisfaction as well as the consumption of goods and services. The preferences of the representative individual in the economy are presumed to follow the function.

$$U(c, q) \tag{2}$$

which satisfies the following assumptions:

A.5. U has positive and diminishing marginal utilities.

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2) As with many other theoretical models in the environmental literature, equation (1) is formulated too broadly to specify the nature and harshness of the effects of wastes on the environment. Mills [19, pp. 62-63] suggests that the equation may be made more accurate by disaggregating e by kinds of materials, such as organic wastes, metals, and the like, and by disaggregating q by the dimensions of air, water and land destinations of discharge.

3) The definition of μ implies that the environmental quality index, q, is homogeneous of degree zero in E and K_g which may be reasonable for some noxious pollutants in water or air. For the other types of indices (color, order, etc.) however, this simplistic dilution technology would be inappropriate.

A.6 . $U_{12} = 0$

A.7 . a) $U_1(o, q) = +\infty$

b) $U_2(c, o) = +\infty$

c) $U(c, q) = -\infty \quad \forall q, \; q < o$

Here, $c = C/N$ is his personal consumption. A.6 assumes that c and q are nonrelated commodities. This additive assumption of the utility function simplifies the analyses in the subsequent sections. It is virtually uncertain whether c and q are complements or substitutes. A.5 and A.6 guarantee that U is strictly concave. a and b in A.7 are designed to compel c and q to be positive in the optimal solution and appear reasonable with respect to human health.[4] A.7.c serves to rationalize A.4 and guarantees the boundedness of (2).[5] Hence, in view of A.7.c, an admissible rate of μ is restricted to

$$o \leq \mu \leq \bar{\mu} \tag{3}$$

substituting (1) into (2) one obatins

$$U[c, Q(\mu)] \equiv u(c, \mu) \tag{4}$$

which is strictly concave,[6] and has negative marginal utility of μ.

The technology for the joint production of output and effluents in the economy follows the specification in Brock [6]. In particular, aggregate output, Y, is presumed to be a strictly quasi-concave and linear homogeneous (hence, concave) function, $F(K_p, E, N)$, with positive and diminishing marginal products, that satisfies the Inada conditions.

Denoting by $k_p = K_p/N$ and $y = Y/N$

private capital and total output, respectively, in per capita terms, it follows that

$$y = F(k_p, e, 1) \equiv f(k_p, e) \tag{5}$$

which satisfies the following conditions:

A.8. $f(k_p, e)$ is a strictly quasi-concave (and concave) function which possesses positve and diminishing marginal products.

A.9. $f(k_p, e)$ satisfies the following:

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4) $U_1(o, q) = +\infty$ can be replaced by $U(o, q) = -\infty$

5) The rason why q requires both A.7.b and c is that, unlike c, q is not naturally bounded from below by zero.

6) That is, $u_{11} = U_{11} < o$, $u_{22} = U_{22}(Q')^2 + U_2 Q'' < o$ and $u_{11}u_{22} - u_{12}^2 = (U_{11}U_{22} - U_{12}^2)(Q')^2 + U_{11}U_2Q'' > o$. Clearly, since (1) is concave, the strict concavity of (2) would imply the same for (4).

a) $f(o,e)=o=f(k_p,o)$
b) $f_1(o,e)=\infty=f_2(k_p,o)$
c) $f_1(\infty,e)=o=f_2(k_p,\infty)$

A.9 (a) and (b) note that both k_p and e are "essential" and "indispensable", respectively, to the production of positive output. A.9 (c) prevents their marginal products form being asymtotically bounded away from zero.

Utilizing the definition of μ, one may rewrite (5) as

$$y = f\ (k_p,\ \mu k_g) \tag{6}$$

which is noticeably similar to the technology discussed in Arrow and Kurz [1, Chapter IV], though generalized to permit a variable utilization of public capital.

Turning to the investment possibilities, let b and d be the amount of the bond issues and the public debt in per capita real terms, respectively. Since interest payments are financed by income taxes, it follows that

$$x = rd \tag{7}$$

where x is the amount of income taxes collected and r denotes the rate of interest on the public debt which is presumed to be equal to the marginal product of private capital, f_1. Then, the personal income would be $y_p = y + rd$, and the disposable income form (7), can be defined as

$$y_d = y_p - x = y + rd - rd = y \tag{8}$$

which means that the disposable income becomes equal to national income. For some fixed constant s, $s \in (o, 1)$, personal saving may be defined as

$$S_p = sy_d = sy \tag{9}$$

Since personal saving is presumed to be a function of current disposable income, all borrowing is taken to fall on personal saving so that the private investment may be reduced by the equal amount of bond issues.[7] Thus, consumption per capita would be defined as

$$c = (1-s)\ y \tag{10}$$

and the gross private investment per capita, i_p, is $i_p = sy-b$, so that per capita private capital accumulation can be written by

$$\dot{k}_p = sy - b - nk_p \tag{11}$$

7) This proposition was discussed by Modigliani [20] and Phelps and Shell [22] in dynamic models.

Since the public investment is financed by only bond issues, the gross public investment per capita, i_g, becomes $i_g = b$ and public capital accumulated per head would be

$$\dot{k}_g = b - nk_g \tag{12}$$

Since both investments are nonmalleable (i. e., $i_p \geq o$, $i_g \geq o$), the admissible amount of bond issues is restricted to

$$b \in [\ o,\ sy\ ]^{8)} \tag{13}$$

We may note that (10), (11) and (12) satisfy the feasibility condition,

$$Y = c + i_p + i_g$$

substituting (6) into (10) and (10) into (4), the individual's utility function becomes

$$u\ \{ (1-s)\ f\ (k_p, \mu k_g)\ ,\ \mu\ \} \tag{14}$$

Finally, we need to mention the growth of public debt caused by issuing bonds. The public debt per head follows the simple law of motion

$$\dot{d} = b - nd \tag{15}$$

where, as in diamond [10], this could be thought to consist of demand loans held by the private sector. Assuming that the planner has positive debt as the initial point in time, the growth of public debt is endogenously determined by the authority's public investment policy, since it is directly affected by the amount of public investment financed by bond issues. This differential equation, therefore, would be a budgetary stability constraint on the debt-finunced public investment policy, which would be determined by public capital accumulation. Budgetary stability may refer to whether the ratio of public debt to national income converges to a certain limit. If it diverges to infinity even in a steady state, it may be said that the debt-financed public investment policy is budgetarily unstable, so that the bond finance may not be a tenable instrument for a public investment policy. Therefore, equation (15) will be used to examine the budgetary stability.

8) As mentioned earlier, the model described in this paper is adopted from the non-environmental investment model in Arrow and Kurz [1]. In their model, however, they allow the size of borrowing to be positive or negative infinite since they allow the reversibility of private capital and public capital.

III. The General Analysis for the Second-best Optimality

The planner's problem in such an economic setting may now be posed. Before proceeding, we define some definitions. Let $I=[o,\infty)$ be the planning period. Define the control vector by $V=(b,\ \mu\ )$ and the admissible region by $V=\{v{:}b \in [o,\ sf(k_p,\ \mu\ k_g)],\ \forall s \in (0,\ 1){:}\ \mu \in [0,\ \bar{\mu}\ ]\}$. Thus, an admissible control may be represented by $v\ \in V$. Denoting the state vector by $a=(k_p,\ k_g)$, define the feasible region by

$$A=\{a:\dot{k}_p= sf(k_p,\mu k_g)-b-nk_p,k_p(o)=\bar{k}^o\ ;\dot{k}_g\ =\ b-nk_g,$$

$$k_g(o)\ =\ \bar{k}^o_g\ ;n,\ \bar{k}^o_p,\ \bar{k}^o_g\ >o\ \text{are given}\ \}\ ;\ \text{whence}\ a\ \in A\ \text{is feasible.}$$

Note $k_p(o)$ and $k_g(o)$ indicate initial states of private and public capital.

Denoting a policy vector by $j=(v,\ a)$, the set of admissible policies may be defined as $J=\{j{:}\ v \in\ V,\ a \in\ A\}$; accordingly, $j\ \in J$ is admissible. Finally, we define an admissible path as a sequence of admissible policies denoted by $Z_\infty=\{j \in J\}^{\infty}_{o}$.

The planner's goal is to find a path Z_∞ that maximizes

$$W=\ \int_{o}^{\infty}\ u\ [\ (1-s)\ f\ (k_p,\mu k_g),\ \ \mu\ ]\ e^{-\rho t}\ dt \tag{16}$$

where $\rho>o$ is taken to be the society's constant rate of time preference. It is noted that all control and state variables are functions of time t.

We define the current-value Lagrangian for the planning problem by the fllowing relations:

$$L(\ b,\mu,k_p\ ,k_g,p_p,p_g,w_1,w_2\ )\ =H(b,\mu,\ k_p,k_g,p_p,p_g)+\ w_1b\ +w_2\{sf(k_p,\mu k_g)\ -b\ \}$$[9]

where the current value Hamiltonian function is defined by

$$H(\cdot)=u\ [\ (1-s)\ f(k_p,\mu k_g)\ ,\mu\ ]+p_p\ \{\ sf\ (k_p,\mu k_g)\ -b-nk_p\ \}+p_g\ (b-nk_g)$$

and w_1 and w_2 are the current-value Lagrangian multipliers associated with $b\geq o$ and $b\leq sf$, respectively. The costate variables, p_p and p_g, are current-vaule shadow prices of private and public investment, respectively, in terms of utils.

As a starting point of the analysis, we first state the maximum principle for the problem. This, however, would be the necessary conditions for

9) In light of A.4. and A.7.b, we know that $\mu<\bar{\mu}$, and from A.9.b, we know that $\mu>0$, hence, we only consider interior values of μ in the Langrangian.

the second-best optimality, for which the reason will be provided later. They are as follows:

$$p_p - p_g \geq o,\ (p_p - p_g)\, b = 0\ ;\ b \in [o, sf),\ \mu \in (o, \bar{\mu}) \tag{17.a}$$

$$p_p - p_g \leq o,\ (p_p - p_g)\,(sf - b) = 0\ ;\ b \in (o, sf\,],\ \mu \in (o, \bar{\mu}) \tag{17.b}$$

$$\{(1-s)\, u_1 + sp_e\}\ f_2 k_g + u_2 = 0\ ;\ b \in [o, sf\,] \tag{18}$$

along with (11) and (12), and the transition equations

$$\dot{p}_p = (\rho + n)\, p_p - \{(1-s) u_1 + sp_e\}\, f_1 \tag{19.a}$$

$$\dot{p}_g = (\rho + n)\, p_g - \{(1-s)\, u_1 + sp_e\}\, f_2 \mu \tag{19.b}$$

where $p_e \equiv p_p$, if $b \in [o, sf)$
$\equiv p_g$, if $b \in (o, sf\,]$

Note that the u_1 is the marginal social value of consumption, or the shadow price of consumption in terms of utils, and u_2 is the marginal social disutility of the utilization of public capital in terms of environmental degradation, viz., marginal external (or, damage) cost (MEC).

It is analytically convenient to decompose optimally into subpolicies: subpolicy 1, characterized by an investment subpolicy of complete specialization ($b = o$ or $b = sf$): subpolicy 2, characterized by investment subpolicy of incomplete specialization. ($o < b < sf$)

Subpolicy 1: Investment subpolicy of complete specialization ($b = o$ or $b = sf$).

This subpolicy is subdivided into two: subpolicy 1.1, viz., complete specialization in private investment ($b = o$), and subpolicy 1.2, viz., complete specialization in public investment ($b = sf$).

Subpolicy 1.1 is mainly represented by (17.a) and

$$\dot{k}_g = -nk_g \tag{20}$$

The nature of this subpolicy is equivalent to the one of complete specialization in private investment which can be seen from Gruver [13]. Condition (17.a) indicates that, in order to maximize social welfare, the public investment in evironmental capacity should not be undertaken as long as the marginal social value of private capital is at least as great as that of public capital. This subpolicy might be implemented in the economy, either where the stock of public capital is initially large enough to keep an admissible level of environmental quality without making any further public investment, or where, as in many underdeveloped countries, the central planner concentrates on economic development without regard to environmental quality. This subpolicy, however, ought to be switched into either subpolicy 1.1. or subpolicy 2 at some point in time,

t^*, $t^* \in I$, since public capital per head will converge to zero in the long run (i.e. $\lim_{t\to\infty} k_g(t) = o$) as it can be easily seen from (20). In other words, a point of time will be reached after which $\mu \geq \bar{\mu}$, so that $q \leq o$. Besides, a steady state does not exist under this subpolicy. From (20), $\dot{k}_g = o$ at a steady state, so that $k_g = o$, a contradiction, in view of A.9.a and b.

On the other hand, the main characteristics of subpolicy 1.2 are (17.b) and

$$\dot{k}_p = -nk_p \tag{21}$$

condition (17.b) reminds us that when the utility price of public investment is greater than the social value of foregone private investment, the planner must induce individuals to purchase bonds with full amount of their savings, so that total output may be allocated only between consumption and public investment. This is again one of the extreme policies, but may be the natural consequence of subpolicy 1.1, which results in an abundant private capital and inadmissible level of environmental quality. If this subpolicy were carried on forever, however, the stock of private capital would continuously decumulate over time and would approach zero in the long run, as we can see from (21).

As a consequence, the economy will collapse in the end, because $\lim_{t\to\infty} f(k_p(t), \mu(t)k_g(t)) = o$, in view of A.9.a. In addition, since total saving would converge to zero, viz., $\lim_{t\to\infty} sf(k_p(t), \mu(t)k_g(t)) = o$, b would eventually become zero, so that k_g would also converge to zero, ending up with $\mu = \infty > \bar{\mu}$, which is not admissible.

Before this occurs, the planner must switch this subpolicy either into subpolicy 1.1 or into subpolicy 2. In the former case of policy switch, the planner may have to switch subpolicy 1.1 either back into subpolicy 1.2 or into subpolicy 2. Thus, "Bang-Bang" policy switches may also be optimal, especially in finite planning horizon. As Comolli [9] showed, however, it may not be a general phenomenon, and subpolicy 2 is most likely to be a terminal one, even though it cannot be shown rigorously in this model. Again, it can be seen from (21), A.9.a and b that there does not exist a steady state under subpolicy 1.2.

Proposition 1. There does not exist a steady state under subpolicy 1. Moreover, subpolicy 1 should not be a permanent one.

Subpolicy 2: Investment of incomplete specialization ($o < b < sf$).

The above subpolicies could be carried out for a while. They, however, should not be permanent policies because they impair either the economy or natural environment or both in the long run. Whatever the initial phase may be, the terminal investment subpolicy is most likely to be the one of incomplete specialization.

The relevant optimality conditions for this subpolicy are as follows:

$$p = p_p = p_g \tag{22}$$

$$f_1 = f_2 \mu \tag{23}$$

$$[(1-s)u_1 + sp]\ f_2 k_g = -u_2 \tag{24}$$

along with (11) and (12) and the transition equation

$$\dot{p} = (\rho+n)p - \{(1-s)u_1 + sp\}\ f_1 \tag{25}$$

where p is the common utility prices of both private capital and public capital.

Condition (22) asserts that the planner should issue outstanding bonds until the utility price of public investment equals the value of foregone private investment to the society. Condition (23), obtained by equating (19.a) and (19.b), indicates that an optimal allocation of personal savings may be conducted by the planner when borrowing is made in such a way that the rate of return on private capital is equated with the rate of return on public capital. This is also a criterion on public investment decisions along the steady state path under the present environmental policy. This criterion implies that public investment in environmental capacity must be undertaken as long as the rate of return on public investment is greater than that on private investment, when borrowing is used for financing public investment and interest payments are financed by income taxes. Clearly, this criterion makes use of condition (22) in the form that the choice of borrowing should be based on the cost-benefit analysis. The opportunity cost of borrowing is the value of the foregone private investment (i.e., pf_1), while its benefit is the value of an increase in the public investment (i.e., pf_2), as reflected in the accretion of environmental quality. Thus, this criterion implies that the optimal amount of borrowing could be attained when its benefit and cost are equal to each other. Condition (24) states that the optimal utilization of the public capital could be accomplished by equating the gain in society's utility through the increased output caused by an additional utilization of public capital, which is distributed among consumption and investment in the proportions, 1-s and s, respectively, with the loss in society's utility by the associated deterioration in evironmental quality, viz. MEC. Moreover, rewriting condition (25) would reduce to the result similar to Brock [6]. That is, $\dot{p}/p + (p_w f_1 - np)/p = \rho$, where $p_w = (1-s)u_1 + sp$. This result implies that capital gains on either k_p or k_g, $\dot{p}/p$, plus net yield on it, viz. $(p_w f_1 - np)/p$ equals the social rate of return, ρ.

IV. A Steady State

It has just been seen in the last section that under subpolicy 1, a steady state does not exist and subpolicy 2 is most likely to be the terminal policy. In this section, therefore, we focus our attention on the analysis of a steady state under subpolicy 2. We first show the existence of a steady state and then characterize it.

***Proposition** 2.* Assuming that ρ is sufficiently small, there exists at least one steady state denoted by $(b^*, \mu^*, k_p^*, k_g^*, p_p^*, p_g^*)$ satisfying conditions (22)-(24), and the following:

$$sf - b = nk_p \tag{26}$$

$$b = nk_g \tag{27}$$

$$(\rho+n)p = \{(1-s)u_1 + sp\} f_1 \tag{28}$$

Proof: The method of proof follows the one proposed by Maler [18, Appendix A pp. 97-101] for his recycling model. At a steady state $\dot{k}_p = o = \dot{k}_g$, so (11) and (12) collapse to (26) and (27), respectively and similarly $\dot{p} = 0$ so that (25) collapses to (28).

Thus conditions (23), (24), (26)-(28) characterize an optimal stationary variables, providing five relations for five stationary variables. To show such a solution does exist, consider the following maximization problem.

$$\max u\{(1-s) f(k_p, \mu k_g), \mu\}$$

$$\text{subject to } -sf(k_p, \mu k_g) + b + nk_p \leq 0$$

$$-b + nk_g \leq 0$$

$$b \in [0, sf(k_p, \mu k_g)], \ \mu \in [0, \bar{\mu}], k_p \in [o, \infty), \ k_g \in [0, \infty)$$

It is obvious that constraints are consistent. In addition, points (b, μ, k_p, k_g) satisfying the constraints are regular points, that is, the Jacobian matrix of the constraints satisfies the rank condition (i.e. Rank 2).[10]

If the problem has a solution, from Hestenes [14, theorem 10.1, p.36], there exists multipliers p_p, p_g such that both are nonnegative and such that with

$$L = u\{(1-s) f, \mu\} - p_p(-sf + b + nk_p) - p_g(-b + nk_g),$$

maximum point satifies

10) See Hesteness [14 pp. 25-34] for a discussion of regular constraints.

$$L_b = -p_p + p_g \leq 0 \qquad \text{if } b \in [0, sf)$$
$$L_b = -p_p + p_g \geq 0 \qquad \text{if } b \in (0, sf]$$
$$L_\mu = u_2 + \{(1-s)u_1 + sp_p\} f_1 k_g \leq 0 \qquad \text{if } \mu \in [0, \bar{\mu})$$
$$L_\mu = u_2 + \{(1-s)u_1 + sp_p\} f_1 k_g \geq 0 \qquad \text{if } \mu \in (0, \bar{\mu}]$$
$$L_{kp} = \{(1-s)u_1 + sp_p\} f_1 - np_p \leq 0 \qquad \text{if } k_p \in [0, \infty)$$
$$L_{kg} = \{(1-s)u_1 + sp_p\} f_2 \mu - np_g \leq 0 \qquad \text{if } k_g \in [0, \infty)$$

with equality when the corresponding variables are strictly positive. It follows from A.4 and A.7. a and b, and A.9.a and b that if a maximum exists, we must have $0 < b < sf$, $0 < \mu < \bar{\mu}$, $k_p > 0$ and $k_g > 0$. All inequalities in the above system thus can be replaced by equalities. Moreover, at the optimum (if it exists) all constraints are satisfied with equality. Assuming ρ is sufficiently small, this system of necessary conditions with equality is indentical to the syste (22)-(24) and (26)-(28). Thus, it suffices to show that the constained maximum problem has a solution for the proof of Proposition 2. μ is naturally bounded from above by $\bar{\mu}$, and b is also bounded from above, which can be seen from the first constraint, viz., $b \leq sf(k_p, \bar{\mu}k_g) - nk_p \leq sf(\bar{k}_p, \bar{\mu}\bar{k}_g) - n\bar{k}_p$, where $\bar{k}_p$ and $\bar{k}_g$ are defined as yielding a maximum of $sf - nk_p$. It follows from A.9 that $sf - nk_p$ has a maximum. Since all variables in u are nonempty and bounded, u is defined as a nonempty bounded set in R^4. As all functions appearing are continuous, this set is closed and thus compact. u has, therefore, a maximum, which means that this maximization problem has a solution. Since the objective function is not necessarily striclty concave in μ, k_p and k_g, multi-solutions may be possible.

Condition (28) implies that at a steady state, the rate of return on public capital net of population growth is not equal to the social rate of time preference, ρ. This means that ρ cannot be the criterion on the public investment decision making under the present public investment policy. Instead, the required rate of return on public investment is the rate of return on private investment. It may be seen in condition (28) that if the utility price of consumption, u_1, were equal to the common utility price of capitals, p, then ρ would also be the required rate of return on public investment. This, however, occurs only when the stationary solutions for the present policy happens to be the ones in the first-best optimal investment policy. In fact, if $p = u_1$, conditions (24) and (28) together with (23), collapse to $f_2 k_g = -u_2$ and $\rho + n = f_1$ respectively, which are virtually the stationary conditions in Comolli's model [9]. p, however, is not necessarily equal to u_1 undewr subpolicy 2 due to the nature of bond financing. We know from the argument in section II that the bond financing displaces only private investment. In other words, the borrowing as a policy instrument can be used to allocate the total capital between private and public sectors, but not to allocate total output between the consumption and investment. This inequality between the prices of consumption and capitals at a steady state indicates that the present environmental policy cannot be the first-best optimal one described in Comolli [9].

Proposition 3. At a steady state of subpolicy 2, the required rate of return on public investment is the rate of return on private investment, regardless of "the social rate of time preference."

This proposition tells us that public investment in environmental capacity should be evaluated at the rate of return on private investment, not at the social rate of time preference. In other words, as long as the rate of return on public capital is greater than that of private capital, the planner should continue to induce people to purchase government bonds to finance the public investment until those two become equal, without regard to social rate of time preference. Kim [16] showed under the same specification of the current model that the required rate of return on public investment equals the social rate of time preference at the steady state, regardless of the market rate of interest, if authority finances public investment in pollution control capital through income taxes only. This proposition, coupled with the result in Kim [16], assures Arrow and Kurz [1] assertion elaborated by Bradford [5] and Boadway [4] that the criterion on public investment decision vary with a given set of policy instruments.

We now examine the dynamic behavior of public debt.
From (12) and (15)

$$\dot{k}_g - \dot{d} = -n(k_g - d) \tag{29}$$

Let $G = k_g - d$ and, at initial time, $\bar{G}^o = k_g^o - \bar{d}^o$, where $\bar{d}^o > o$.
Then (29) can be expressed by $\dot{G} = -nG$, so that

$$\lim_{t\to\infty} G(t) = \lim_{t\to\infty} \bar{G}^o e^{-nt} = 0. \text{ That is,}$$

$$\lim_{t\to\infty} k_g(t) = \lim_{t\to\infty} d(t) \tag{30}$$

(30) indicates that the convergency of public debt depends solely on that of public capital. At a steady state, k_g converges to a positive constant, and so does d. Moreover, at a steady state, k_p and μ also converges to a finite constant, thus so does y. Therefore the ratio of public debt to

national income, d/y converges to a certain limit.
Thus we have the following:

Proposition 4. At a steady state under subpolicy 2, the planner's budget is stable. That is, d/y converges to a positive constant.

This result may convince us that debt finance is one of the tenable financing instruments for public investment policy. Moreover, this proposition may justify our model in which bonds are not retired. Feldstein [12]] asserts that the public debt need not be paid off through future taxes as long as the ratio of debt to national income do not diverge. Long ago, Domar [11] also suggested that debt should not be retired. He showed that the tax rates necessary to service a given public debt must diminish with rising national income. He then concluded that the solution to debt problem lies not in efforts toward retirement, but in trying to find ways of achieving a growing national income.

V. Analysis under a Separable Production Economy

We now specialize the general model to a separable production model in order to examine the dynamic nature of the optimal path as well as the characteristics of a steady state more rigorously.

Suppose that the production function $f(k_p, \mu k_g)$ takes the following form:

$$y = f(k_p, \mu k_g) = \gamma \log k_p + \delta \log \mu k_g] \tag{31}$$

where γ and δ are positive constants. This production function is arbitrary, but it satisfies all the assumptions made for the general production function, i.e. A.8 and A.9, so that results may be considered suggestive.

Under this specification of production structure, the planning problem and corresponding optimality conditions can be stated equally to those for the general model. This production model, however, allows us to show propositon 2 without the assumption that ρ is sufficiently small, and to analyze stability of the steady state. We can state the following.

Proposition 5. Under Subpolicy 2 in a separable production economy, there exists a unique steady state, which is a saddle point, if $n \geq sf_1$.[11)]

11) We need to assume $n \geq sf_1$ to prove proposition 5. We, therefore, restrict our analysis to what Phelps and Shell [22] call 'classical range.' In order to facilitate the analysis of the stability, we introduce a new state variable, $k = k_p + k_g$, with k_g replacing b as a control variable. The current-value Hamiltonian under this transformation, $u\{(1-s)f(k-k_g, \mu k_g), \mu\} + p\{sf(k-k_g, \mu k_g) - nk\}$ has the same maximum principles (22) and (24) and the transition equation (25) as the original system. The proof of this proposition will be provided upon request.

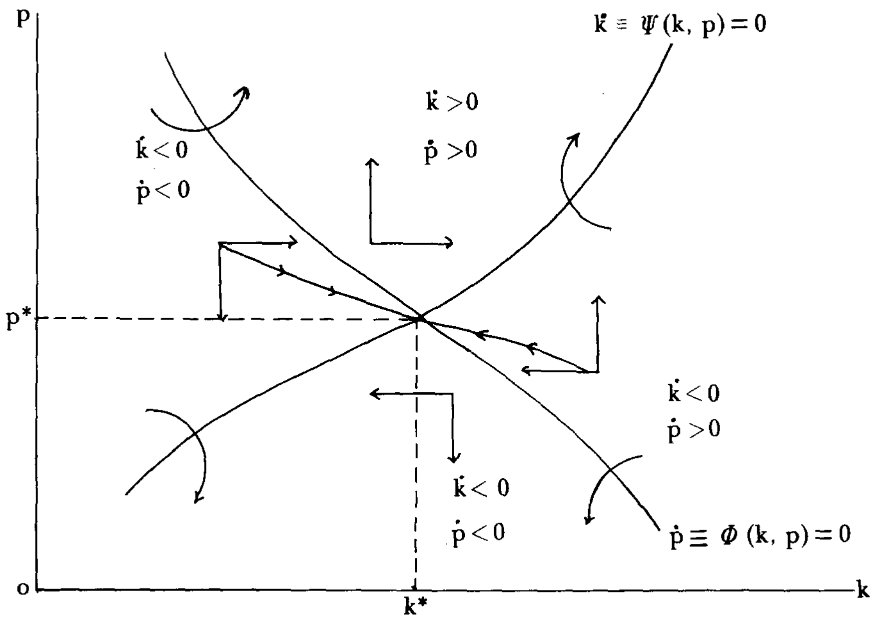

Figure 1. Illustration of Subpolicy 2

Figure 1 reflects the proposition 5 and shows the optimal path. It can be seen, for example, from the figure that when $k < k^*$, k is rising and p is falling along the optimal path; thus, net saving is positive. Moreover, the stock of public capital is increasing, while the economy's utilization rate of public capital is falling.[12] In view of (30), the size of public debt is also rising since k_g is rising.

So far we have assumed that exogeneous variables such as s, n, ρ are held constant. Variations in these variables, however, will move the steady state path from one position to another, resulting in changes in all endogeneous variables at the steady state, viz. ($b^*, \mu^*, k_p^*, k_g^*, P_p^*, P_g^*, y^*, c^*, q^*$).

Table 1 shows the summary of the results of comparative statics analysis in the neighborhood of the unique steady state.[13]

We now want to interpret only the signs of the second column of the table, since it may be obvious to interpret other signs. If the utility price

12) In the proof of proposition 5, it is seen that $\partial k_g / \partial k > 0$, $\partial \mu / \partial k < 0$.

13) Calculation process for these results will be provided upon request.

of investment is at least as great as that of consumption, an increase in the marginal propensity to save (MPS) will raise the quilibrium borrowing, private capital stock and public capital stock, while its increase will lower the equilbrium utilization rate of public capital and the equilibrium utility price of investment. It may be deduced from these results as well as from (30) that an increase in MPS causes a rise in public debt. Since the equilibrium utilization rate of public capital is falling the environmental quality will be improved, in view of A.1. We know from the production function that the qualitative change in equilbrium output

[Table 1] Summary of Comparative Statics Under Subpolicy 2

Derivatives of	with respect to		
	s	n	ρ
b	$+^{1}$	-	-
μ	$-^{1}$	+	-
k_p	$+^{1}$	-	-
k_g	$+^{1}$	-	-
d	$+^{1}$	-	-
p	$-^{1}$	+	-
y	$+^{1,2}$	-	-
c	?	-	-
q	$+^{1}$	-	+

Note: 1. $u_l \leq p$[14]
2. $s \leq (\gamma+\delta)/(\gamma+2\delta)$

depends on the degree of variations of k_p, k_g and μ, which do not move in the same direction. It, however, could be easily seen that the incremental effect of an increase in MPS on equilibrium output requires $s \leq (\gamma+\delta)/(\gamma+2\delta)$,[15] where the upper bound of MPS is always less than one. From the consumption function (10), we may see that $\partial c/\partial s = -y + (1-s)\partial y/\partial s$. This implies that an increase in MPS has two effects on consumption—that is, a direct negative consumption effect and an

14) This condition can be replaced by $f_l - n > 0$. From (28), $\{(1-s)u_l + sp\}f_l - np = \rho p$. If $u_l \leq p$, $(f_l - n)p \geq \rho p > 0$ so that $f_l - n > 0$. Thus the assumption of $u_l \leq p$ implies that the rate of interest (f_l) on debt is greater than the natural rate of growth, n. In discussion Barro [2, 3] Burdidge [7] shows that in his model, f_l must exceed n.

15) This result has a useful policy implication, empirically. Since one can estimate γ and δ, one may calculate the upper bound of MPS for increasing output. So if the goal of a society is economic growth, the planner should induce individual's MPS not to exceed this level under subpolicy 2.

indirect effect through variations in output. If the output effect is positive, the total consumption effect is uncertain. Thus, the change in MPS on the social welfare is also uncertain.

VI. Concluding Remarks

In this paper, we have seen that the hybrid environmental policy considered cannot achieve the first-best optimal policy even though the number of targets equals that of the policy instruments. This is because the income tax is used only for financing interest payments and any policy instruments used in this program cannot play a role in the allocation of output between consumption and investment. Thus, as we have seen, the utility prices of consumption and investment are not necessarily equal, which is an indicator of the failure of achieving a publicly optimal investment policy.

Nevertheless, we see that the environmental authority in this prototype economy can achieve a socially optimal level of environmental quality as in a perfectly palnned economy. This is true because we assume that the authority exerts direct control on polluting activities which permits it to set up an optimal pollution standard and to regulate directly firms' utilization rate of public capital. The authority can accomplish this by allowing the production sector to utilize public capital to the point where the marginal net social benefit (MNSB) equates the marginal external cost (MEC). It is seen, however, that this direct regulation must be accompanied by the appropriate public investment in environmental capacity. In proposition 1, we see that the subpolicy of complete specialization in private investment must not be carried on forever. This result confirms the indispensability of public investment in environmental capacity in an environmental policy, reminding us, "Pollution makes the ecosystem less capable of withstanding further pollution."[16)]

There is another comment that is worth making. We have seen in section IV, that at a steady state, the required rate of return on public investment should be qual to that on private investment, irrespective of the social rate of time preference. Comparing this result with the ones in Kim [16, 17] and Comolli [9], we may conclude that the criterion for the evaluation of public investment must vary with every change in economic environment, viz., of policy instruments available to the policy maker.

There are essentially two areas to be covered in future research. One

16) Pearce [21], p. 62.

may consider the situation when both income taxes and bond issues are available as financial instruments for environmental authority to finance public investment in environmental capacity. Taking into account the direct control over pollution activities, the authority has three instruments for three targets. Consequently, the nature of the optimal plan should be "the first-best" as in the Comolli [9].

The other area for future research involves a hybrid environmental policy with an effluent tax scheme. A tax on effluents not only raises revenue for public investment, but also serves to regulate indirectly to firms' utilization rate of public capital so that direct regulation may be disregarded in such a model.

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